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EE-606: Solid State Devices
Lecture 8: Density of States

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Outline

1) **Calculation of density of states**

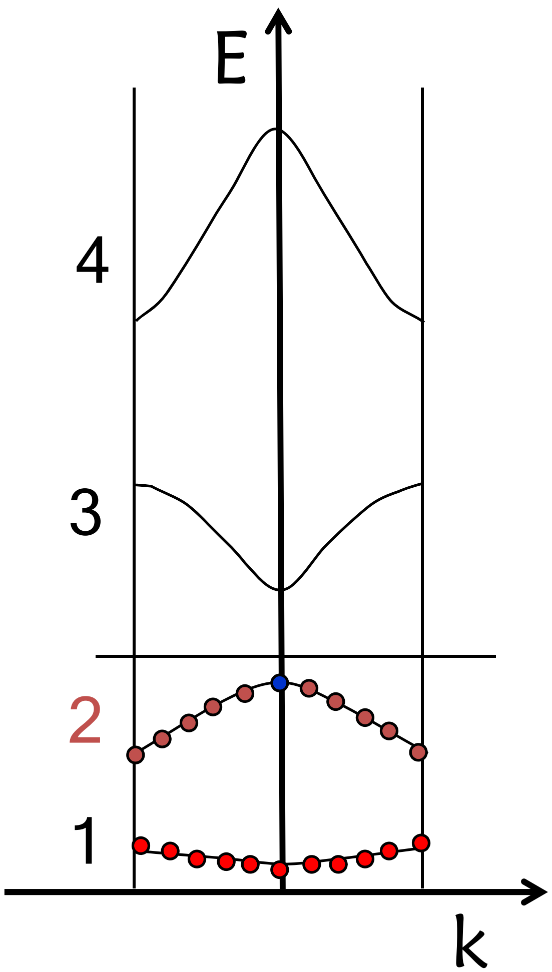
2) Density of states for specific materials

3) Characterization of Effective Mass

4) Conclusions

Reference: Vol. 6, Ch. 3 (pages 88-96)

Density of States



A single band has total of N-states

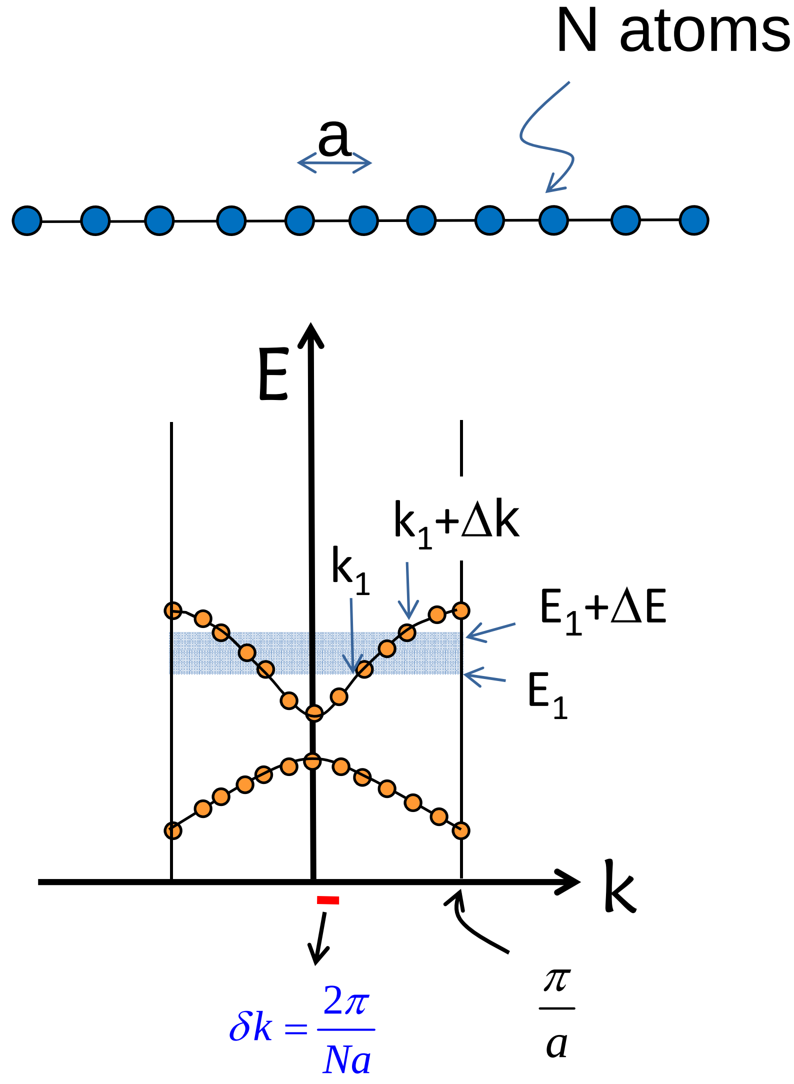
Only a fraction of states are occupied

How many states are occupied upto E?

Or equivalently…

How many states per unit energy ? (DOS)

Density of States in 1-D Semiconductors



$$\text{States between } E_1+\Delta E \ \& \ E_1 = 2 \times \frac{\Delta k}{\delta k}$$

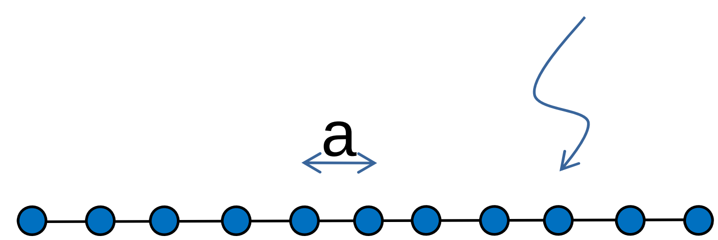
$$= 2 \times \frac{\Delta k}{2\pi/Na}$$

$$\text{States/unit energy @ } E_1 = \frac{Na}{\pi}\frac{\Delta k}{\Delta E}$$

1D-DOS

L

$$\text{States/unit energy @ } E = \frac{Na}{\pi}\frac{\Delta k}{\Delta E}$$



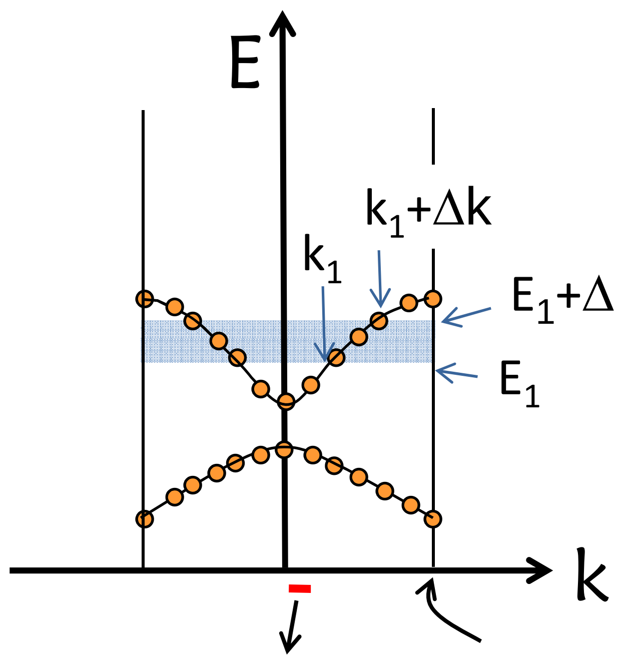
$$E - E_0 = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k = \sqrt{\frac{2m^*(E-E_0)}{\hbar^2}}$$

$$\frac{dk}{dE} = \sqrt{\frac{m^*}{2\hbar^2(E-E_0)}}$$

$$\text{States/unit energy @ } E = \frac{L}{\pi}\sqrt{\frac{m^*}{2\hbar^2(E-E_0)}}$$

