Thermodynamics: An Engineering Approach
Seventh Edition
Yunus A. Cengel, Michael A. Boles
McGraw-Hill, 2011

Chapter 9
GAS POWER CYCLES

BASIC CONSIDERATIONS IN THE ANALYSIS OF POWER CYCLES

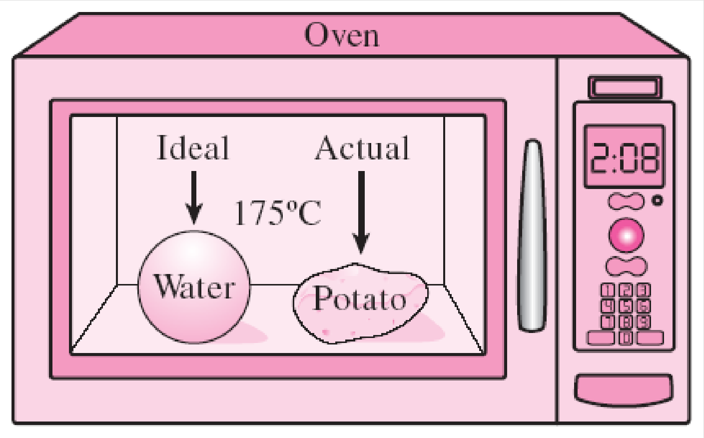
Thermal efficiency of heat engines:

$$\eta_{th} = \frac{W_{net}}{Q_{in}} \quad \text{or} \quad \eta_{th} = \frac{w_{net}}{q_{in}}$$

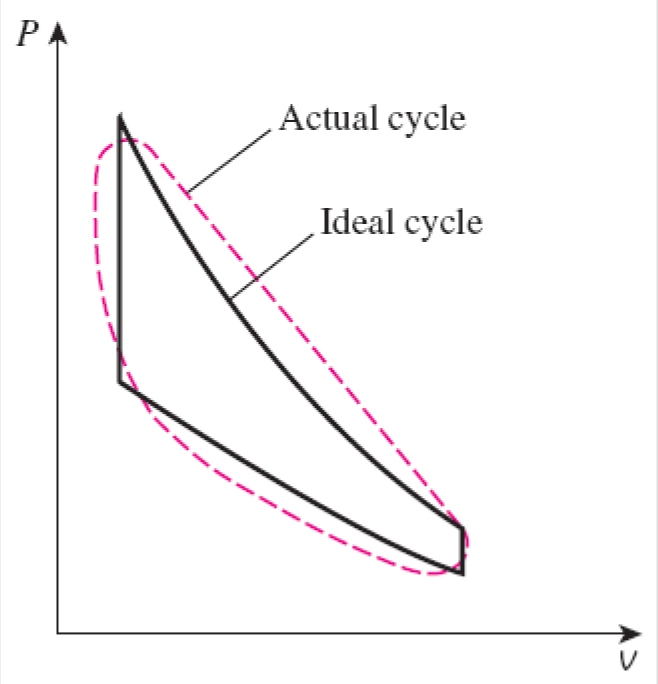
Most power-producing devices operate on cycles.

Ideal cycle: A cycle that resembles the actual cycle closely but is made up totally of internally reversible processes.

Reversible cycles such as **Carnot cycle** have the highest thermal efficiency of all heat engines operating between the same temperature levels. Unlike ideal cycles, they are totally reversible, and unsuitable as a realistic model.



Modeling is a powerful engineering tool that provides great insight and simplicity at the expense of some loss in accuracy.



The analysis of many complex processes can be reduced to a manageable level by utilizing some idealizations.

The ideal cycles are *internally reversible,* but, unlike the Carnot cycle, they are not necessarily externally reversible. Therefore, the thermal efficiency of an ideal cycle, in general, is less than that of a totally reversible cycle operating between the same temperature limits. However, it is still considerably higher than the thermal efficiency of an actual cycle because of the idealizations utilized.

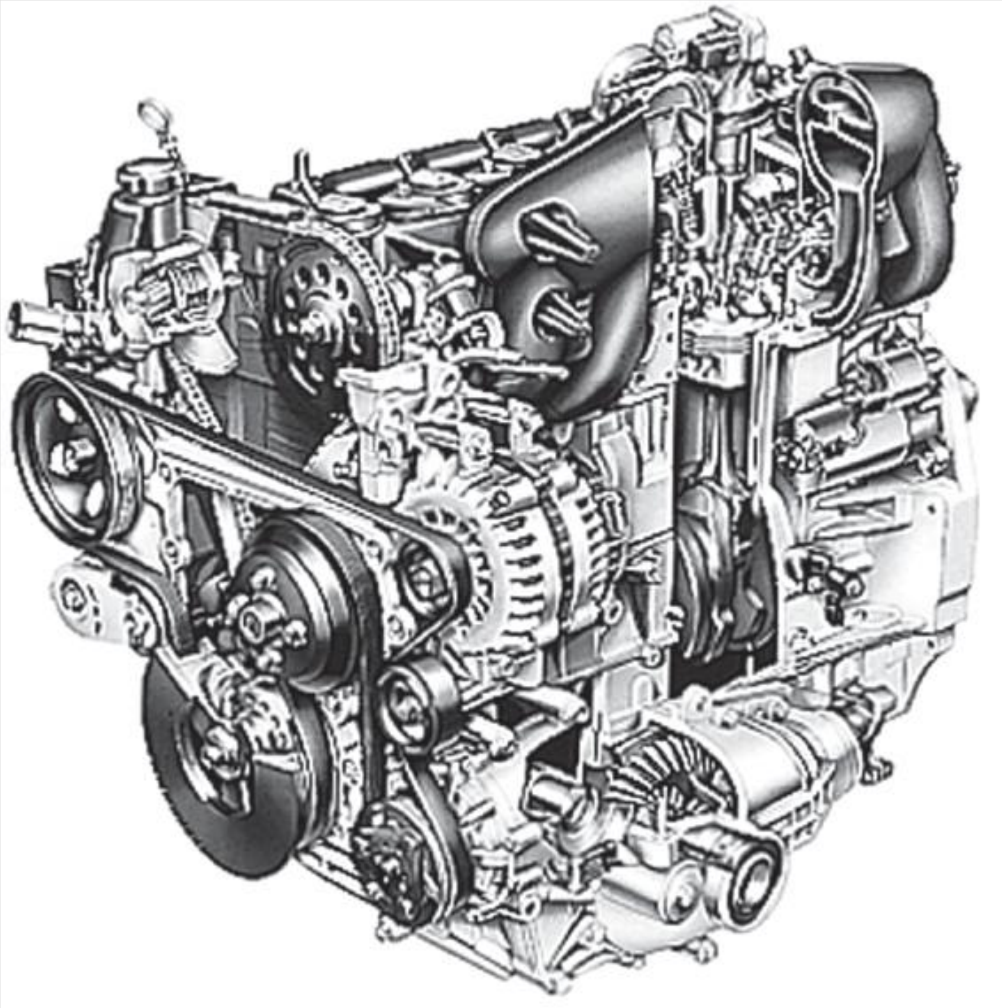


FIGURE 9–4

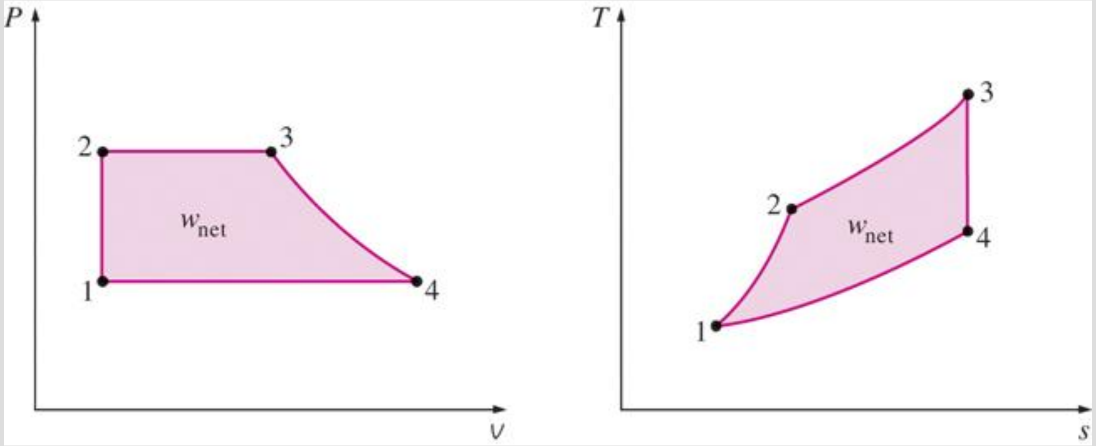
An automotive engine with the combustion chamber exposed.

On a *T-s* diagram, the ratio of the area enclosed by the cyclic curve to the area under the heat-addition process curve represents the thermal efficiency of the cycle. Any modification that increases the ratio of these two areas will also increase the thermal efficiency of the cycle.

The idealizations and simplifications in the analysis of power cycles:

1. The cycle does not involve any *friction.* Therefore, the working fluid does not experience any pressure drop as it flows in pipes or devices such as heat exchangers.
2. All expansion and compression processes take place in a *quasi-equilibrium* manner.
3. The pipes connecting the various components of a system are well insulated, and *heat transfer* through them is negligible.

Care should be exercised in the interpretation of the results from ideal cycles.

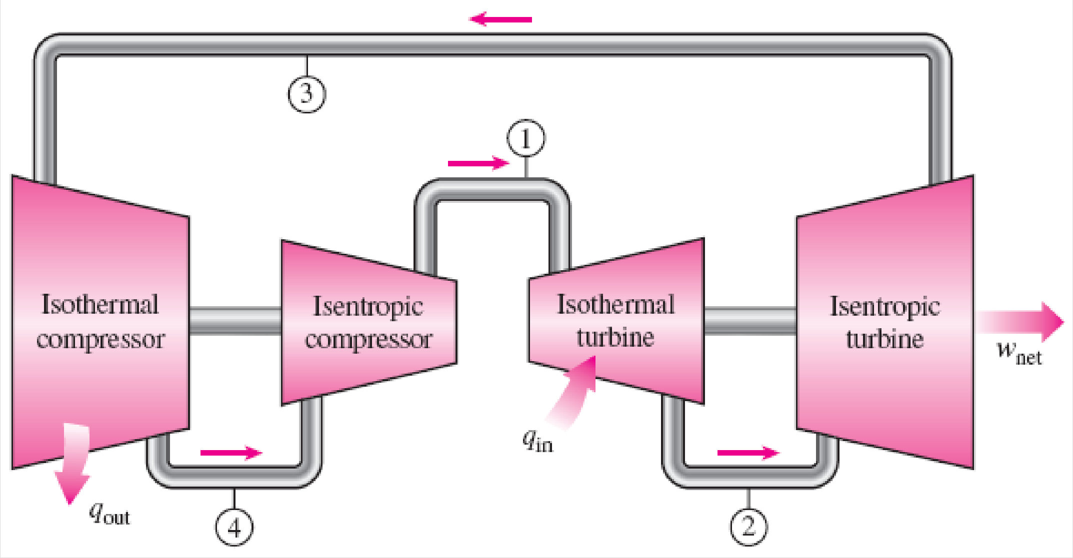


On both *P-v* and *T-s* diagrams, the area enclosed by the process curve represents the net work of the cycle.

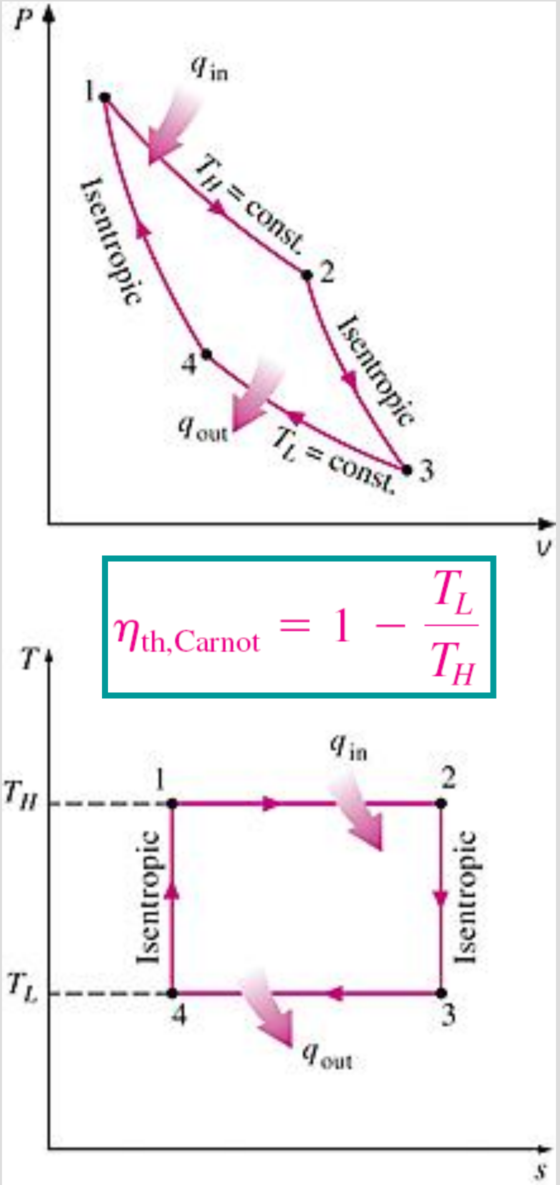
THE CARNOT CYCLE AND ITS VALUE IN ENGINEERING

The Carnot cycle is composed of four totally reversible processes: isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression.

For both ideal and actual cycles: *Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.*



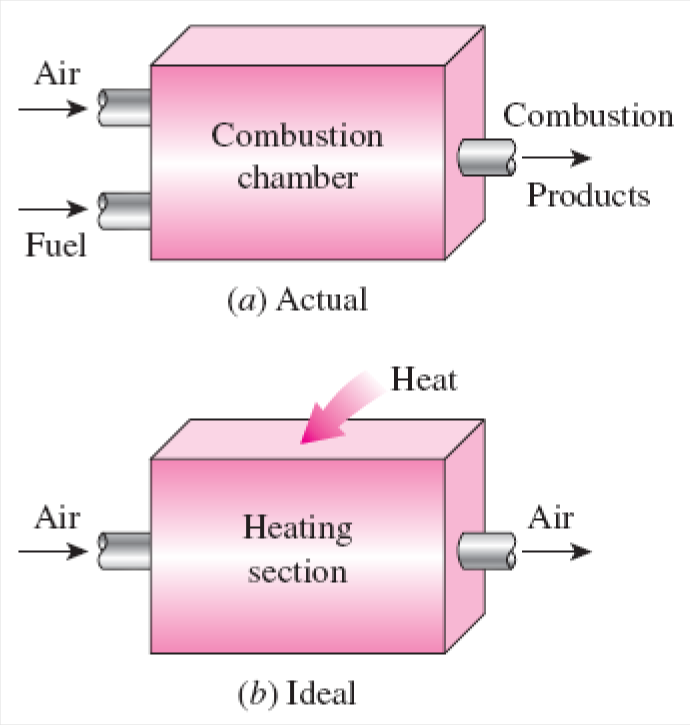
A steady-flow Carnot engine.



$$\eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

P-v and *T-s* diagrams of a Carnot cycle.

