Notes for ECE-606: Spring 2013

L2: Miller Indices

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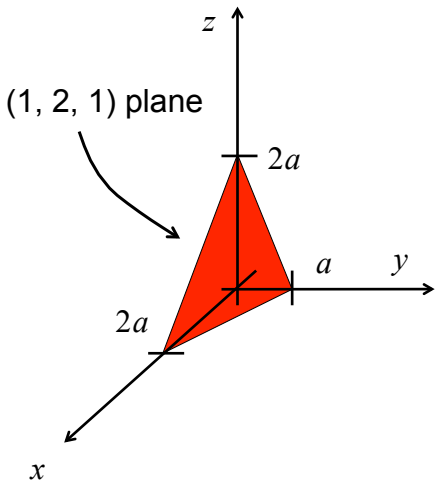
prescription for describing planes



x, y, and z-axis intercepts:

2a, 1a, 2a
2, 1, 2

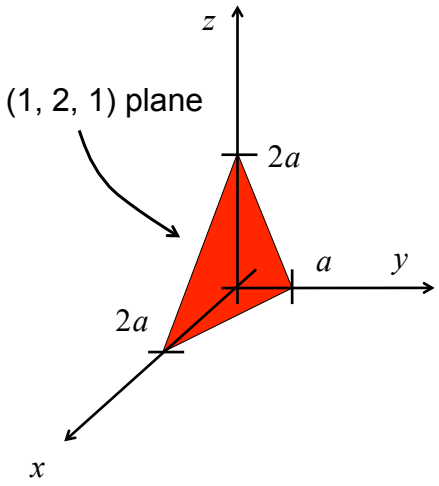
invert:

½. 1, ½

Rationalize:

1, 2, 1

where it comes from



equation of a plane:

$$\frac{x}{x_{int}}+\frac{y}{y_{int}}+\frac{z}{z_{int}}=1$$

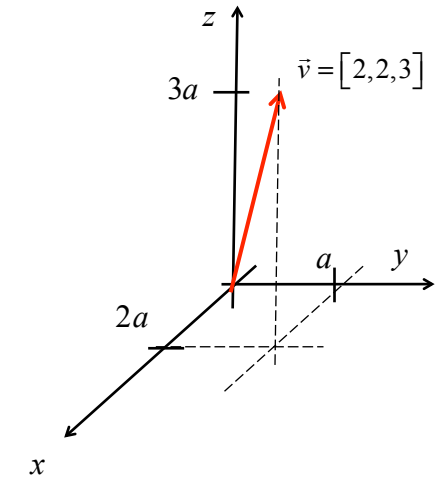
describe with numbers:

$$\frac{1}{x_{int}},\frac{1}{y_{int}},\frac{1}{z_{int}}$$

equivalent to:

$$\frac{1}{x_{int}/a},\frac{1}{y_{int}/a},\frac{1}{z_{int}/a}$$

prescription for describing directions



equation of a vector:

$$\vec{v}=2a\hat{x}+2a\hat{y}+3a\hat{z}$$

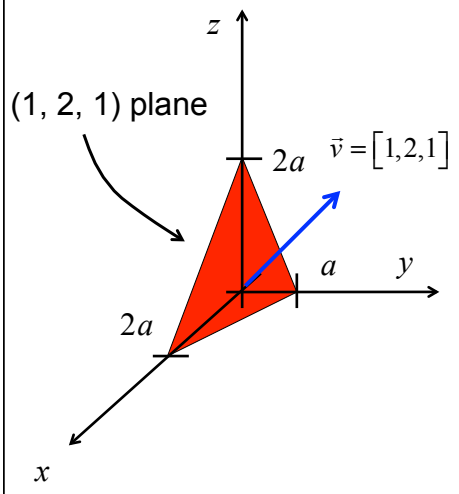
describe with components:

$$2a,2a,3a$$

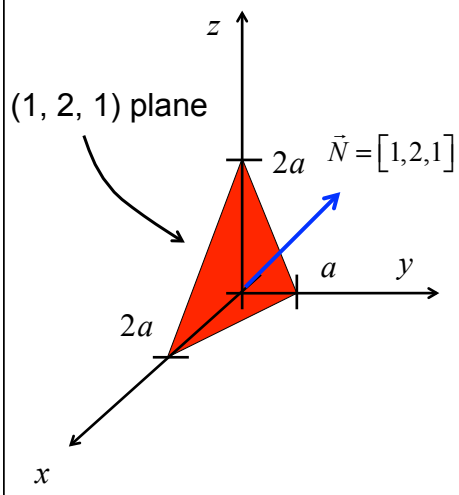
equivalent to:

$$2,2,3$$

direction normal to a plane



where it comes from



equation of a plane:

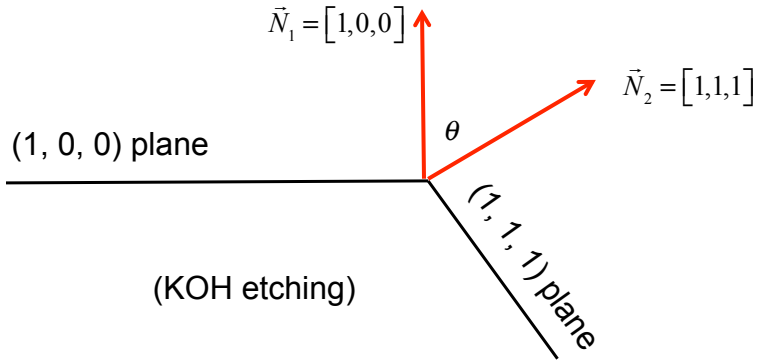
$$f(x,y,z) = \frac{x}{x_{int}} + \frac{y}{y_{int}} + \frac{z}{z_{int}} = 1$$

normal to a plane:

$$\vec{N} = \nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial x}\hat{z}$$

$$\vec{N} = \frac{1}{x_{int}}\hat{x} + \frac{1}{y_{int}}\hat{y} + \frac{1}{z_{int}}\hat{z}$$

angle between planes



$$\vec{N}_1 \bullet \vec{N}_2 = N_1 N_2 \cos\theta$$

angle between planes

$$\cos\theta = \frac{\vec{N}_1 \bullet \vec{N}_2}{N_1 N_2}$$

$$\vec{N}_1 = [h_1, k_1, l_1]$$

$$\vec{N}_2 = [h_2, k_2, l_2]$$

$$\cos\theta = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{h_1^2 + k_1^2 + l_1^2}\sqrt{h_2^2 + k_2^2 + l_2^2}}$$

$$\vec{N}_1 = [1,0,0]$$

$$\vec{N}_2 = [1,1,1]$$

$$\cos\theta = \frac{1+0+0}{\sqrt{1^2+0^2+0^2}\sqrt{1_2^2+1_2^2+1_2^2}}$$

$$\cos\theta = \frac{1}{\sqrt{3}}$$

$$\theta = 54.7^\circ$$

distance between adjacent planes

(h,k,l)

$[h,k,l]$

$$\vec{N} = ha\hat{x} + ka\hat{y} + la\hat{z}$$

$$d = \frac{1}{|\vec{N}|}$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

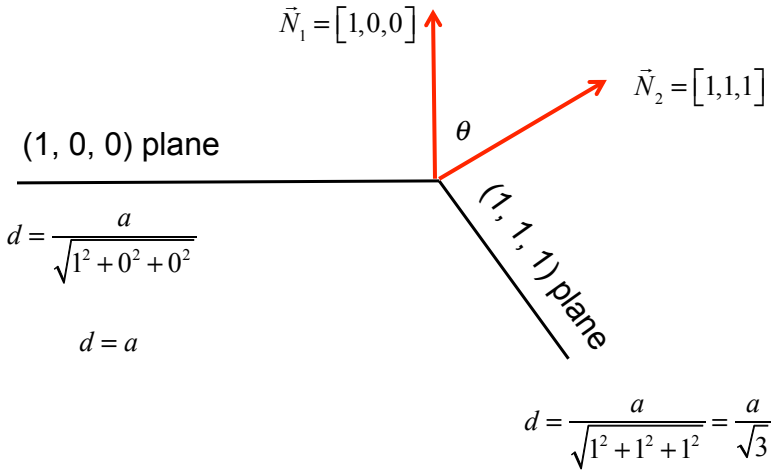
distance between adjacent planes

References:

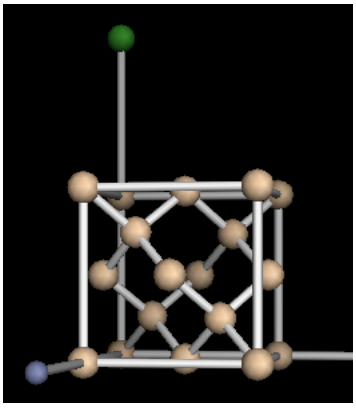
Shyh Wang, *Fundamentals of Semiconductor Theory and Device Physics*, Prentice-Hall, 1989. pp. 44-46.