States/unit energy/unit length @ E

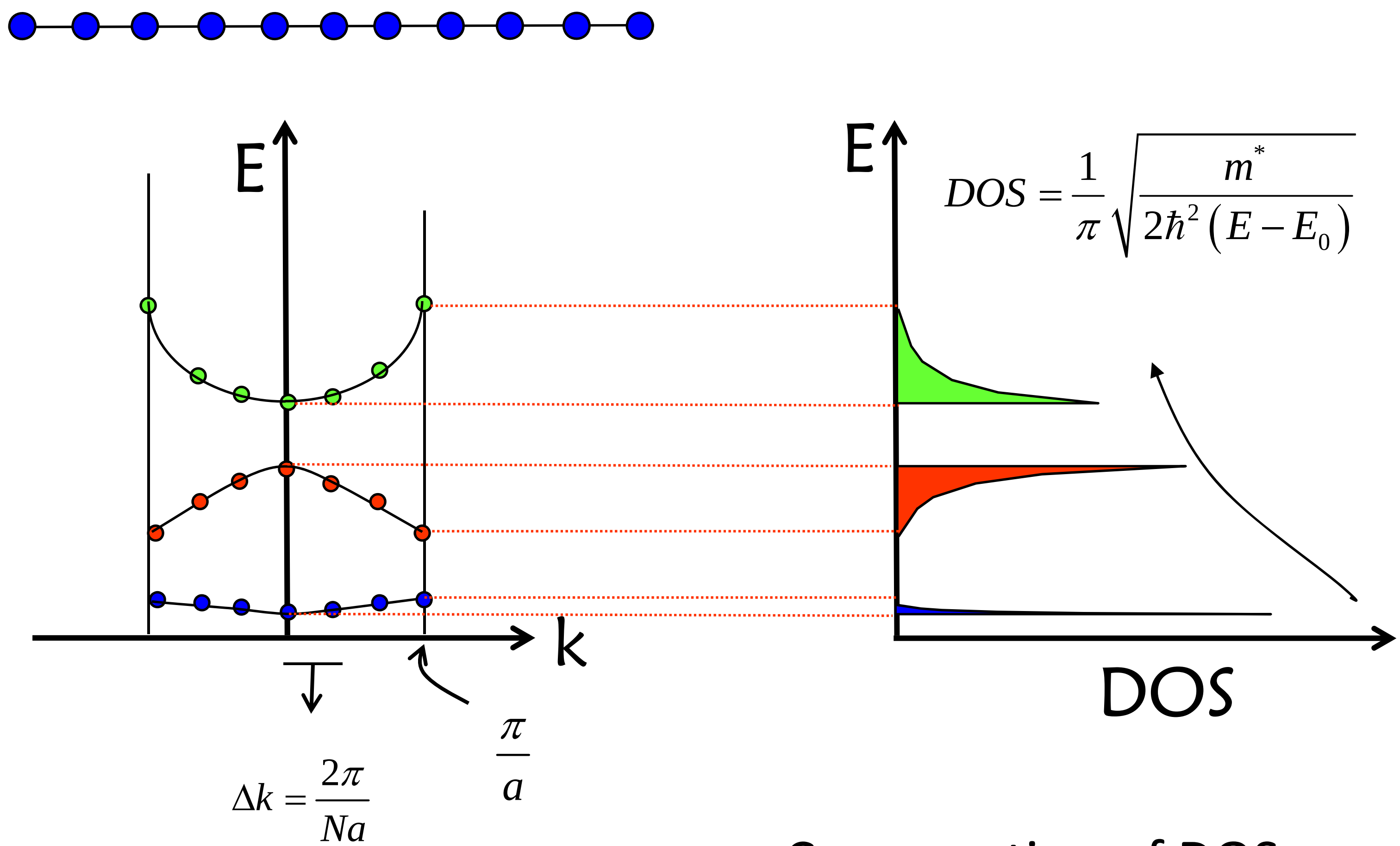
$$\equiv DOS = \frac{1}{\pi}\sqrt{\frac{m^*}{2\hbar^2(E-E_0)}}$$



$$\delta k = \frac{2\pi}{Na}$$

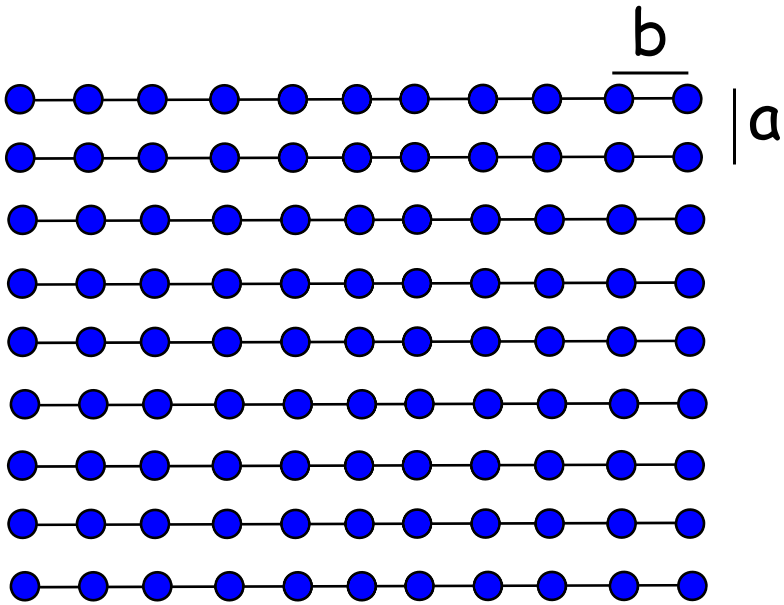
$$\frac{\pi}{a}$$

1D-DOS



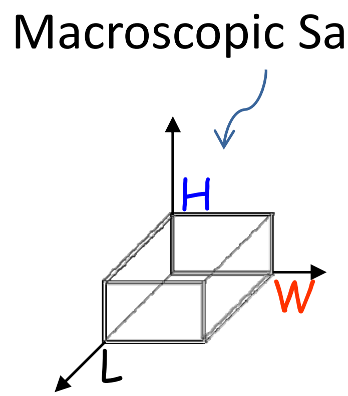
Conservation of DOS

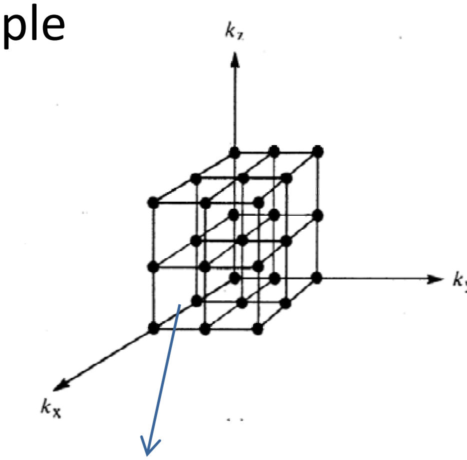
Density of States in 2D Semiconductors



Show that 2D DOS is a constant independent of energy!