AIR-STANDARD ASSUMPTIONS



The combustion process is replaced by a heat-addition process in ideal cycles.

Air-standard assumptions:

1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are internally reversible.
3. The combustion process is replaced by a heat-addition process from an external source.
4. The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

Cold-air-standard assumptions: When the working fluid is considered to be air with constant specific heats at room temperature (25°C).

Air-standard cycle: A cycle for which the air-standard assumptions are applicable.

AN OVERVIEW OF RECIPROCATING ENGINES

Compression ratio

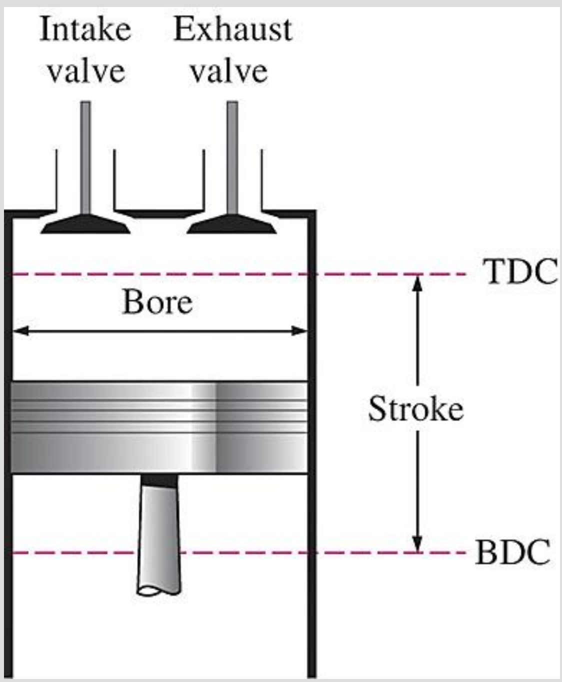
$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$

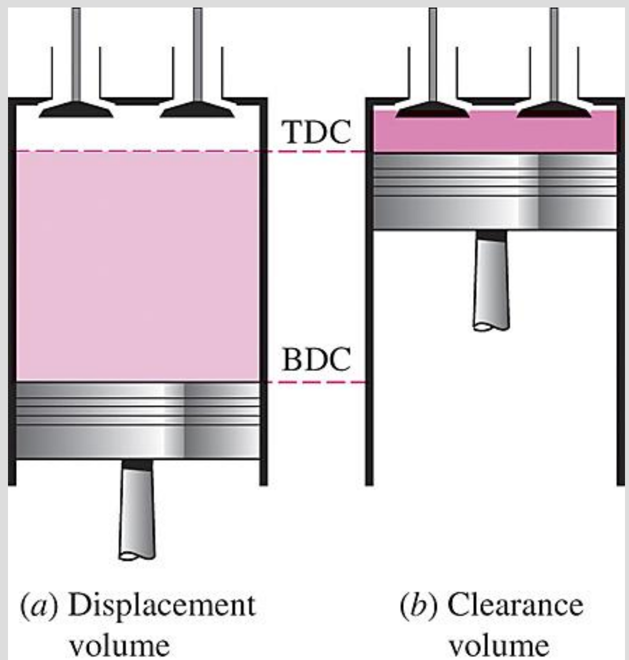
$$W_{net} = \text{MEP} \times \text{Piston area} \times \text{Stroke} = \text{MEP} \times \text{Displacement volume}$$

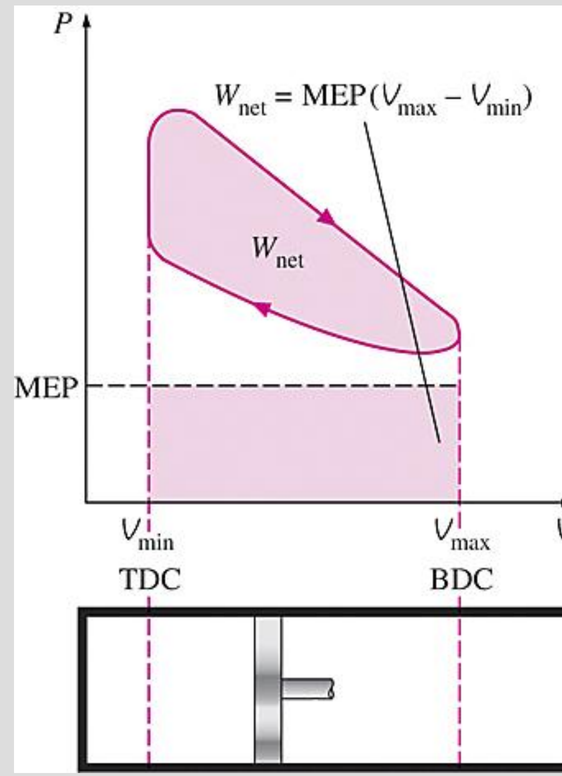
Mean effective pressure

$$\text{MEP} = \frac{W_{net}}{V_{max} - V_{min}} = \frac{w_{net}}{v_{max} - v_{min}} \quad \text{(kPa)}$$

- **Spark-ignition (SI) engines**
- **Compression-ignition (CI) engines**

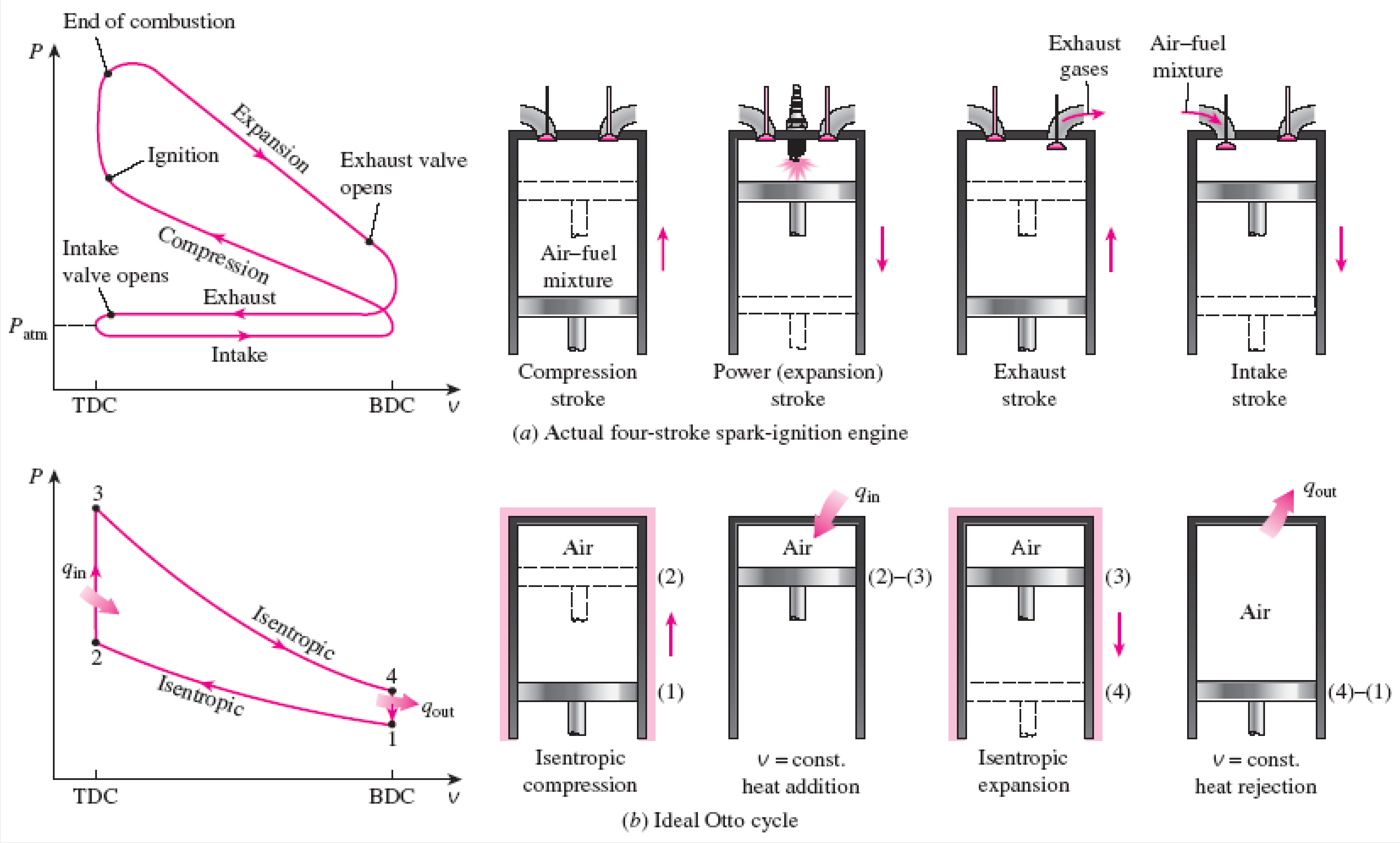






Nomenclature for reciprocating engines.

OTTO CYCLE: THE IDEAL CYCLE FOR SPARK-IGNITION ENGINES



Actual and ideal cycles in spark-ignition engines and their *P-v* diagrams.

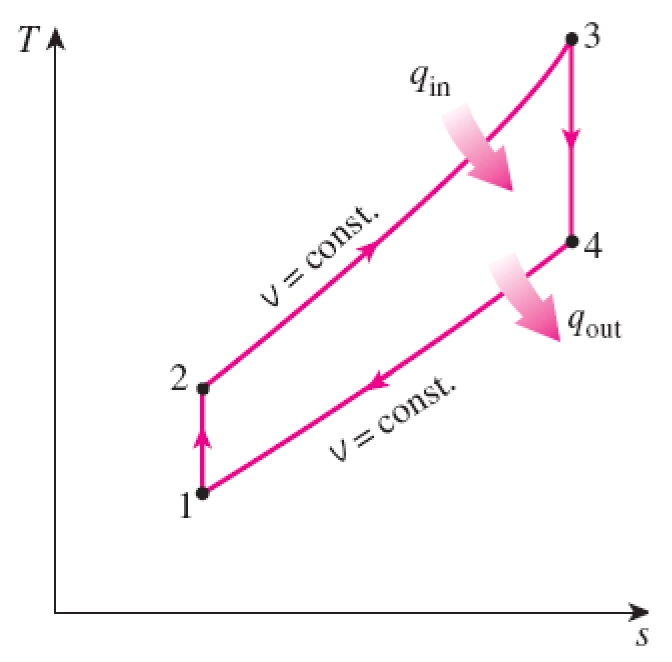
Four-stroke cycle

1 cycle = 4 stroke = 2 revolution

Two-stroke cycle

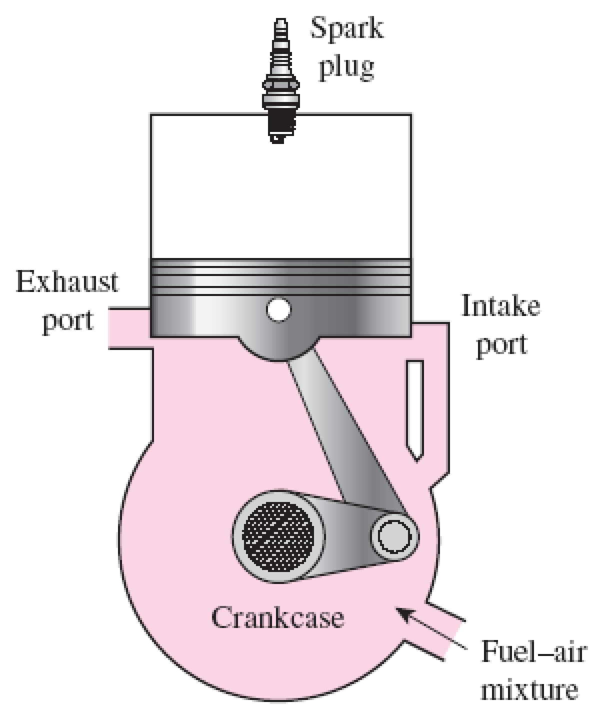
1 cycle = 2 stroke = 1 revolution

1-2	Isentropic compression
2-3	Constant-volume heat addition
3-4	Isentropic expansion
4-1	Constant-volume heat rejection



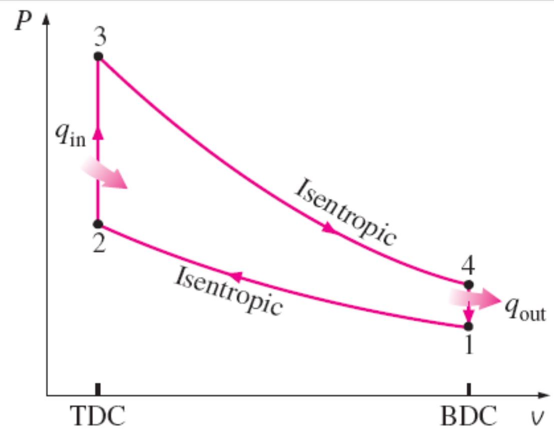
T-s diagram of the ideal Otto cycle.

The two-stroke engines are generally less efficient than their four-stroke counterparts but they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios.



Schematic of a two-stroke reciprocating engine.

FIGURE 9–15
Two-stroke engines are commonly used in motorcycles and lawn mowers.