J.P. McKelvey, *Solid State and Semiconductor Physics*, Harper and Rowe, 1966. pp. 12-15.

spacing of (100) and (111)



The diamond lattice

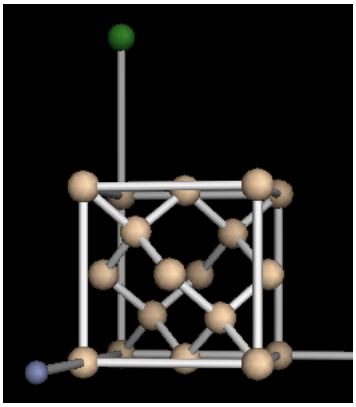


FCC Bravais lattice

Basis of 2 atoms per site
(000) and (¼ ¼ ¼)

https://nanohub.org/tools/crystal_viewer

The diamond lattice



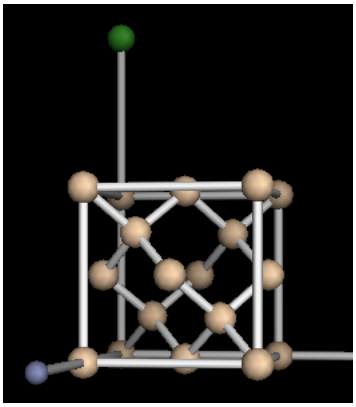
Atoms per unit cell

8 times 1/8 + 6 times ½ + 4

8 atoms per unit cell

https://nanohub.org/tools/crystal_viewer

Silicon: density



Lattice constant: a = 5.4307 Ang

Density = total mass/vol. of unit cell.

Atomic mass of Si: 28.0855 amu

1 amu = 1.6605 x 10^{-27} kg

$$\rho = \frac{8 \times 28.0855 \times 1.6605 \times 10^{-27}}{\left(5.4307 \times 10^{-10}\right)^3} \quad \text{kg/m}^3$$

$$\rho = 2.3296 \text{ g/cm}^3$$

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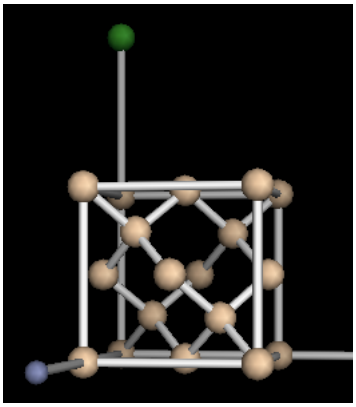
Silicon: NN spacing



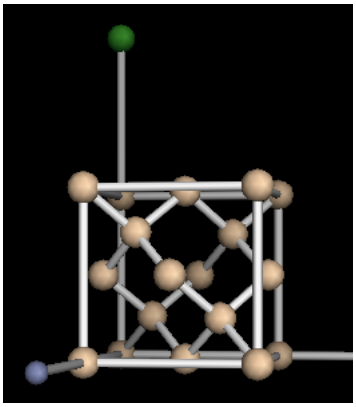
Lattice constant: a = 5.4307 Ang

Body diagonal = sqrt(3) a.

NN spacing = sqrt(3)a/4

https://nanohub.org/tools/crystal_viewer

Silicon: packing density



Lattice constant: a = 5.4307 Ang

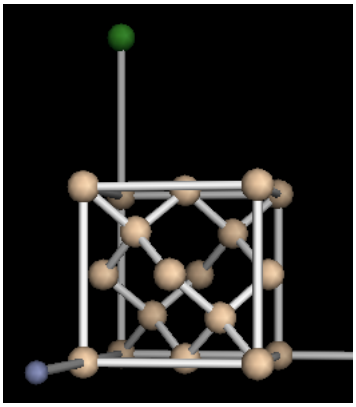
NN spacing = sqrt(3)a/4

Radius of atom = ½ NN spacien
=sqrt(3)a/8

Vol of atom = (4/3) pi R^3

$$PF = \frac{8 \times \frac{4}{3}\pi R^3}{(a)^3} = \frac{\sqrt{3}\pi}{16} = 34\%$$

Silicon: atoms / cm2 on (100)



Lattice constant: 5.4307 Ang

Atoms on face = 4 times ¼ +1 = 2

Ns = 2/ a^2

Ns = 6.81x 10^{14} /cm^2

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