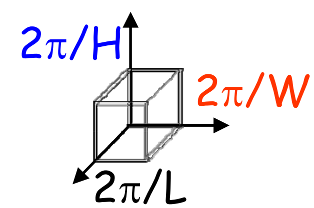
Density of States in 3D Semiconductors





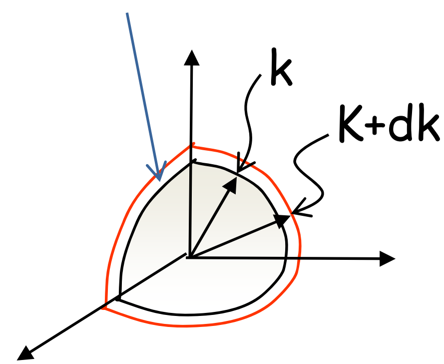
States between $E_1 + \Delta E$ & E_1

$$= \frac{\frac{4}{3}\pi(k+dk)^3 - \frac{4}{3}\pi k^3}{\frac{2\pi}{L}\frac{2\pi}{W}\frac{2\pi}{H}} = \frac{V}{2\pi^2}k^2\Delta k$$



States/unit energy @ E $= \dfrac{V}{2\pi^2}k^2\dfrac{\Delta k}{dE}$

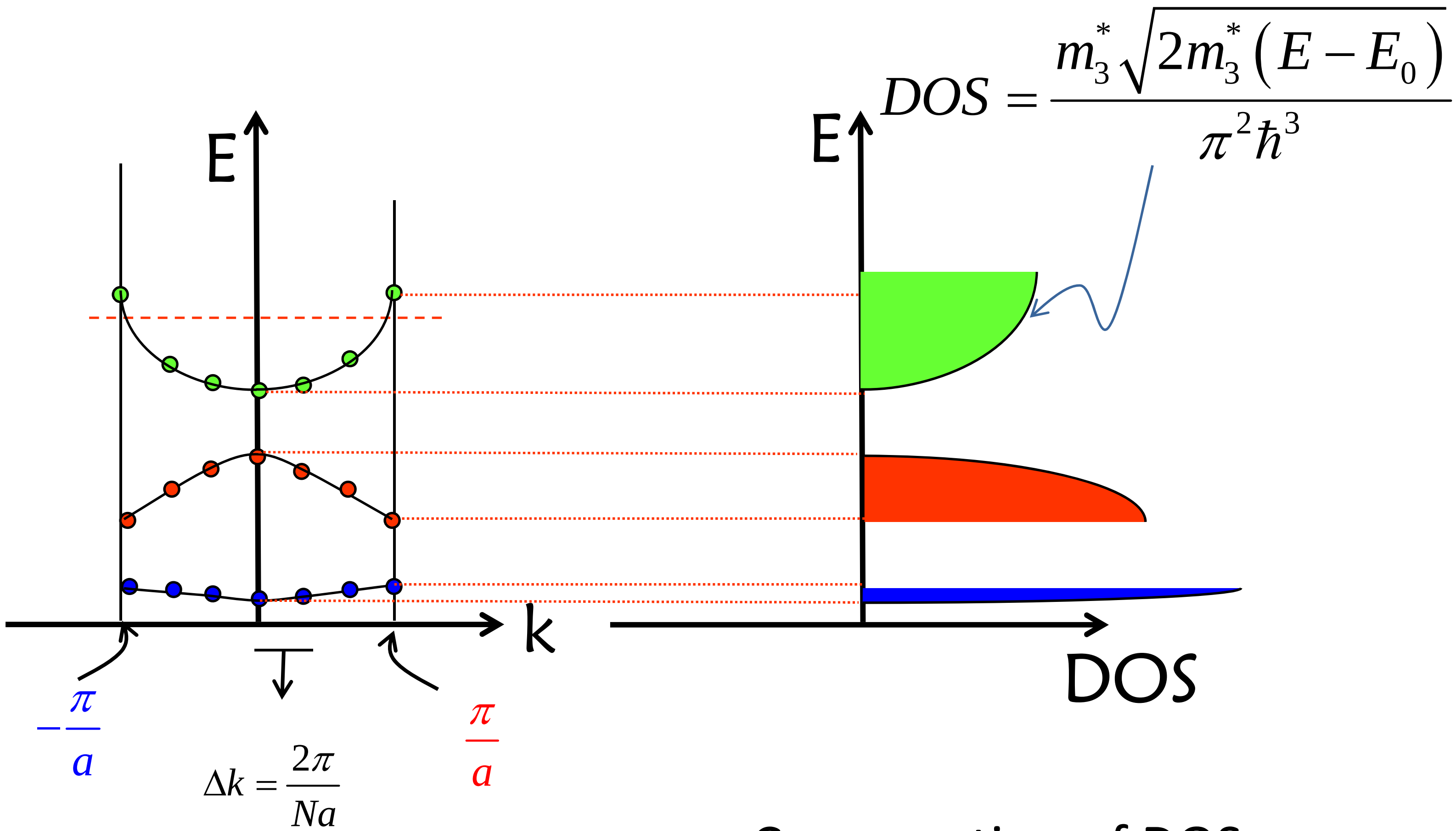
$$E - E_0 = \frac{\hbar^2 k^2}{2m^*} \Rightarrow k = \sqrt{\frac{2m^*(E-E_0)}{\hbar^2}} \Rightarrow \frac{dk}{dE} = \sqrt{\frac{m^*}{2\hbar^2(E-E_0)}}$$