$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

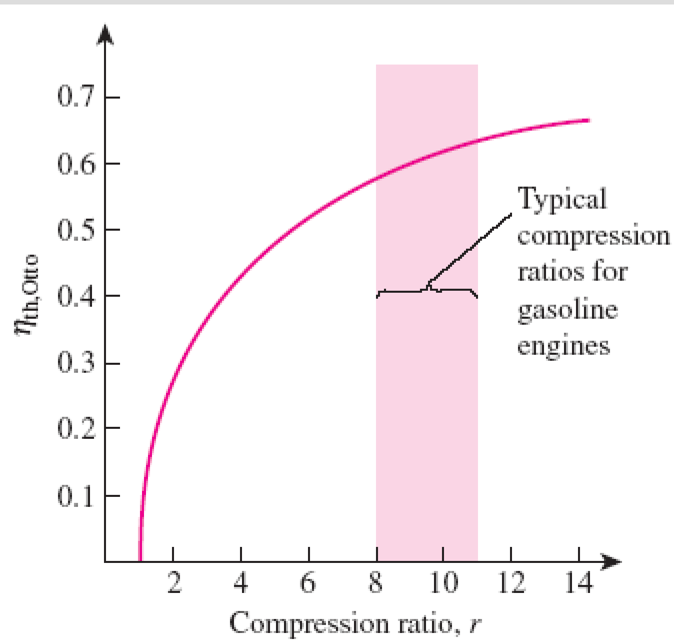
$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Otto} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$

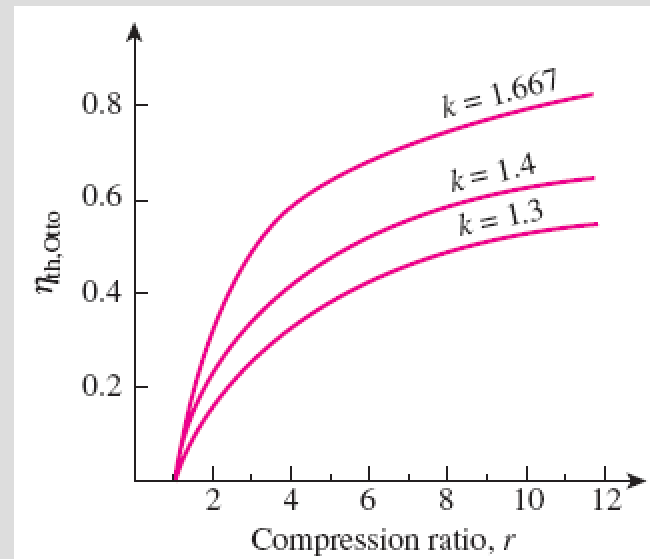
$$r = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{v_1}{v_2}$$



Thermal efficiency of the ideal Otto cycle as a function of compression ratio (k = 1.4).

$$\eta_{th,Otto} = 1 - \frac{1}{r^{k-1}}$$

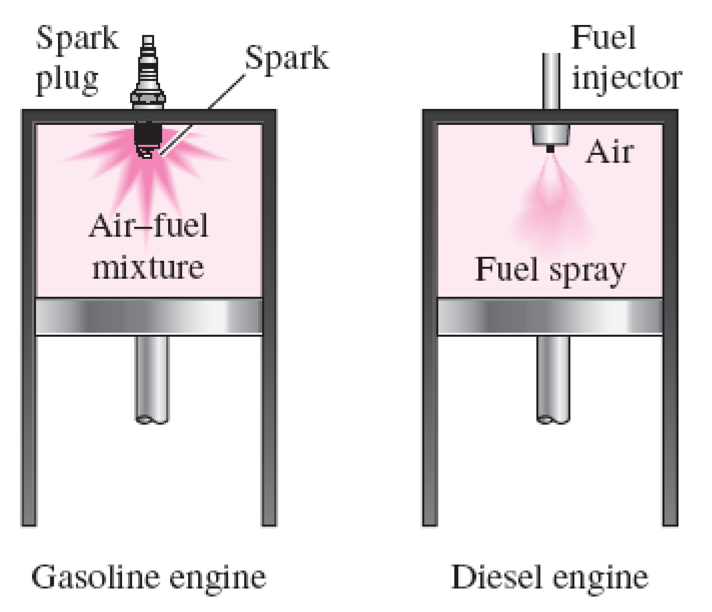
In SI engines, the compression ratio is limited by **autoignition** or **engine knock**.



The thermal efficiency of the Otto cycle increases with the specific heat ratio *k* of the working fluid.

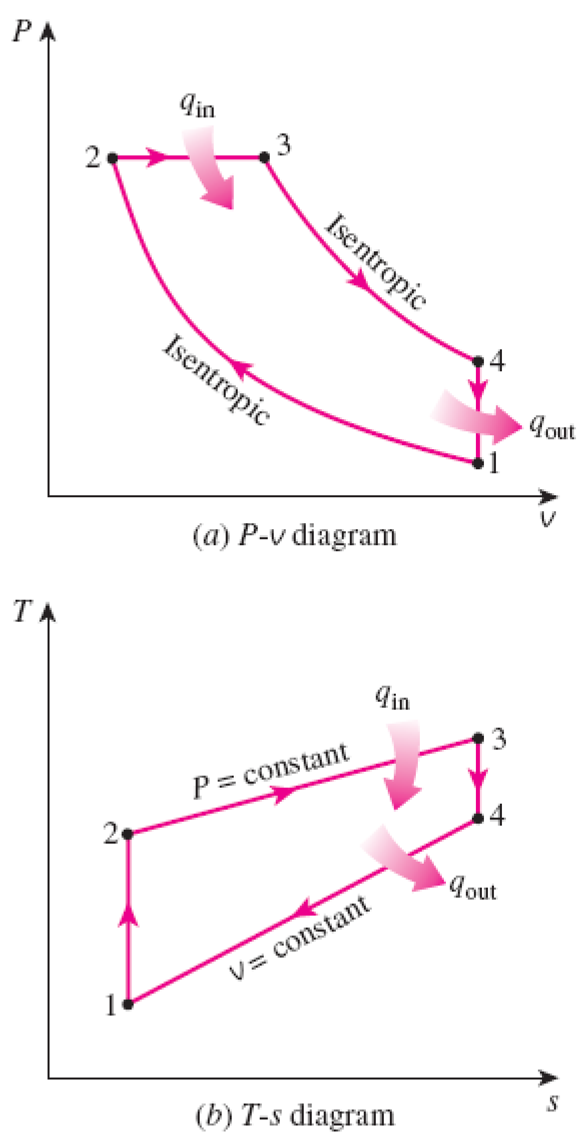
DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION ENGINES

In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of autoignition (engine knock). Therefore, diesel engines can be designed to operate at much higher compression ratios than SI engines, typically between 12 and 24.



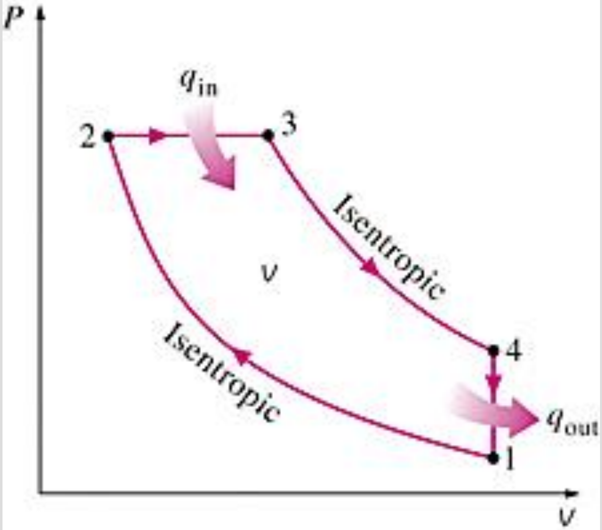
1-2 isentropic compression
2-3 constant-volume heat addition
3-4 isentropic expansion
4-1 constant-volume heat rejection.

In diesel engines, the spark plug is replaced by a fuel injector, and only air is compressed during the compression process.

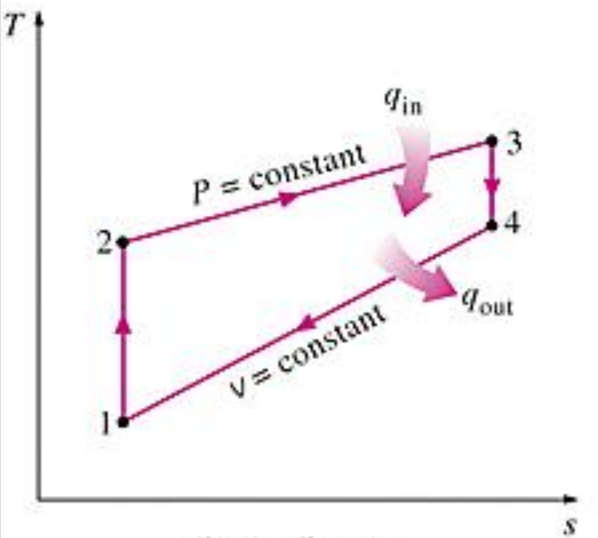


(a) P-v diagram

(b) T-s diagram



(a) P- v diagram



(b) T-s diagram

$$q_{in} - w_{b,out} = u_3 - u_2 \rightarrow q_{in} = P_2(v_3 - v_2) + (u_3 - u_2)$$
$$= h_3 - h_2 = c_p(T_3 - T_2)$$

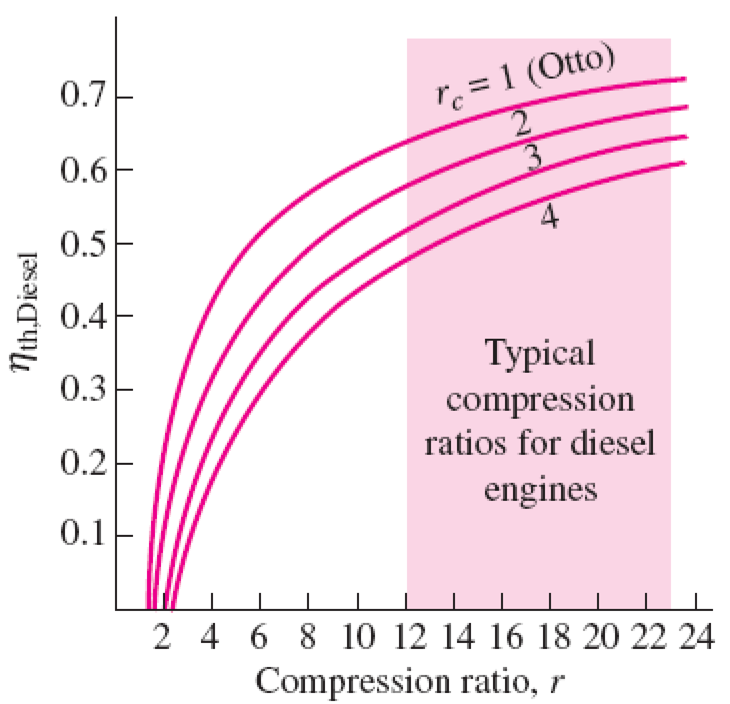
$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,Diesel} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}$$

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$ Cutoff ratio

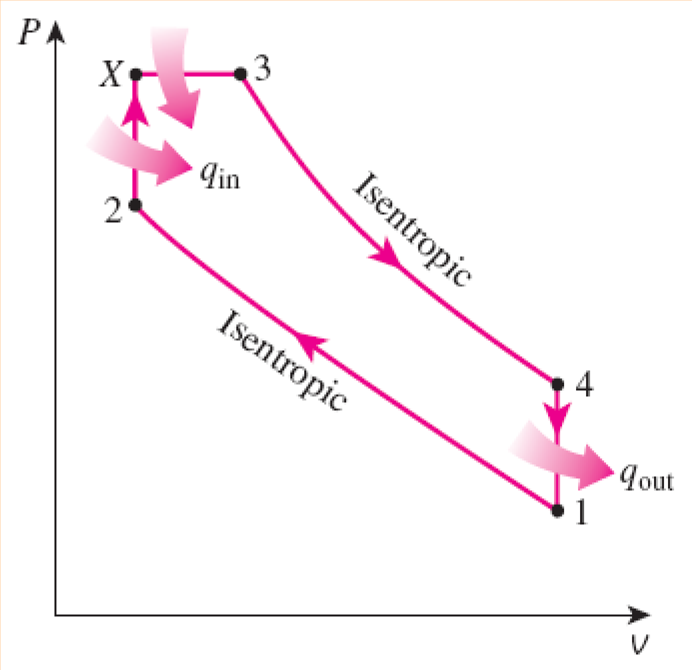
$$\eta_{th,Diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

$\eta_{th,Otto} > \eta_{th,Diesel}$ for the same compression ratio



Thermal efficiency of the ideal Diesel cycle as a function of compression and cutoff ratios (k=1.4).

Dual cycle: A more realistic ideal cycle model for modern, high-speed compression ignition engine.



P-v diagram of an ideal dual cycle.

QUESTIONS ???

Diesel engines operate at higher air-fuel ratios than gasoline engines. Why?

Despite higher power to weight ratios, two-stroke engines are not used in automobiles. Why?

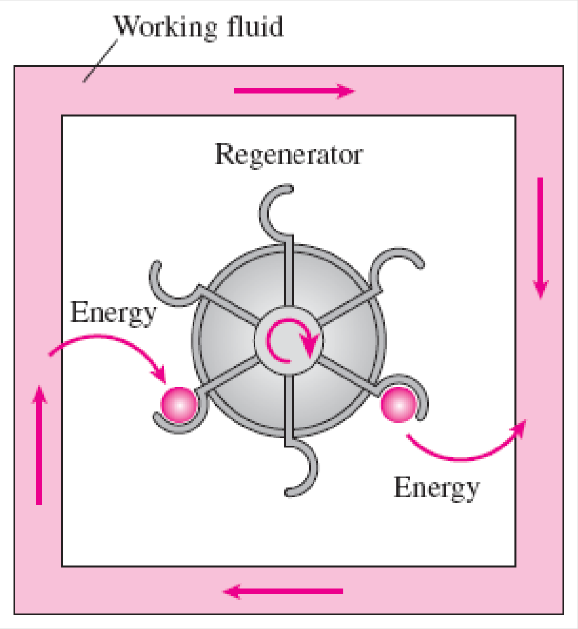
The stationary diesel engines are among the most efficient power producing devices (about 50%). Why?

What is a turbocharger? Why are they mostly used in diesel engines compared to gasoline engines.

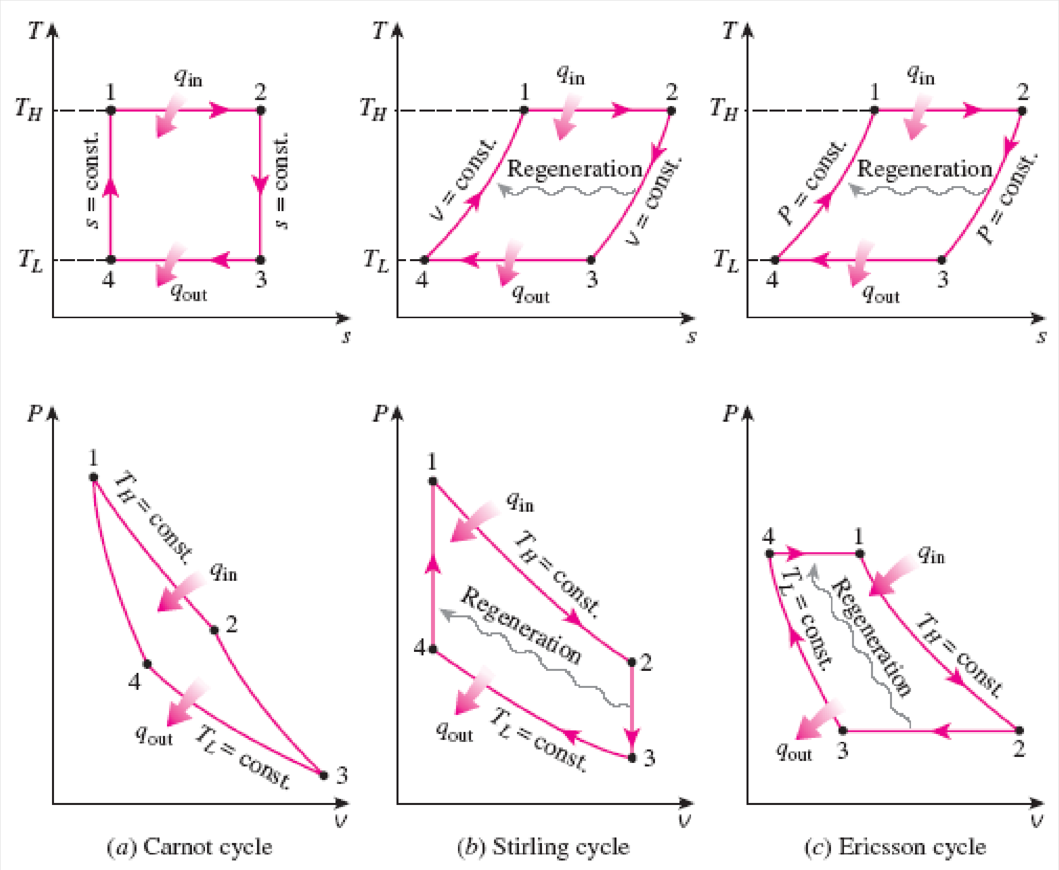
STIRLING AND ERICSSON CYCLES

Stirling cycle

1-2 *T = constant* expansion (heat addition from the external source)
2-3 *v = constant* regeneration (internal heat transfer from the working fluid to the regenerator)
3-4 *T = constant* compression (heat rejection to the external sink)
4-1 *v = constant* regeneration (internal heat transfer from the regenerator back to the working fluid)



A regenerator is a device that borrows energy from the working fluid during one part of the cycle and pays it back (without interest) during another part.