States/unit energy/unit volume @ E_1

$$DOS = \frac{m^*}{2\pi^2\hbar^3}\sqrt{2m^*(E-E_0)}$$

3D-DOS



Conservation of DOS

Outline

1) Calculation of density of states

2) Density of states for specific materials

3) Characterization of Effective Mass

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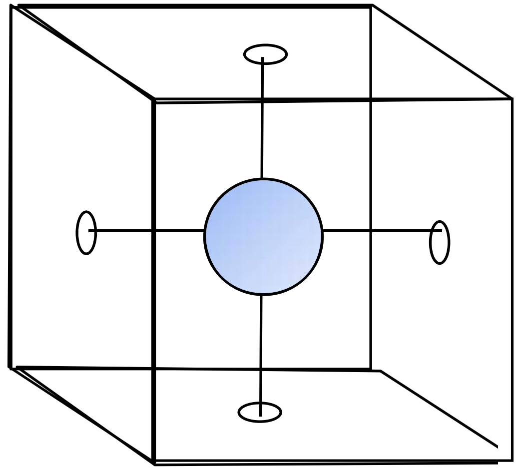
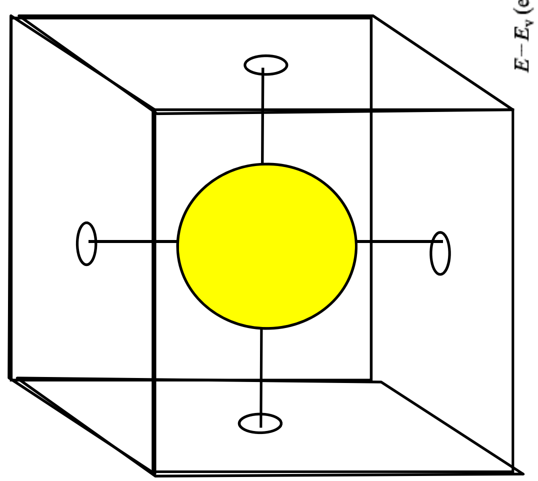
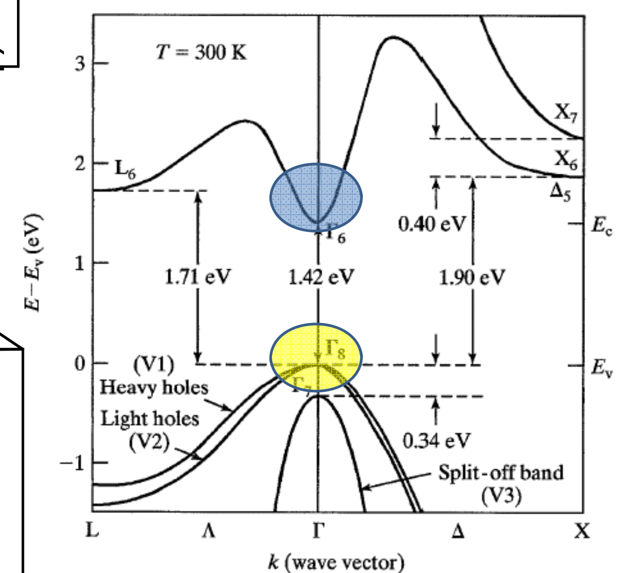
Density of States of GaAs: Conduction/Valence Bands

$$g_c(E) = \frac{m_n^* \sqrt{2m_n^*(E - E_c)}}{2\pi^2\hbar^3}$$

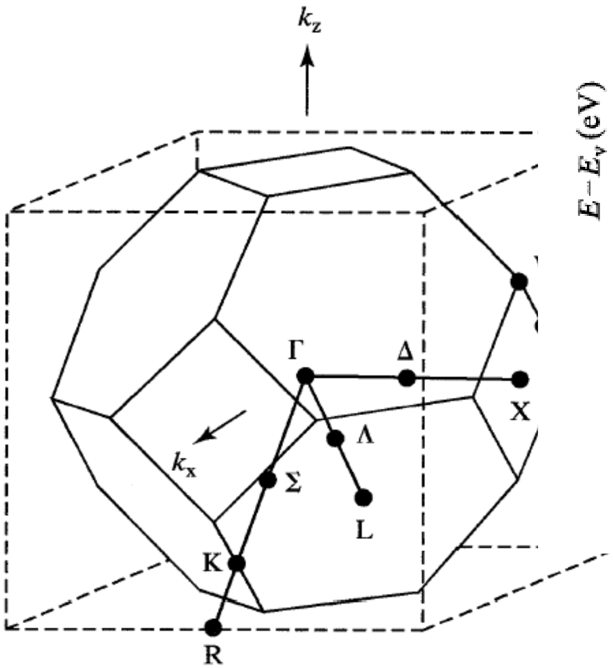


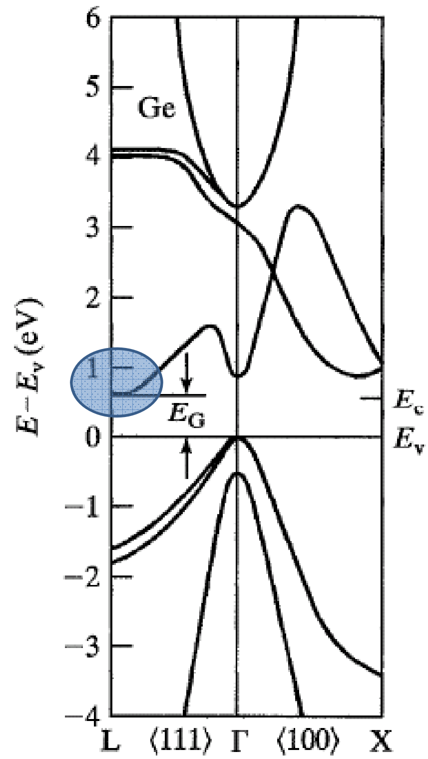
$$g_\upsilon(E) = \begin{cases} \dfrac{m_{hh}^* \sqrt{2m_{hh}^*(E - E_\upsilon)}}{2\pi^2\hbar^3} \\ \\ \dfrac{m_{lh}^* \sqrt{2m_{lh}^*(E - E_\upsilon)}}{2\pi^2\hbar^3} \end{cases}$$

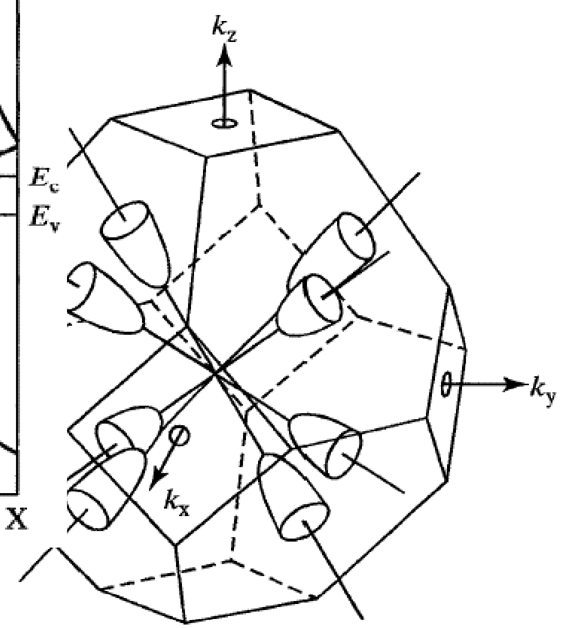




Four valleys inside BZ for Germanium



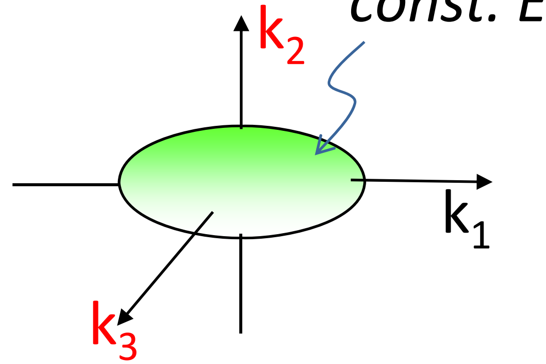




Ellipsoidal Bands and DOS Effective Mass

$$E - E_C = \frac{\hbar^2 k_1^{\,2}}{2m_l^*} + \frac{\hbar^2 k_2^{\,2}}{2m_t^*} + \frac{\hbar^2 k_3^{\,2}}{2m_t^*}$$