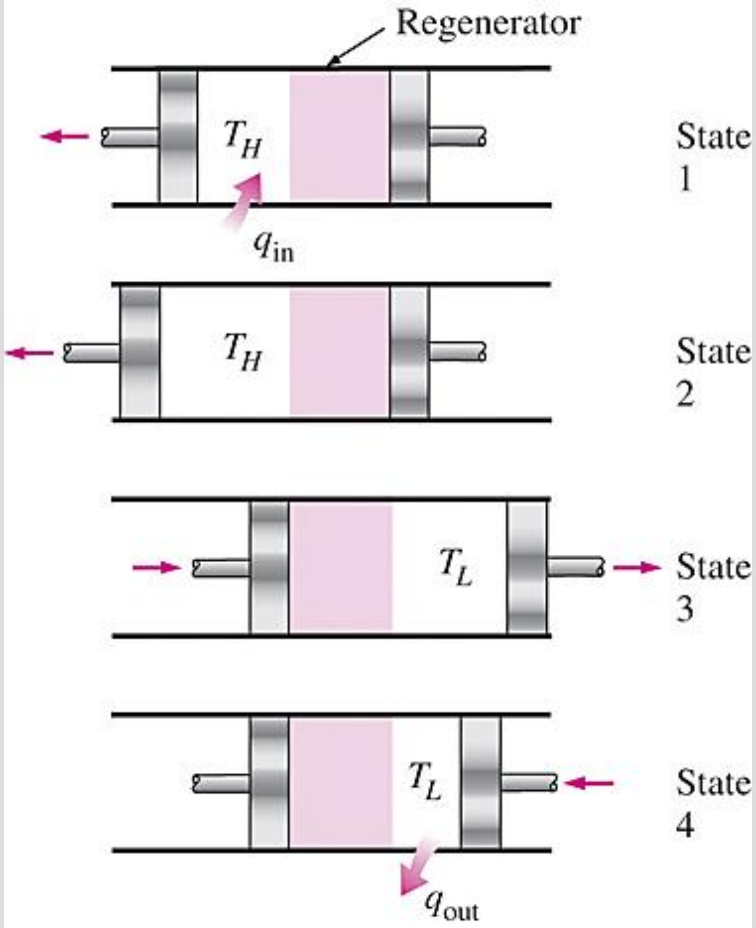


T-s and *P-* *v*diagrams of Carnot, Stirling, and Ericsson cycles.

The Stirling and Ericsson cycles give a message: *Regeneration can increase efficiency.*

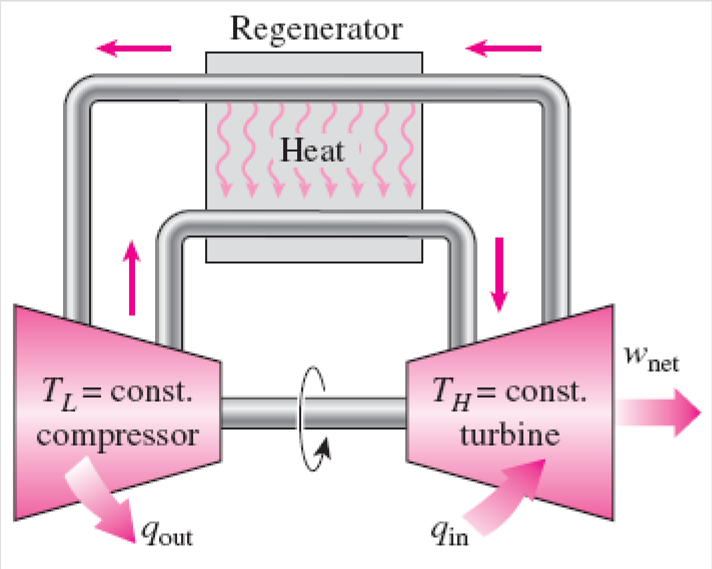
Both the Stirling and Ericsson cycles are totally reversible, as is the Carnot cycle, and thus:

$$\eta_{th,Stirling} = \eta_{th,Ericsson} = \eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$



The execution of the Stirling cycle.

The Ericsson cycle is very much like the Stirling cycle, except that the two constant-volume processes are replaced by two constant-pressure processes.

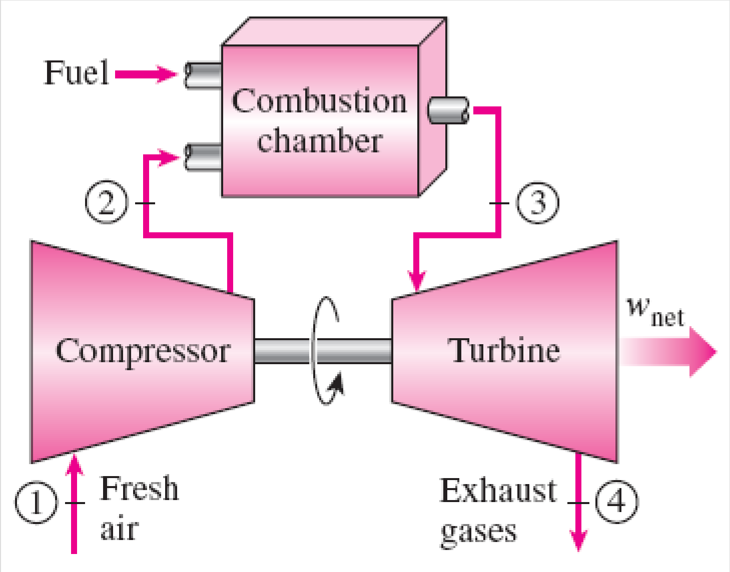


A steady-flow Ericsson engine.

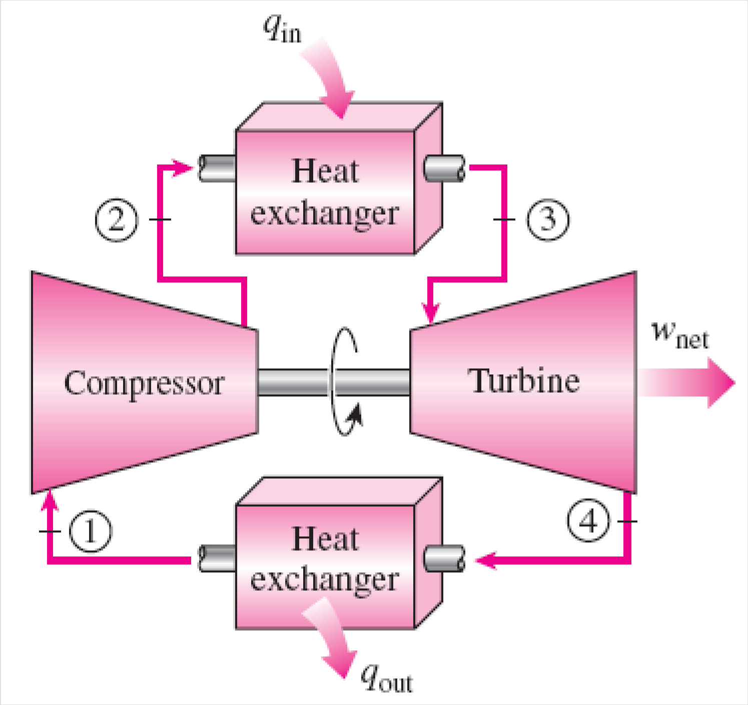
BRAYTON CYCLE: THE IDEAL CYCLE FOR GAS-TURBINE ENGINES

The combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air.

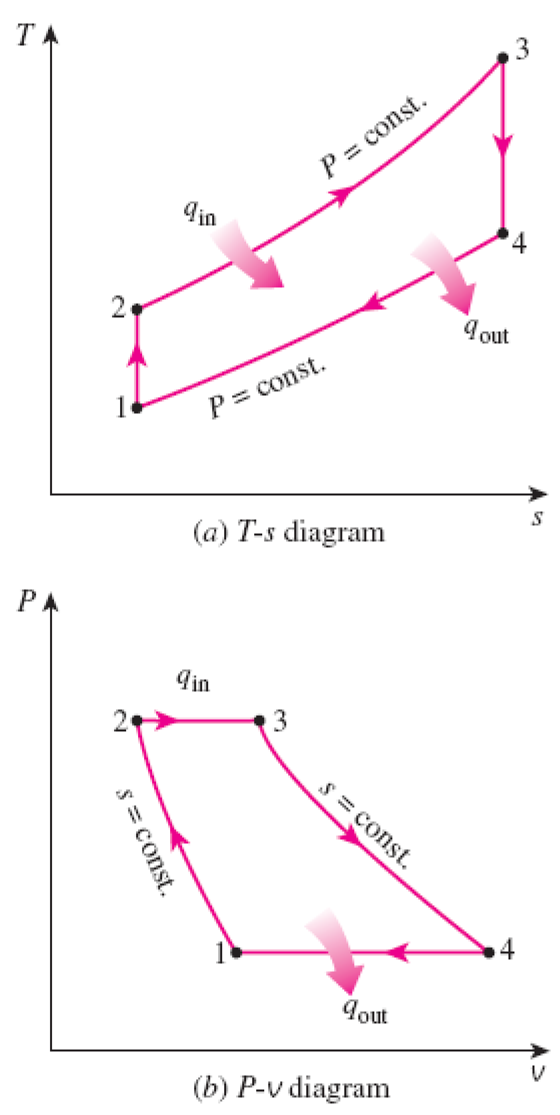
1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection



An open-cycle gas-turbine engine.



A closed-cycle gas-turbine engine.



T-s and *P-v* diagrams for the ideal Brayton cycle.

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

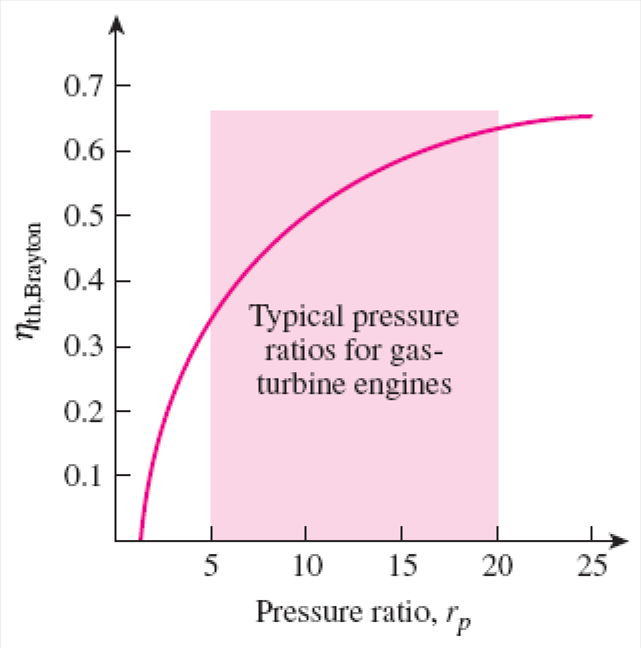
$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

$$\eta_{th,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

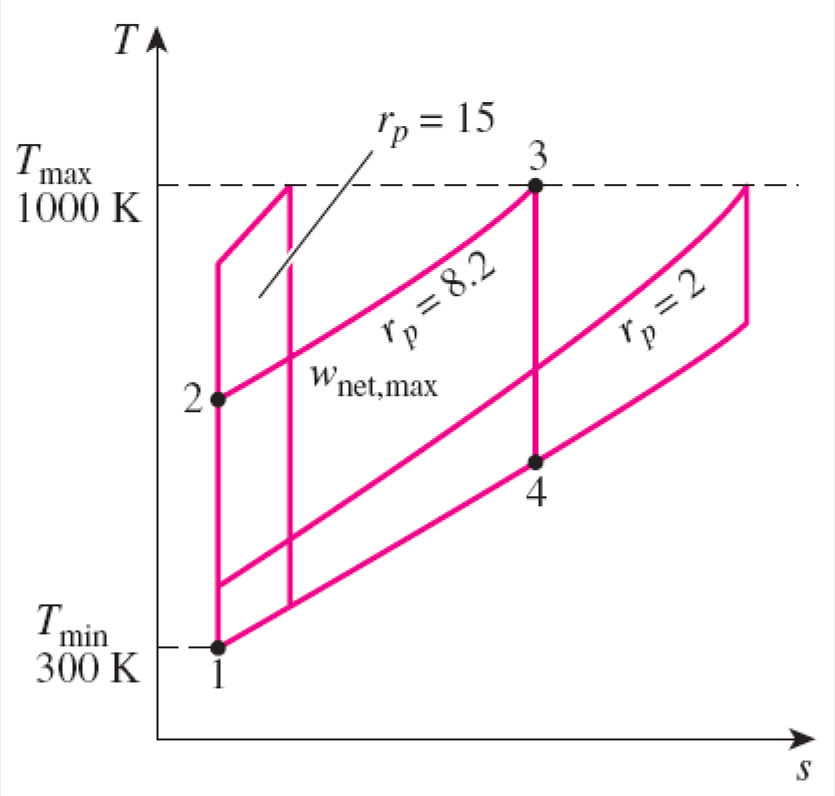
$$r_p = \frac{P_2}{P_1}$$ Pressure ratio

$$\eta_{th,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$



Thermal efficiency of the ideal Brayton cycle as a function of the pressure ratio.

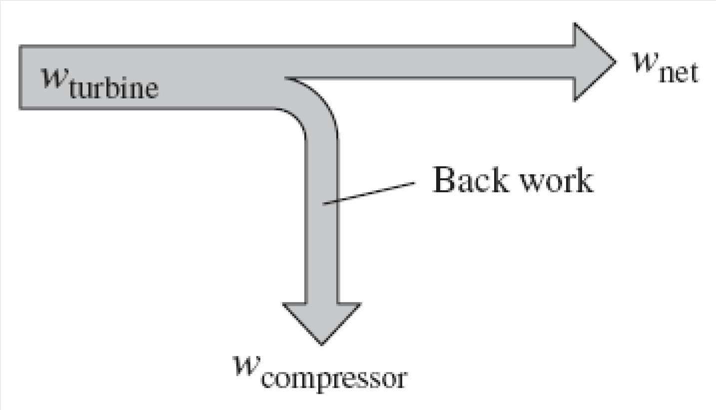
The two major application areas of gas-turbine engines are *aircraft propulsion* and *electric power generation*.



For fixed values of T_{min} and T_{max}, the net work of the Brayton cycle first increases with the pressure ratio, then reaches a maximum at $r_p = (T_{max}/T_{min})^{k/[2(k-1)]}$, and finally decreases.

The highest temperature in the cycle is limited by the maximum temperature that the turbine blades can withstand. This also limits the pressure ratios that can be used in the cycle.

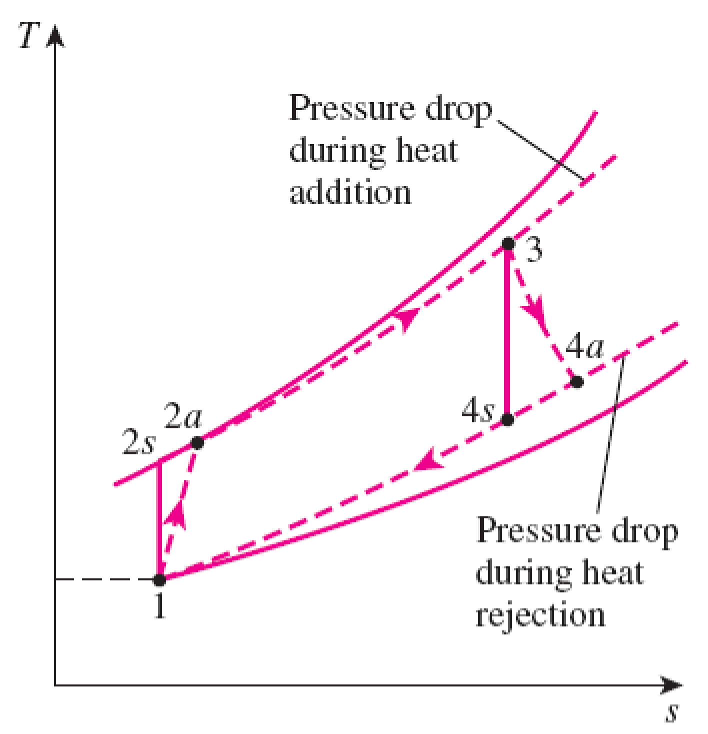
The air in gas turbines supplies the necessary oxidant for the combustion of the fuel, and it serves as a coolant to keep the temperature of various components within safe limits. An air–fuel ratio of 50 or above is not uncommon.



The fraction of the turbine work used to drive the compressor is called the back work ratio.

Development of Gas Turbines

1. Increasing the turbine inlet (or firing) temperatures
2. Increasing the efficiencies of turbomachinery components (turbines, compressors):
3. Adding modifications to the basic cycle (intercooling, regeneration or recuperation, and reheating).



The deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.

Deviation of Actual Gas-Turbine Cycles from Idealized Ones

Reasons: Irreversibilities in turbine and compressors, pressure drops, heat losses

Isentropic efficiencies of the compressor and turbine

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

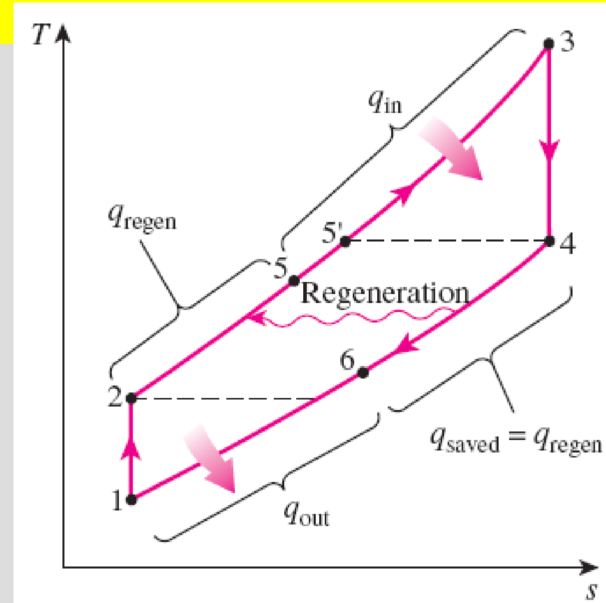
$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$

THE BRAYTON CYCLE WITH REGENERATION

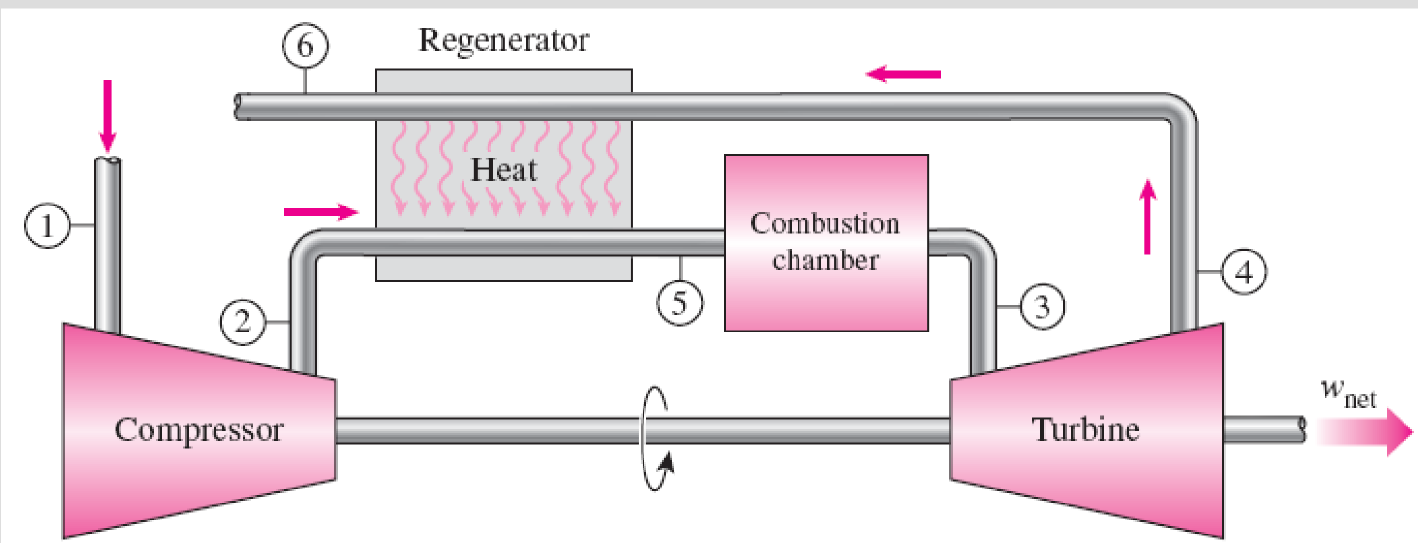
In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor.

Therefore, the high-pressure air leaving the compressor can be heated by the hot exhaust gases in a counter-flow heat exchanger (a *regenerator* or a *recuperator*).

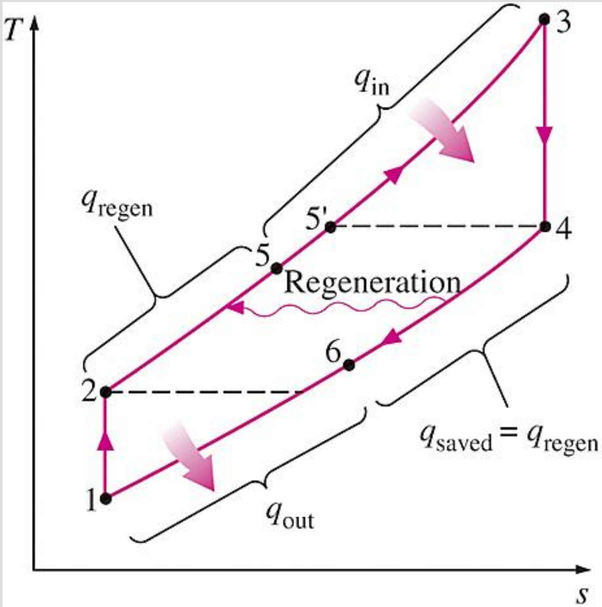
The thermal efficiency of the Brayton cycle increases as a result of regeneration since less fuel is used for the same work output.



T-s diagram of a Brayton cycle with regeneration.



A gas-turbine engine with regenerator.



T-s diagram of a Brayton cycle with regeneration.

$$q_{\text{regen,act}} = h_5 - h_2$$

$$q_{\text{regen,max}} = h_{5'} - h_2 = h_4 - h_2$$

$$\epsilon = \frac{q_{\text{regen,act}}}{q_{\text{regen,max}}} = \frac{h_5 - h_2}{h_4 - h_2}$$

Effectiveness of regenerator

$$\epsilon \cong \frac{T_5 - T_2}{T_4 - T_2}$$

Effectiveness under cold-air standard assumptions

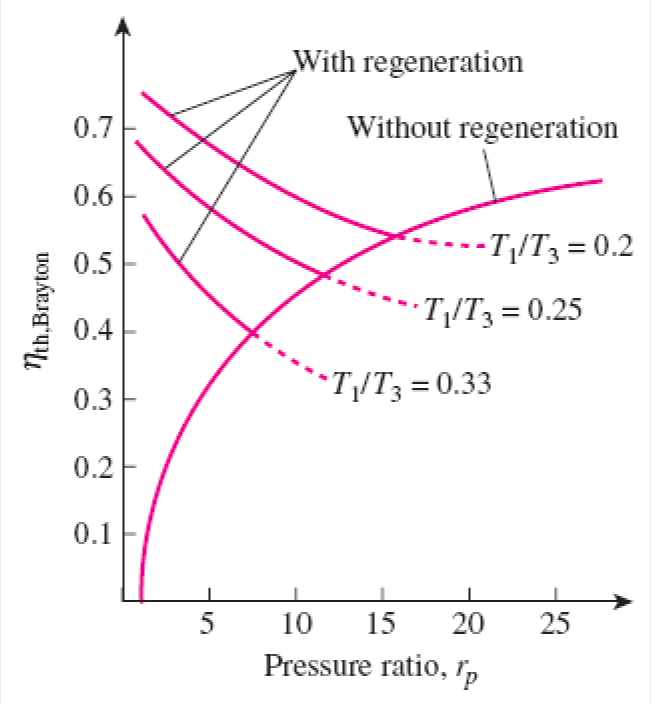
$$\eta_{\text{th,regen}} = 1 - \left(\frac{T_1}{T_3}\right)(r_p)^{(k-1)/k}$$

Under cold-air standard assumptions

The thermal efficiency depends on the ratio of the minimum to maximum temperatures as well as the pressure ratio.

Regeneration is most effective at lower pressure ratios and low minimum-to-maximum temperature ratios.

Can regeneration be used at high pressure ratios?

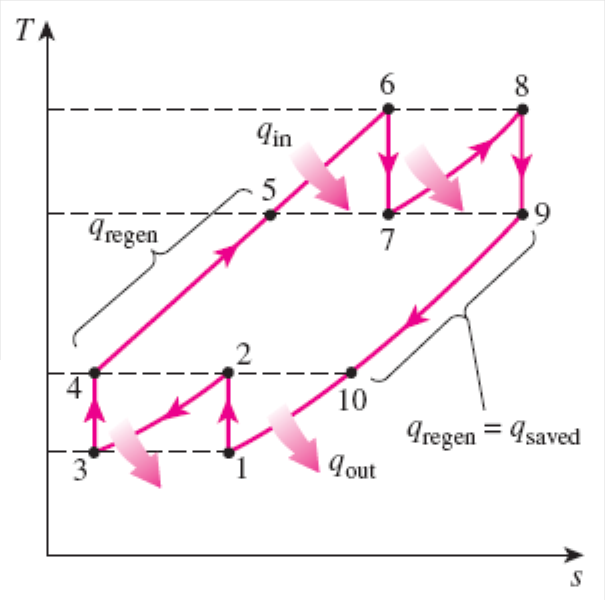


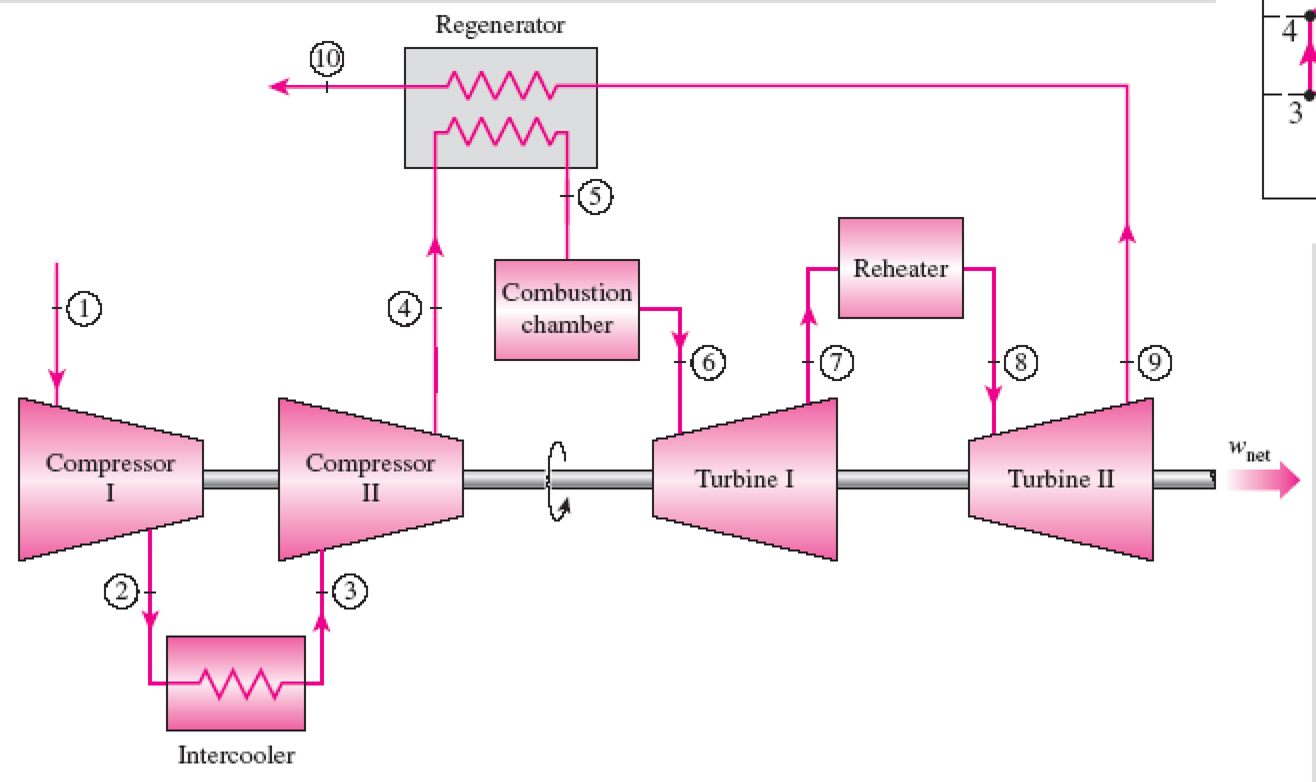
Thermal efficiency of the ideal Brayton cycle with and without regeneration.