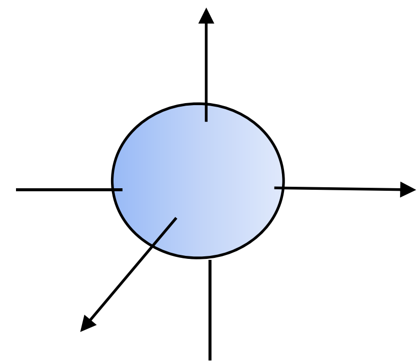
E=const. ellipsoid



$$1 = \frac{k_1^{\,2}}{\left[\frac{2m_l^*\left(E-E_C\right)}{\hbar^2}\right]} + \frac{k_2^{\,2}}{\left[\frac{2m_t^*\left(E-E_C\right)}{\hbar^2}\right]} + \frac{k_3^{\,2}}{\left[\frac{2m_t^*\left(E-E_C\right)}{\hbar^2}\right]}$$

α^2 $\qquad$ β^2

Transform into ...



$$\mathcal{V}_k = N_{el}\left(\frac{4}{3}\pi\alpha\beta^2\right) \equiv \frac{4}{3}\pi k_{eff}^3$$

$$N_{el}\,\frac{4}{3}\pi\sqrt{\frac{2m_l^*\left(E-E_c\right)}{\hbar^2}}\sqrt{\frac{2m_t^*\left(E-E_c\right)}{\hbar^2}}\sqrt{\frac{2m_t^*\left(E-E_c\right)}{\hbar^2}} \equiv \frac{4}{3}\pi\left[\sqrt{\frac{2m_{eff}^*\left(E-E_c\right)}{\hbar^2}}\right]^3$$

$$m_{eff}^* = N_{el}^{2/3}\left(m_l^* m_t^{*2}\right)^{1/3}$$

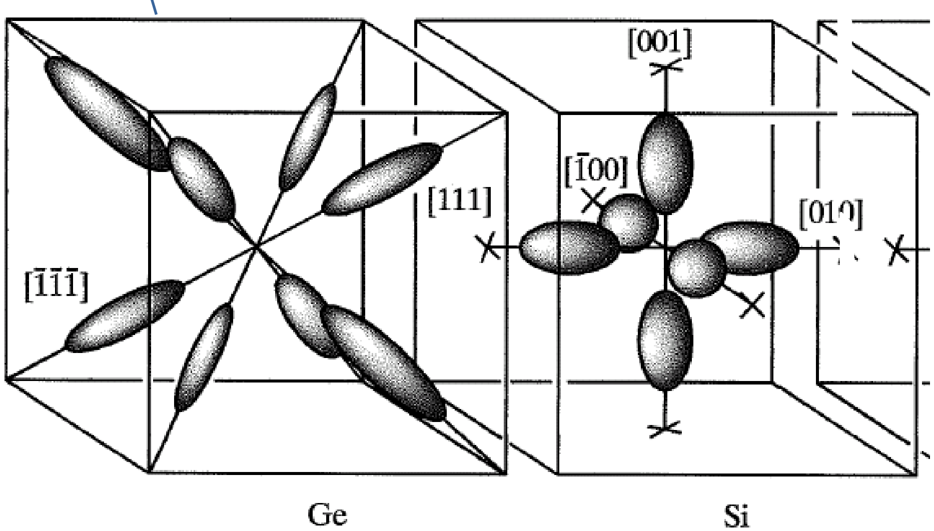
DOS Effective Mass for Conduction Band

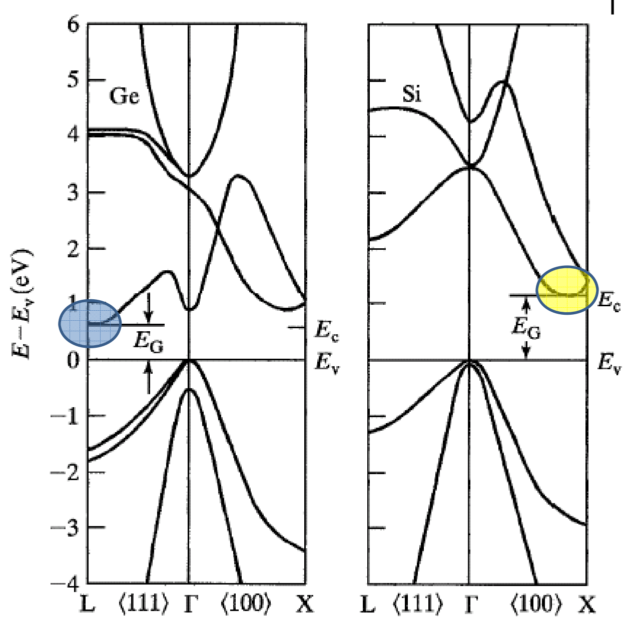
$$m_{eff}^{*} = 4^{2/3}\left(m_l^{*} m_t^{*2}\right)^{1/3}$$

$$g_c(E) = \frac{m_{eff}^{*}\sqrt{2m_{eff}^{*}\left(E - E_c\right)}}{2\pi^2\hbar^3}$$

$$m_{eff}^{*} = 6^{2/3}\left(m_l^{*} m_t^{*2}\right)^{1/3}$$

$$g_c(E) = \frac{m_{eff}^{*}\sqrt{2m_{eff}^{*}\left(E - E_c\right)}}{2\pi^2\hbar^3}$$





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3) **Characterization of Effective Mass**

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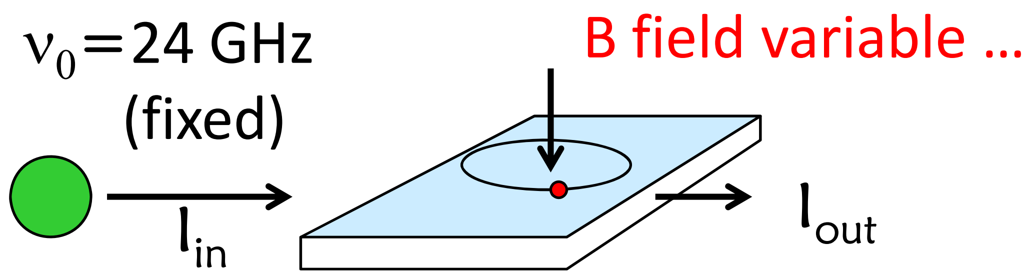
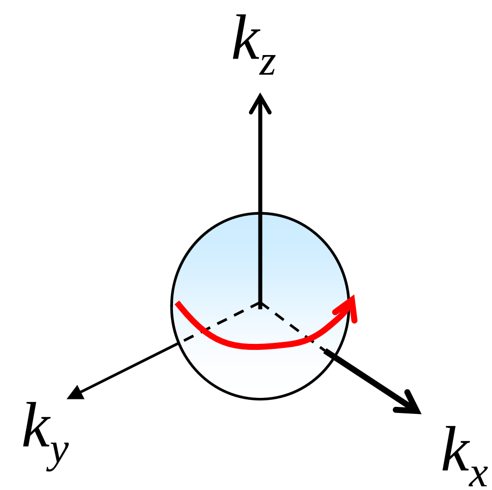
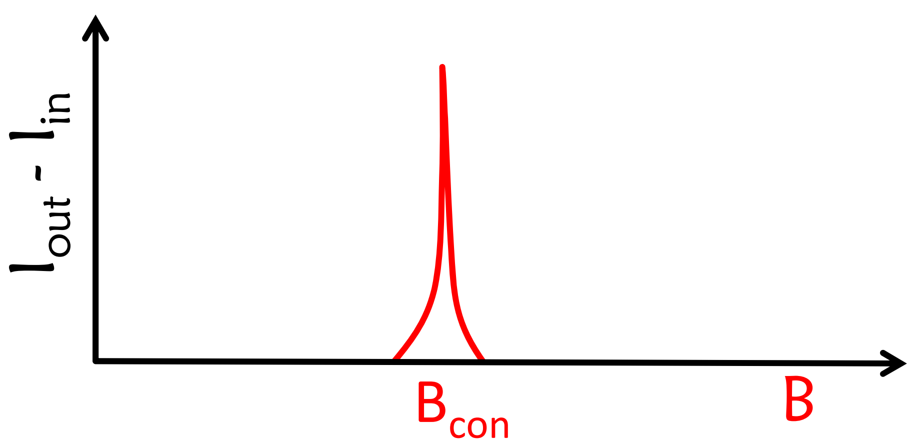
Measurement of Effective Mass