THE BRAYTON CYCLE WITH INTERCOOLING, REHEATING, AND REGENERATION

For minimizing work input to compressor and maximizing work output from turbine:

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} \quad \text{and} \quad \frac{P_6}{P_7} = \frac{P_8}{P_9}$$



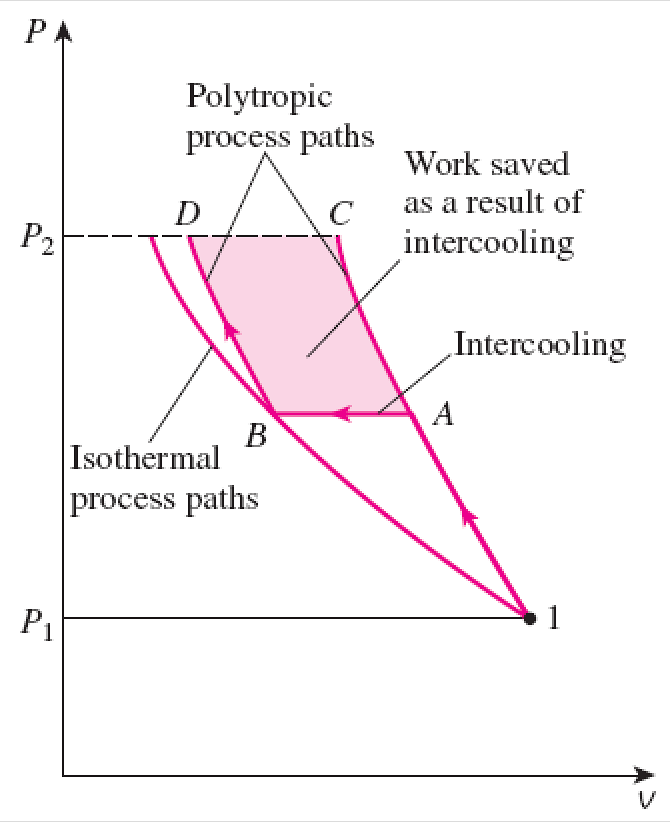


A gas-turbine engine with two-stage compression with intercooling, two-stage expansion with reheating, and regeneration and its *T*-*s* diagram.

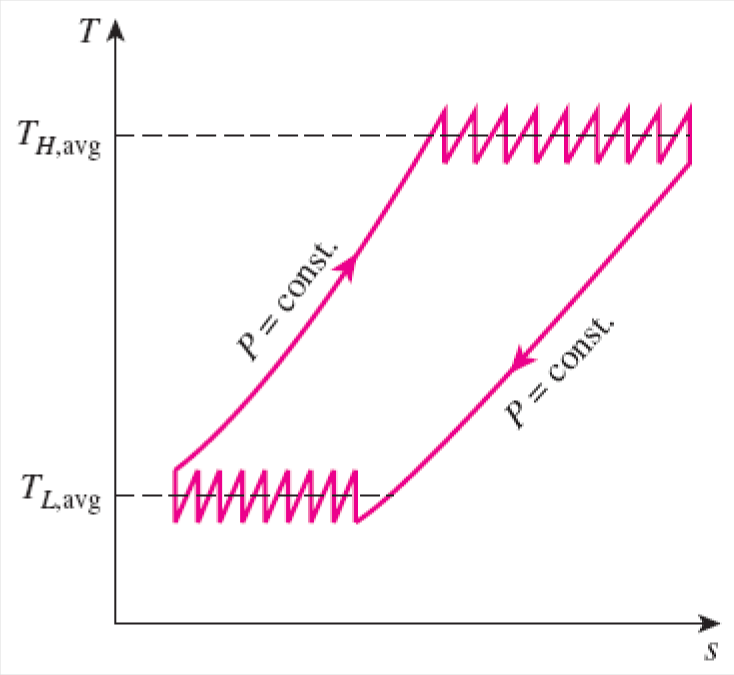
Multistage compression with intercooling: The work required to compress a gas between two specified pressures can be decreased by carrying out the compression process in stages and cooling the gas in between. This keeps the specific volume as low as possible.

Multistage expansion with reheating keeps the specific volume of the working fluid as high as possible during an expansion process, thus maximizing work output.

Intercooling and reheating always decreases the thermal efficiency unless they are accompanied by regeneration. **Why?**



Comparison of work inputs to a single-stage compressor (1*AC*) and a two-stage compressor with intercooling (1*ABD*).



As the number of compression and expansion stages increases, the gas-turbine cycle with intercooling, reheating, and regeneration approaches the Ericsson cycle.

IDEAL JET-PROPULSION CYCLES

Gas-turbine engines are widely used to power aircraft because they are light and compact and have a high power-to-weight ratio.

Aircraft gas turbines operate on an open cycle called a **jet-propulsion cycle**.

The ideal jet-propulsion cycle differs from the simple ideal Brayton cycle in that the gases are not expanded to the ambient pressure in the turbine. Instead, they are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment.

The net work output of a jet-propulsion cycle is zero. The gases that exit the turbine at a relatively high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the aircraft.

Aircraft are propelled by accelerating a fluid in the opposite direction to motion. This is accomplished by either slightly accelerating a large mass of fluid (***propeller-driven engine***) or greatly accelerating a small mass of fluid (***jet or turbojet engine***) or both (***turboprop engine***).

In jet engines, the high-temperature and high-pressure gases leaving the turbine are accelerated in a nozzle to provide thrust.

Thrust (propulsive force)

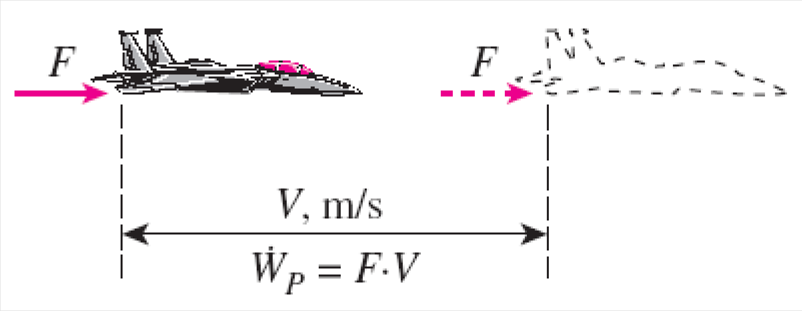
$$F = (\dot{m}V)_{exit} - (\dot{m}V)_{inlet} = \dot{m}(V_{exit} - V_{inlet}) \quad \text{(N)}$$

Propulsive power

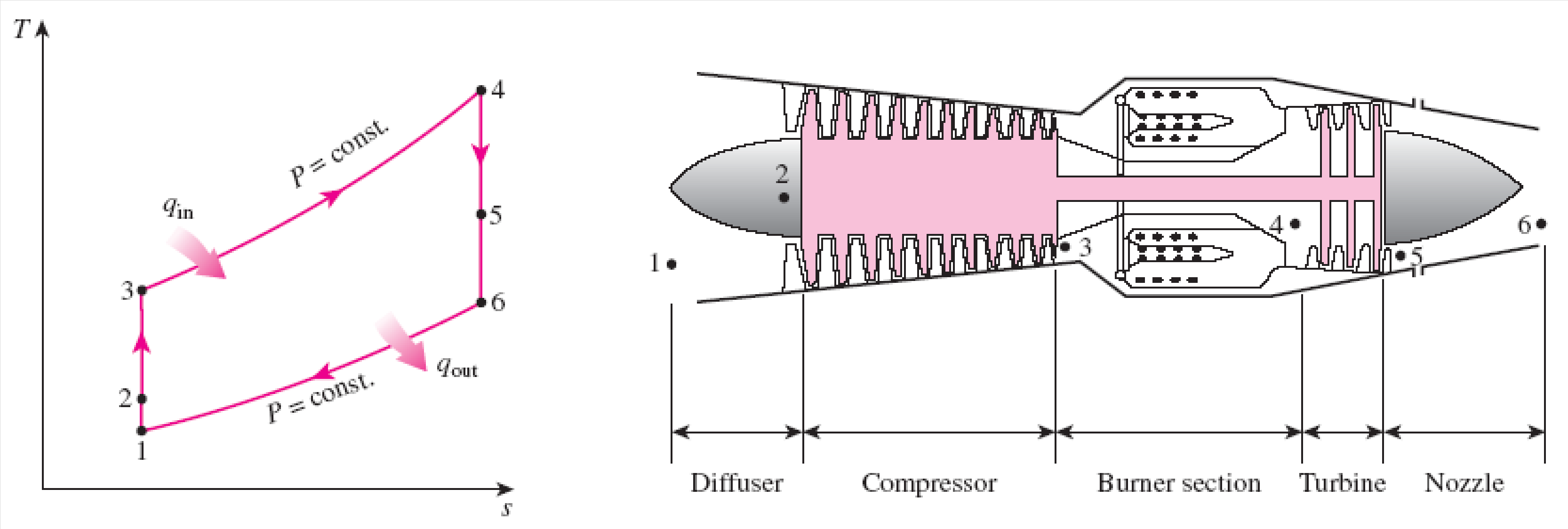
$$\dot{W}_P = FV_{aircraft} = \dot{m}(V_{exit} - V_{inlet})V_{aircraft} \quad \text{(kW)}$$

Propulsive efficiency

$$\eta_P = \frac{\text{Propulsive power}}{\text{Energy input rate}} = \frac{\dot{W}_P}{\dot{Q}_{in}}$$



Propulsive power is the thrust acting on the aircraft through a distance per unit time.



Basic components of a turbojet engine and the *T*-*s* diagram for the ideal turbojet cycle.

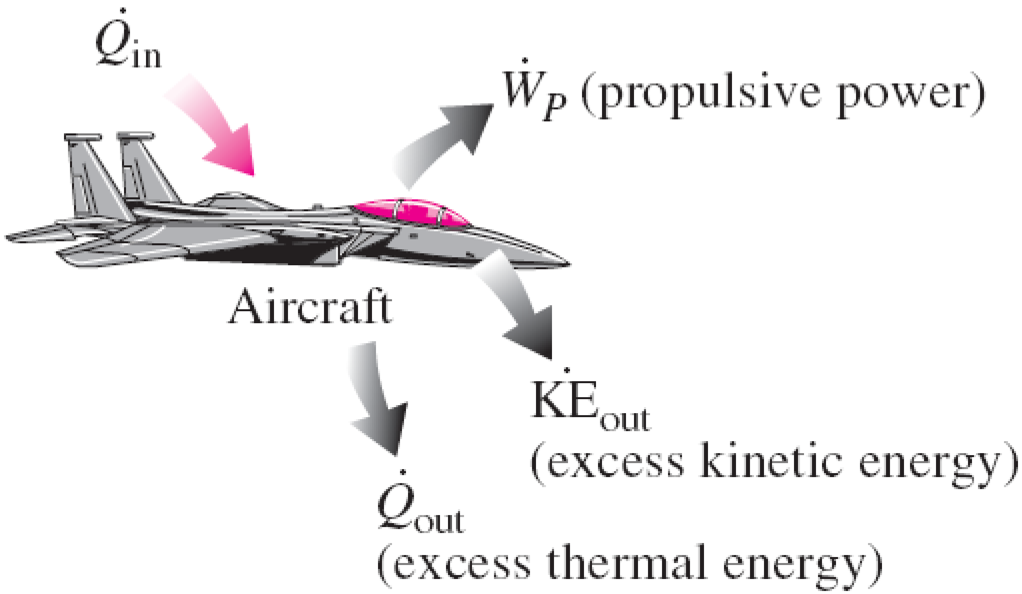


FIGURE 9–51

Energy supplied to an aircraft (from the burning of a fuel) manifests itself in various forms.

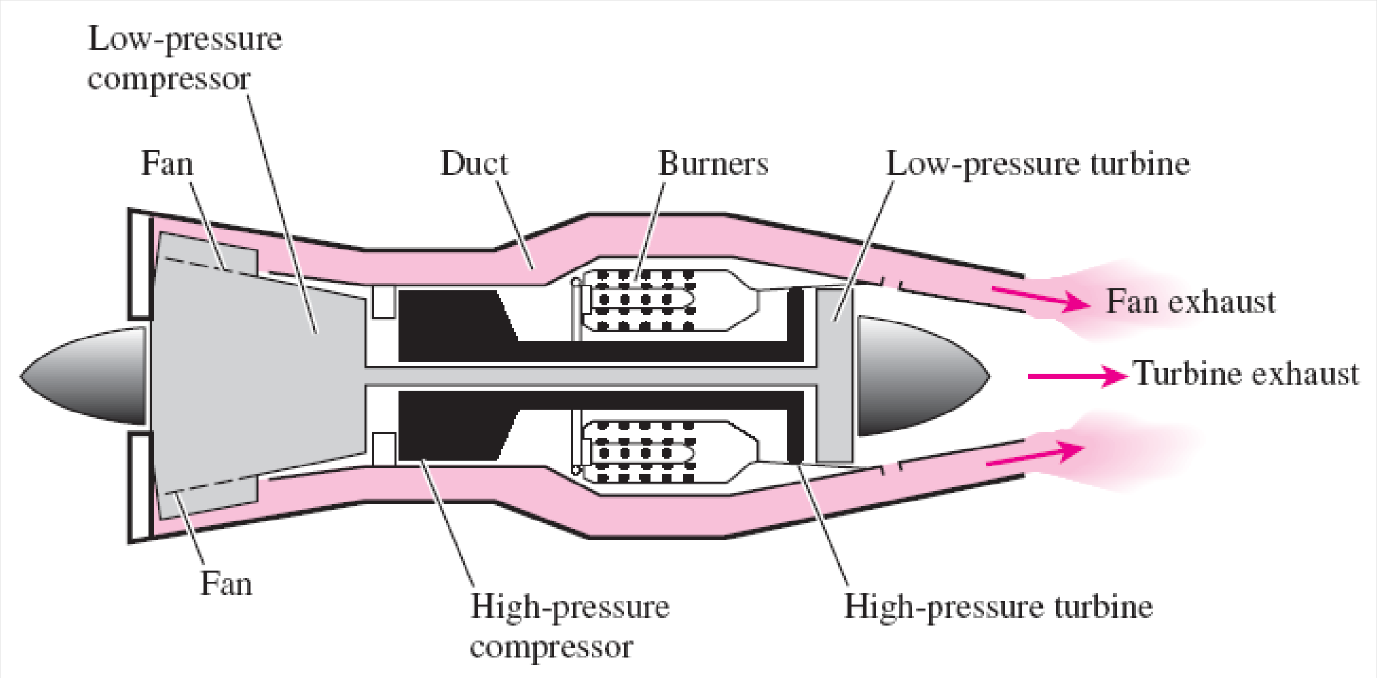
Modifications to Turbojet Engines

The first airplanes built were all propeller-driven, with propellers powered by engines essentially identical to automobile engines.