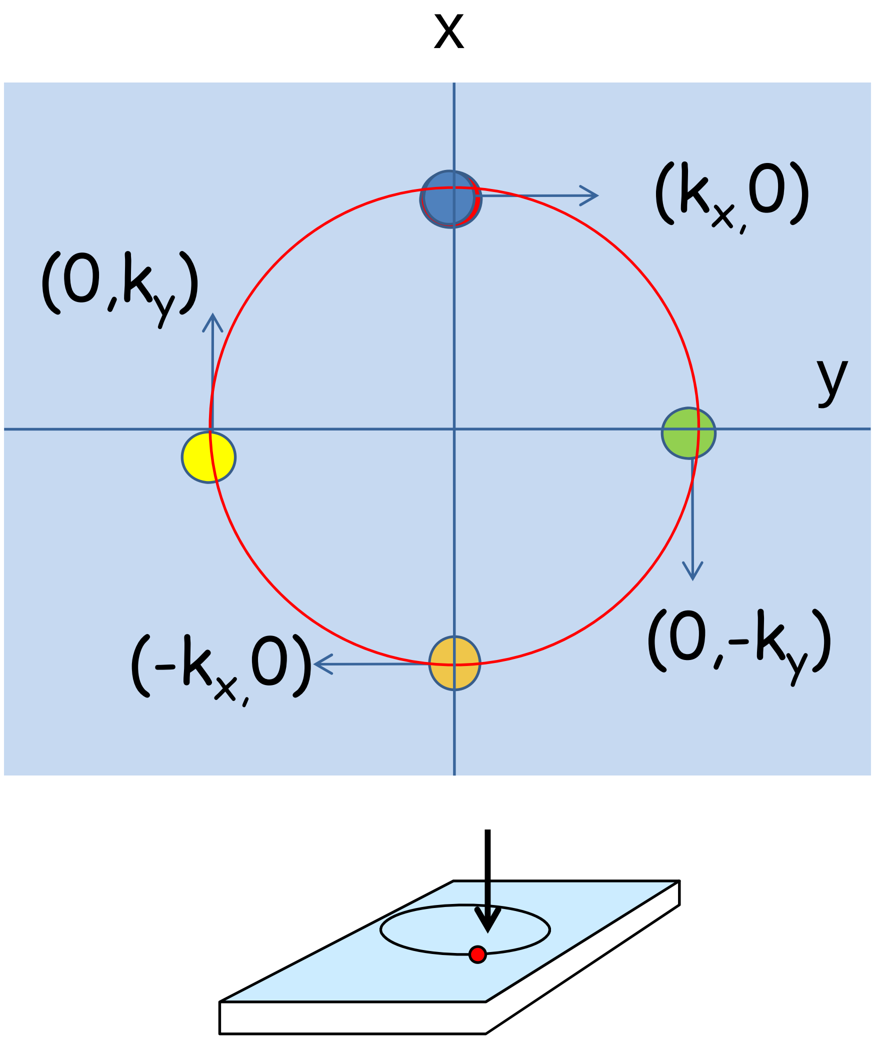




$$m^* = \frac{qB_{con}}{2\pi\nu_0}$$

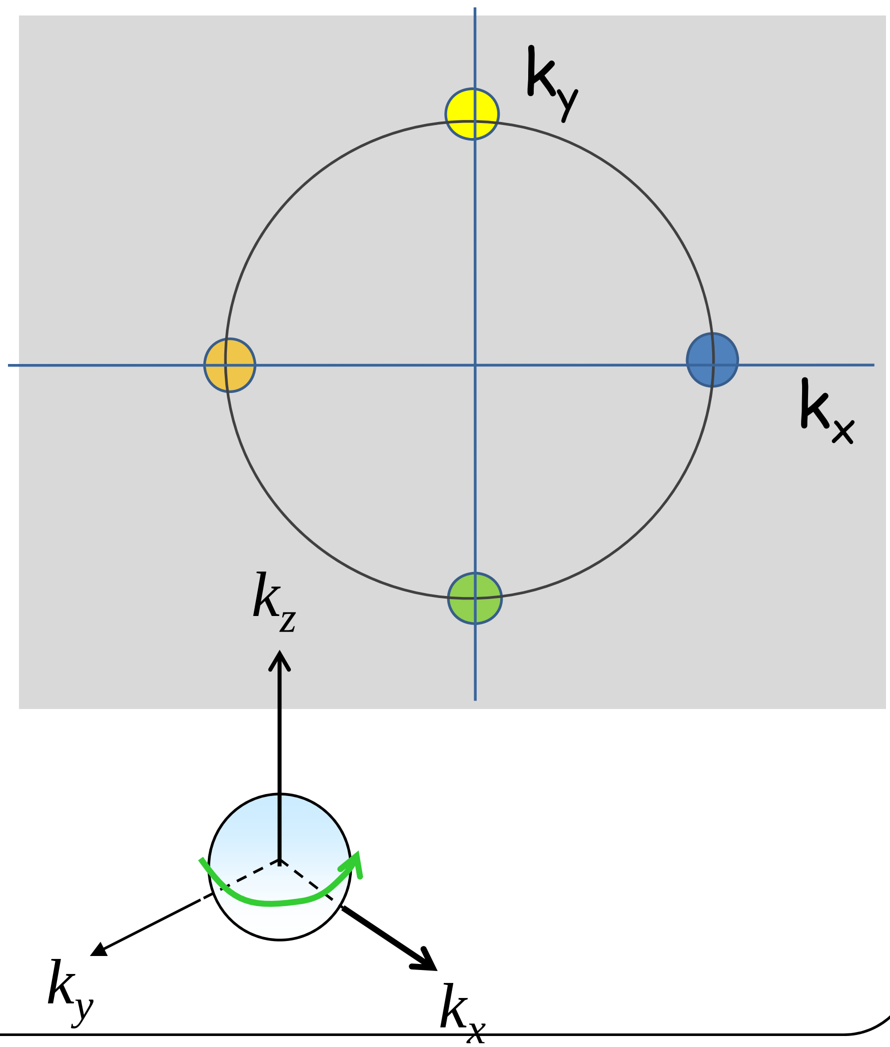


Motion in Real Space and Phase Space



Energy=constant.

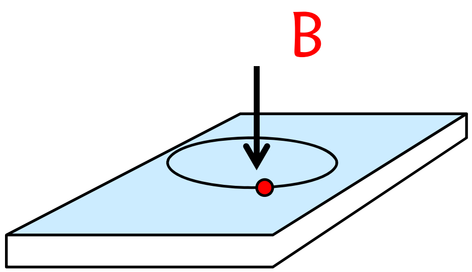
Liquid He temperature …



Derive the Cyclotron Formula $m^* = \frac{qB_0}{2\pi v_0}$

For an particle in (x-y) plane with B-field in z-direction, the Lorentz force is …

$$\frac{m^* \upsilon^2}{r_0} = q\upsilon \times B_z = q\upsilon B_z$$

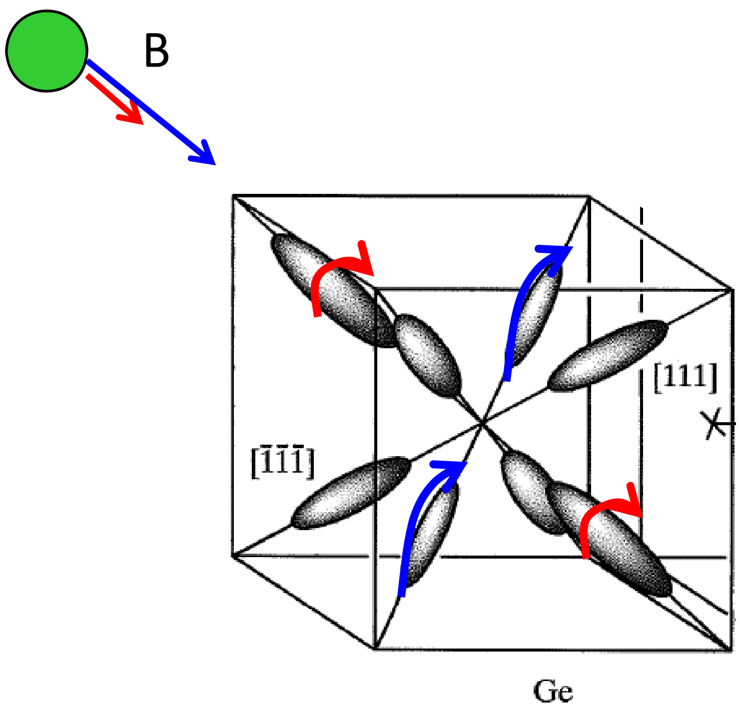


$$\upsilon = \frac{qB_0 r_0}{m^*}$$

$$\tau = \frac{2\pi r_0}{\upsilon} = \frac{2\pi m^*}{qB_0}$$

$$v_0 \equiv \frac{1}{\tau} = \frac{qB_0}{2\pi m^*}$$

Effective mass in Ge



$[\bar{1}\bar{1}\bar{1}]$ $[\bar{1}\bar{1}1]$ $[\bar{1}1\bar{1}]$ $[1\bar{1}\bar{1}]$

$[1\bar{1}\bar{1}]$ $[1\bar{1}\bar{1}]$ $[\bar{1}1\bar{1}]$ $[111]$

4 angles between B field and the ellipsoids …
Recall the HW1

Derivation for the Cyclotron Formula

Show that
$$\frac{1}{m_c^2} = \frac{cos^2\theta}{m_t^2} + \frac{sin^2\theta}{m_l m_t}$$

Given three m_c and three θ, we will Find m_t, and m_l

The Lorentz force on electrons in a B-field

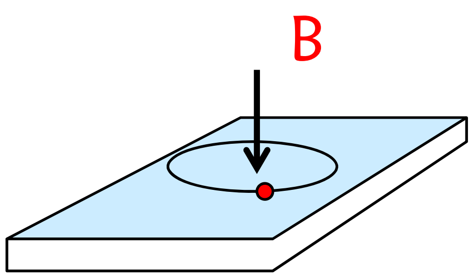
$$F = q\upsilon \times B = [M]\frac{d\upsilon}{dt}$$

In other words,

$$F_x = q(\upsilon_y B_z - \upsilon_z B_y) = m_t^* \frac{d\upsilon_x}{dt}$$

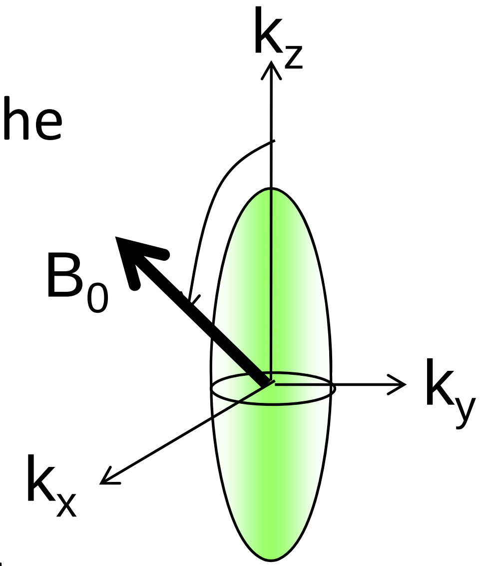
$$F_y = q(\upsilon_z B_x - \upsilon_x B_z) = m_t^* \frac{d\upsilon_y}{dt}$$

$$F_z = q(\upsilon_x B_y - \upsilon_y B_x) = m_l^* \frac{d\upsilon_z}{dt}$$



Continued …

Let (B) make an angle (θ) with longitudinal axis of the ellipsoid (ellipsoids oriented along k_z)