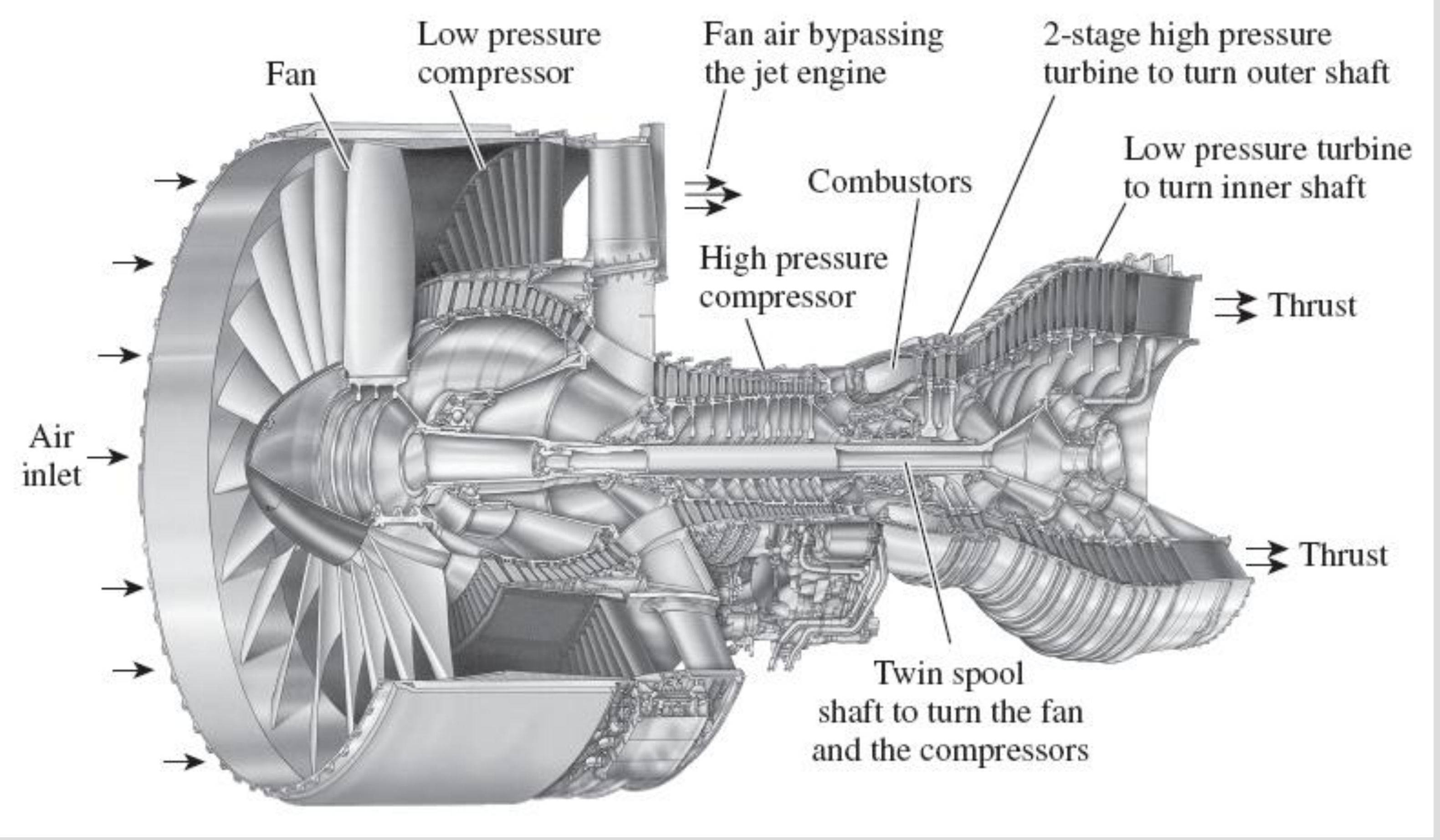
Both propeller-driven engines and jet-propulsion-driven engines have their own strengths and limitations, and several attempts have been made to combine the desirable characteristics of both in one engine.

Two such modifications are the ***propjet engine*** and the ***turbofan engine***.

The most widely used engine in aircraft propulsion is the **turbofan** (or *fanjet*) engine wherein a large fan driven by the turbine forces a considerable amount of air through a duct (cowl) surrounding the engine.



A turbofan engine.

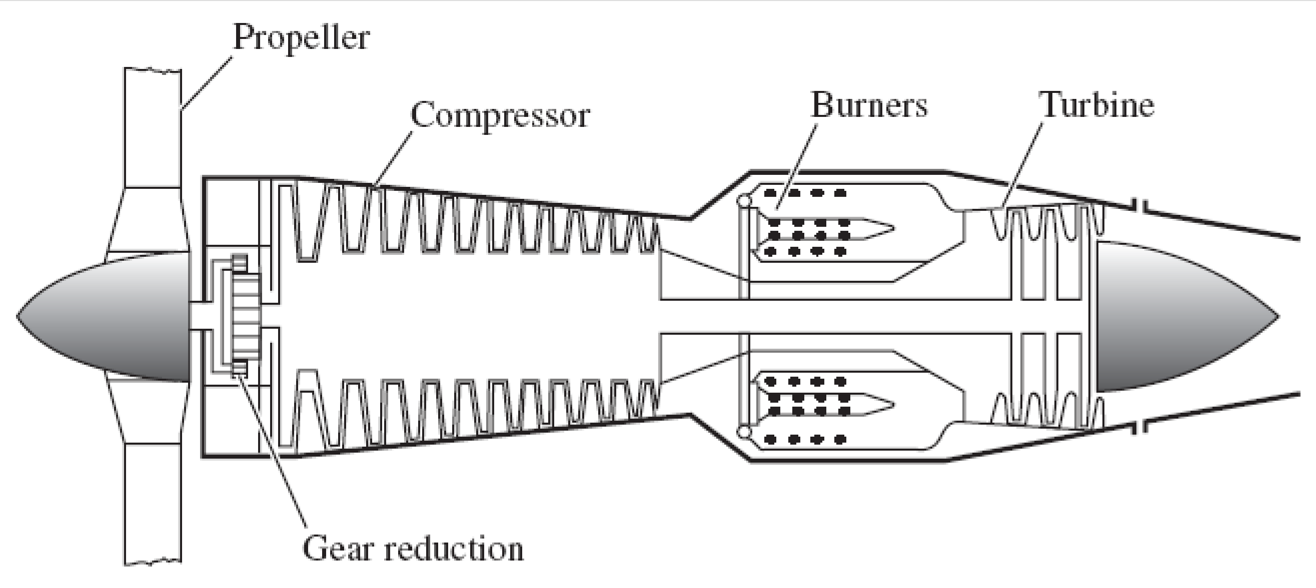


A modern jet engine used to power Boeing 777 aircraft. This is a Pratt & Whitney PW4084 turbofan capable of producing 84,000 pounds of thrust. It is 4.87 m (192 in.) long, has a 2.84 m (112 in.) diameter fan, and it weighs 6800 kg (15,000 lbm).

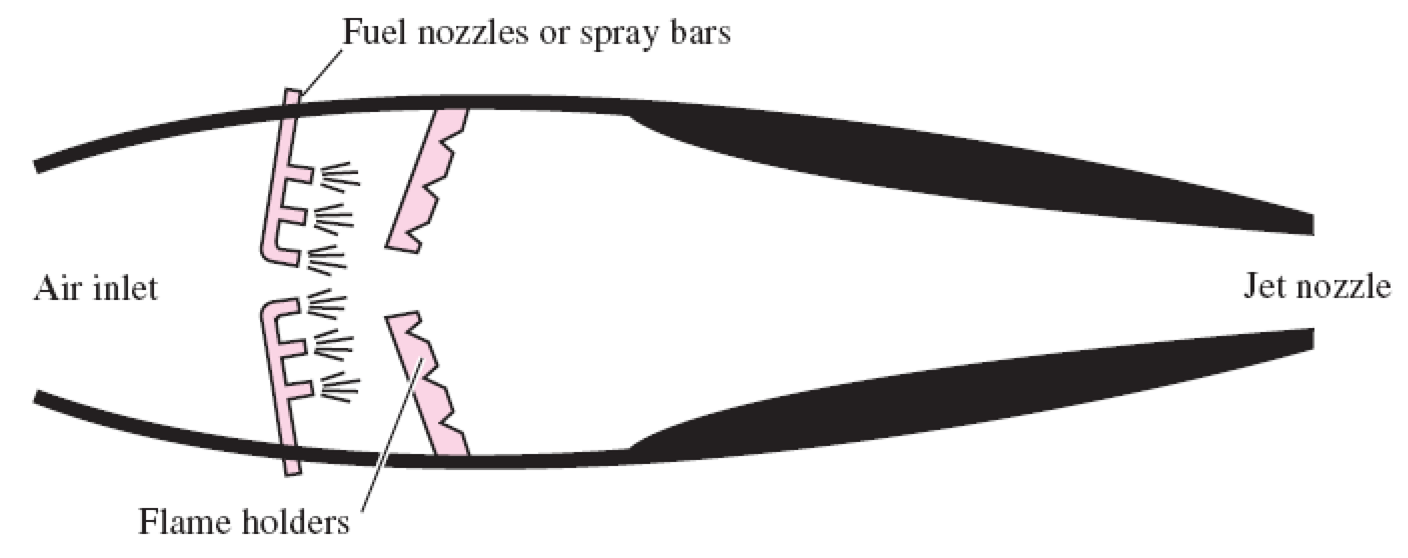
Various engine types:

Turbofan, Propjet, Ramjet, Sacramjet, Rocket

A turboprop engine.



A ramjet engine.



SECOND-LAW ANALYSIS OF GAS POWER CYCLES

$$X_{dest} = T_0 S_{gen} = T_0(\Delta S_{sys} - S_{in} + S_{out})$$
$$= T_0\left[(S_2 - S_1)_{sys} - \frac{Q_{in}}{T_{b,in}} + \frac{Q_{out}}{T_{b,out}}\right] \quad \text{(kJ)}$$

Exergy destruction for a closed system

$$\dot{X}_{dest} = T_0 \dot{S}_{gen} = T_0(\dot{S}_{out} - \dot{S}_{in}) = T_0\left(\sum_{out} \dot{m}s - \sum_{in} \dot{m}s - \frac{\dot{Q}_{in}}{T_{b,in}} + \frac{\dot{Q}_{out}}{T_{b,out}}\right) \quad \text{(kW)}$$

For a steady-flow system

$$X_{dest} = T_0 s_{gen} = T_0\left(s_e - s_i - \frac{q_{in}}{T_{b,in}} + \frac{q_{out}}{T_{b,out}}\right) \quad \text{(kJ/kg)}$$

Steady-flow, one-inlet, one-exit

$$x_{dest} = T_0\left(\sum \frac{q_{out}}{T_{b,out}} - \sum \frac{q_{in}}{T_{b,in}}\right) \quad \text{(kJ/kg)}$$

Exergy destruction of a cycle

$$x_{dest} = T_0\left(\frac{q_{out}}{T_L} - \frac{q_{in}}{T_H}\right) \quad \text{(kJ/kg)}$$

For a cycle with heat transfer only with a source and a sink

$$\phi = (u - u_0) - T_0(s - s_0) + P_0(v - v_0) + \frac{V^2}{2} + gz$$

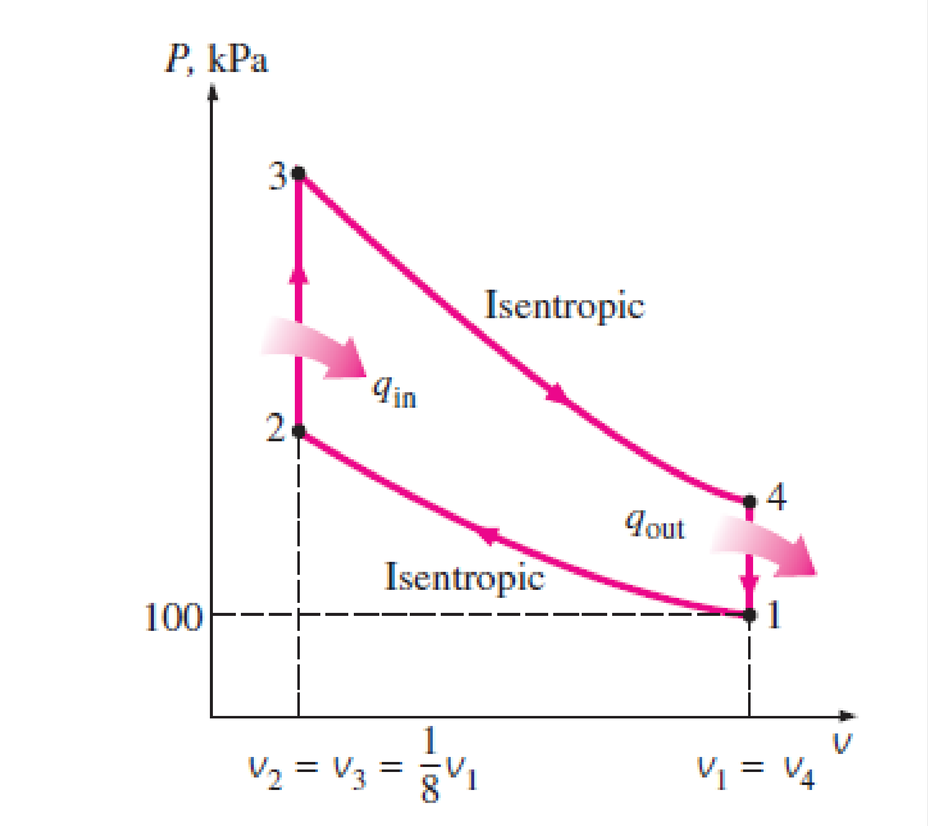
Closed system exergy

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

Stream exergy

A second-law analysis of these cycles reveals where the largest irreversibilities occur and where to start improvements.

Example 1: An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (a) the maximum temperature and pressure that occur during the cycle, (b) the net work output, (c) the thermal efficiency, and (d) the mean effective pressure for the cycle.



$$T_1 = 290 \text{ K} \rightarrow u_1 = 206.91 \text{ kJ/kg}$$
$$v_{r1} = 676.1$$

Process 1-2 (isentropic compression of an ideal gas):

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{676.1}{8} = 84.51 \rightarrow T_2 = 652.4 \text{ K}$$
$$u_2 = 475.11 \text{ kJ/kg}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$$
$$= (100 \text{ kPa})\left(\frac{652.4 \text{ K}}{290 \text{ K}}\right)(8) = 1799.7 \text{ kPa}$$

Process 2-3 (constant-volume heat addition):

$$q_{in} = u_3 - u_2$$
$$800 \text{ kJ/kg} = u_3 - 475.11 \text{ kJ/kg}$$
$$u_3 = 1275.11 \text{ kJ/kg} \rightarrow T_3 = \mathbf{1575.1 \text{ K}}$$
$$v_{r3} = 6.108$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \rightarrow P_3 = P_2\left(\frac{T_3}{T_2}\right)\left(\frac{v_2}{v_3}\right)$$
$$= (1.7997 \text{ MPa})\left(\frac{1575.1 \text{ K}}{652.4 \text{ K}}\right)(1) = \mathbf{4.345 \text{ MPa}}$$

Process 3-4 (isentropic expansion of an ideal gas):

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = r \rightarrow v_{r4} = rv_{r3} = (8)(6.108) = 48.864 \rightarrow T_4 = 795.6 \text{ K}$$

$$u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-q_{\text{out}} = u_1 - u_4 \rightarrow q_{\text{out}} = u_4 - u_1$$

$$q_{\text{out}} = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$$

Thus,

$$w_{\text{net}} = q_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 800 - 381.83 = \mathbf{418.17\ kJ/kg}$$

(*c*) The thermal efficiency of the cycle is determined from its definition:

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{418.17 \text{ kJ/kg}}{800 \text{ kJ/kg}} = \mathbf{0.523\ or\ 52.3\%}$$

Under the cold-air-standard assumptions (constant specific heat values at room temperature), the thermal efficiency would be (Eq. 9–8)

$$\eta_{\text{th,Otto}} = 1 - \frac{1}{r^{k-1}} = 1 - r^{1-k} = 1 - (8)^{1-1.4} = 0.565 \text{ or } 56.5\%$$

(*d*) The mean effective pressure is determined from its definition, Eq. 9–4:

$$\text{MEP} = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{w_{\text{net}}}{v_1 - v_1/r} = \frac{w_{\text{net}}}{v_1(1 - 1/r)}$$

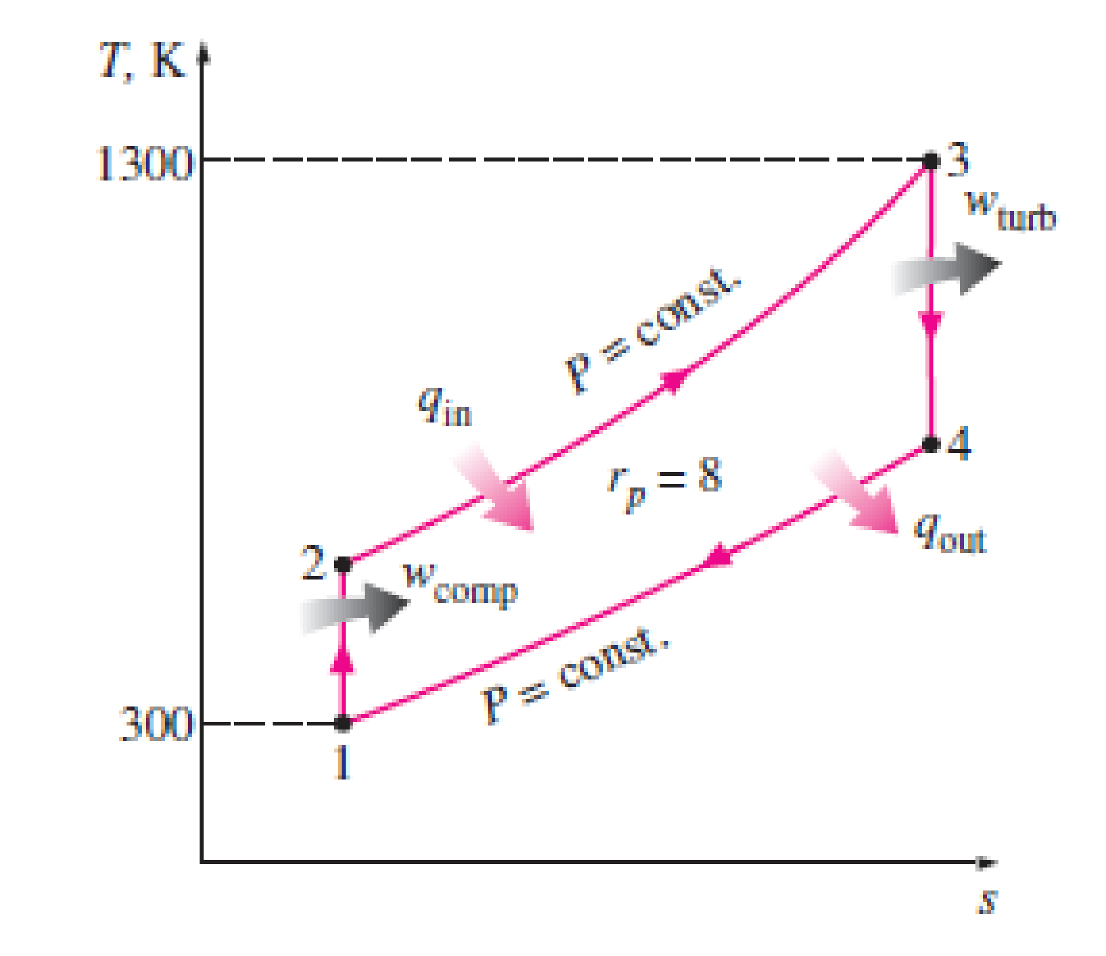
where

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\ \text{K})}{100\ \text{kPa}} = 0.832\ \text{m}^3/\text{kg}$$

Thus,

$$\text{MEP} = \frac{418.17\ \text{kJ/kg}}{(0.832\ \text{m}^3/\text{kg})(1 - \frac{1}{8})}\left(\frac{1\ \text{kPa}\cdot\text{m}^3}{1\ \text{kJ}}\right) = \mathbf{574\ kPa}$$

Example 2: A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (a) the gas temperature at the exits of the compressor and the turbine, (b) the back work ratio, and (c) the thermal efficiency.



Process 1-2 (isentropic compression of an ideal gas):

$$T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg}$$

$$P_{r1} = 1.386$$

$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09 \rightarrow T_2 = \mathbf{540\ K} \qquad \text{(at compressor exit)}$$

$$h_2 = 544.35 \text{ kJ/kg}$$

Process 3-4 (isentropic expansion of an ideal gas):

$$T_3 = 1300 \text{ K} \rightarrow h_3 = 1395.97 \text{ kJ/kg}$$

$$P_{r3} = 330.9$$

$$P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{8}\right)(330.9) = 41.36 \rightarrow T_4 = \mathbf{770\ K} \qquad \text{(at turbine exit)}$$

$$h_4 = 789.37 \text{ kJ/kg}$$

$$w_{comp,in} = h_2 - h_1 = 544.35 - 300.19 = 244.16 \text{ kJ/kg}$$

$$w_{turb,out} = h_3 - h_4 = 1395.97 - 789.37 = 606.60 \text{ kJ/kg}$$

Thus,

$$r_{bw} = \frac{w_{comp,in}}{w_{turb,out}} = \frac{244.16 \text{ kJ/kg}}{606.60 \text{ kJ/kg}} = \mathbf{0.403}$$

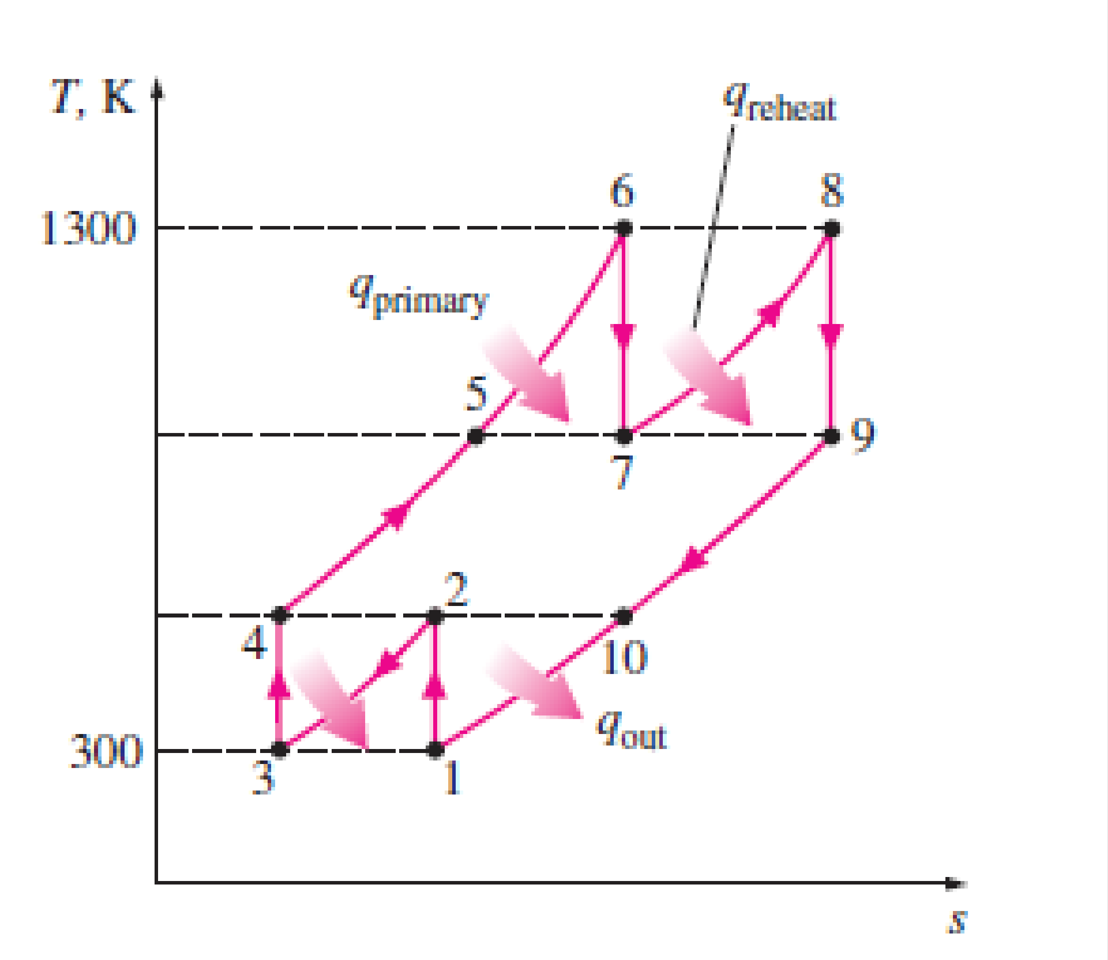
$$q_{in} = h_3 - h_2 = 1395.97 - 544.35 = 851.62 \text{ kJ/kg}$$

$$w_{net} = w_{out} - w_{in} = 606.60 - 244.16 = 362.4 \text{ kJ/kg}$$

Thus,

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{362.4 \text{ kJ/kg}}{851.62 \text{ kJ/kg}} = \mathbf{0.426 \text{ or } 42.6\%}$$

Example 3: An ideal gas-turbine cycle with two stages of compression and two stages of expansion has an overall pressure ratio of 8. Air enters each stage of the compressor at 300 K and each stage of the turbine at 1300 K. Determine the back work ratio and the thermal efficiency of this gas-turbine cycle, assuming (a) no regenerators and (b) an ideal regenerator with 100 percent effectiveness.



For two-stage compression and expansion, the work input is minimized and the work output is maximized when both stages of the compressor and the turbine have the same pressure ratio. Thus,

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{8} = 2.83 \quad \text{and} \quad \frac{P_6}{P_7} = \frac{P_8}{P_9} = \sqrt{8} = 2.83$$

Air enters each stage of the compressor at the same temperature, and each stage has the same isentropic efficiency (100 percent in this case). Therefore, the temperature (and enthalpy) of the air at the exit of each compression stage will be the same. A similar argument can be given for the turbine. Thus,

At inlets: $T_1 = T_3, \quad h_1 = h_3 \quad \text{and} \quad T_6 = T_8, \quad h_6 = h_8$

At exits: $T_2 = T_4, \quad h_2 = h_4 \quad \text{and} \quad T_7 = T_9, \quad h_7 = h_9$

Under these conditions, the work input to each stage of the compressor will be the same, and so will the work output from each stage of the turbine.

(*a*) In the absence of any regeneration, the back work ratio and the thermal efficiency are determined by using data from Table A–17 as follows:

$$T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg}$$

$$P_{r1} = 1.386$$

$$P_{r2} = \frac{P_2}{P_1} P_{r1} = \sqrt{8}(1.386) = 3.92 \rightarrow T_2 = 403.3 \text{ K}$$

$$h_2 = 404.31 \text{ kJ/kg}$$

$$T_6 = 1300 \text{ K} \rightarrow h_6 = 1395.97 \text{ kJ/kg}$$

$$P_{r6} = 330.9$$

$$P_{r7} = \frac{P_7}{P_6} P_{r6} = \frac{1}{\sqrt{8}}(330.9) = 117.0 \rightarrow T_7 = 1006.4 \text{ K}$$

$$h_7 = 1053.33 \text{ kJ/kg}$$

Then

$$w_{\text{comp,in}} = 2(w_{\text{comp,in,I}}) = 2(h_2 - h_1) = 2(404.31 - 300.19) = 208.24 \text{ kJ/kg}$$

$$w_{\text{turb,out}} = 2(w_{\text{turb,out,I}}) = 2(h_6 - h_7) = 2(1395.97 - 1053.33) = 685.28 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{turb,out}} - w_{\text{comp,in}} = 685.28 - 208.24 = 477.04 \text{ kJ/kg}$$

$$q_{\text{in}} = q_{\text{primary}} + q_{\text{reheat}} = (h_6 - h_4) + (h_8 - h_7)$$

$$= (1395.97 - 404.31) + (1395.97 - 1053.33) = 1334.30 \text{ kJ/kg}$$

Thus,

$$r_{\text{bw}} = \frac{w_{\text{comp,in}}}{w_{\text{turb,out}}} = \frac{208.24 \text{ kJ/kg}}{685.28 \text{ kJ/kg}} = \mathbf{0.304 \text{ or } 30.4\%}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{477.04 \text{ kJ/kg}}{1334.30 \text{ kJ/kg}} = \mathbf{0.358 \text{ or } 35.8\%}$$

(*b*) The addition of an ideal regenerator (no pressure drops, 100 percent effectiveness) does not affect the compressor work and the turbine work. Therefore, the net work output and the back work ratio of an ideal gas-turbine cycle are identical whether there is a regenerator or not. A regenerator, however, reduces the heat input requirements by preheating the air leaving the compressor, using the hot exhaust gases. In an ideal regenerator, the compressed air is heated to the turbine exit temperature T_9 before it enters the combustion chamber. Thus, under the air-standard assumptions, $h_5 = h_7 = h_9$.

The heat input and the thermal efficiency in this case are

$$q_{in} = q_{primary} + q_{reheat} = (h_6 - h_5) + (h_8 - h_7)$$
$$= (1395.97 - 1053.33) + (1395.97 - 1053.33) = 685.28 \text{ kJ/kg}$$

and

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{477.04 \text{ kJ/kg}}{685.28 \text{ kJ/kg}} = \mathbf{0.696 \text{ or } 69.6\%}$$