$$B_x = B_0 \cos(\theta),\ B_y = 0,\ B_z = B_0 \sin(\theta),$$

Differentiate (v_y) and use other equations to find …

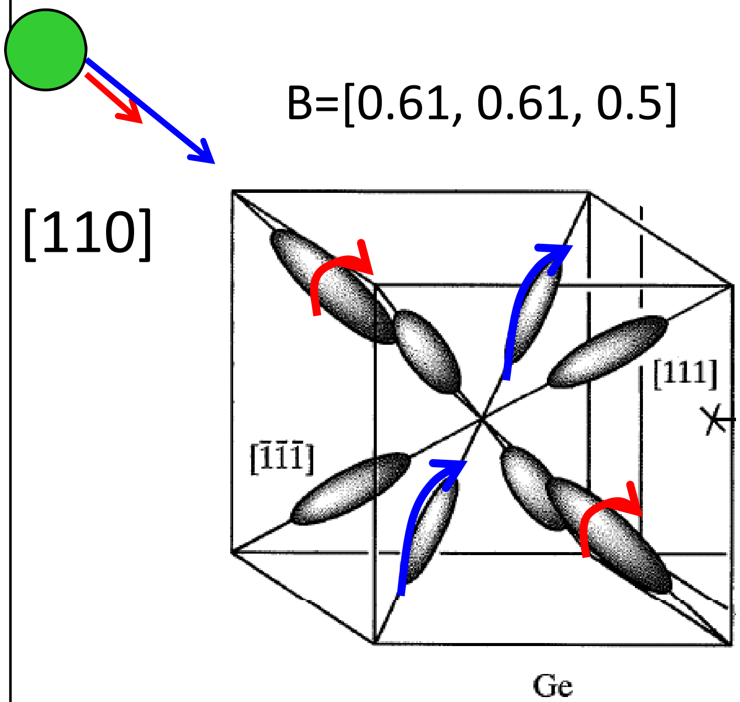
$$\frac{d^2 \upsilon_y}{dt^2} + \upsilon_y \omega^2 = 0 \quad \text{with} \quad \omega^2 \equiv \left[\omega_t w_l \sin^2\theta + \omega_t^{\,2} \cos^2\theta\right]$$

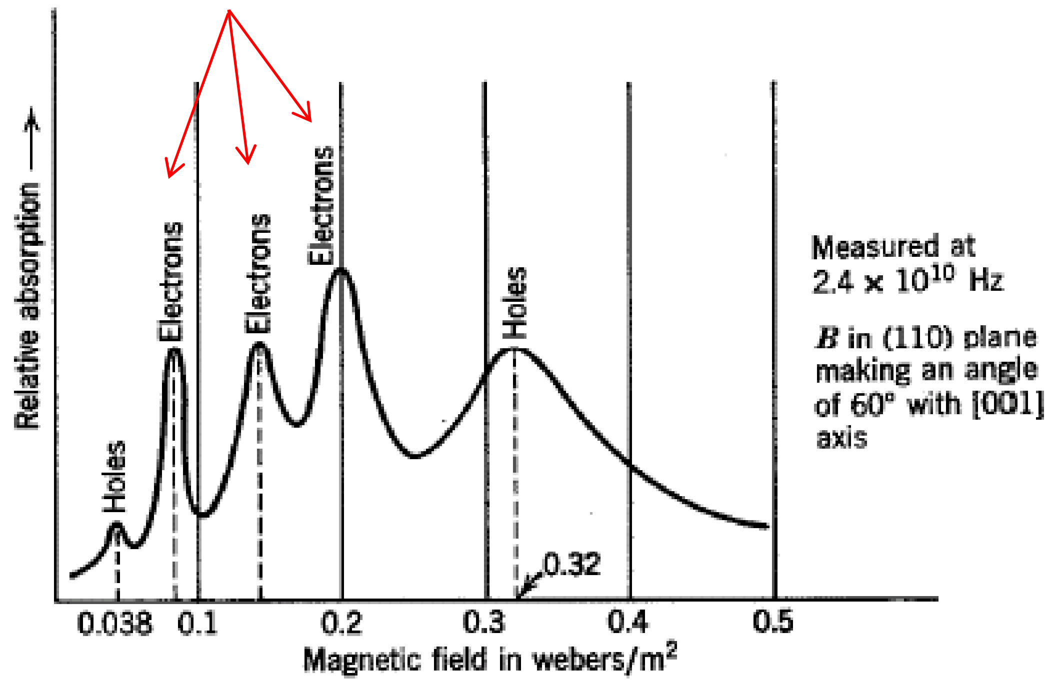
$$\omega_0 \equiv \frac{qB_0}{m_c^*} \qquad \omega_t \equiv \frac{qB_0}{m_t^*} \qquad \omega_l \equiv \frac{qB_0}{m_l^*}$$

so that …

$$\frac{1}{\left(m_c^*\right)^2} = \frac{\sin^2\theta}{m_l m_t} + \frac{\cos^2\theta}{m_t^{\,2}}$$

Measurement of Effective Mass





$$\frac{1}{m_c^2} = \frac{\cos^2\theta}{m_t^2} + \frac{\sin^2\theta}{m_l m_t}$$

Three peaks B_1, B_2, B_3

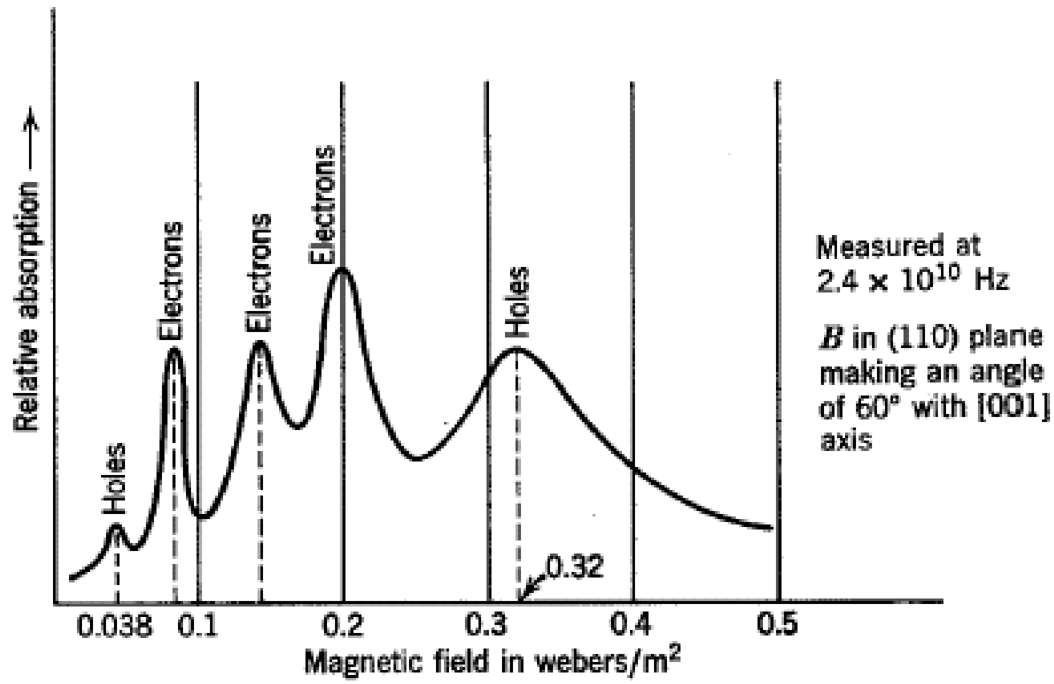
Three masses m_{c1}, m_{c2}, m_{c3}

Three unique angles: 7, 65, 73

$$m_c = \frac{qB_1}{2\pi\nu_0}$$

Known θ and m_c allows calculation of m_t and m_l.

Valence Band Effective Mass



HW. Which peaks relate to valence band?
Why are there two valence band peaks?

Conclusions

1) Measurement of Effective mass and band gaps define the energy-band of a material.

2) Only a fraction of the available states are occupied. The number of available states change with energy. DOS captures this variation.

3) DOS is an important and useful characteristic of a material that should be understood carefully.