

# Thermodynamics: An Engineering Approach

Seventh Edition

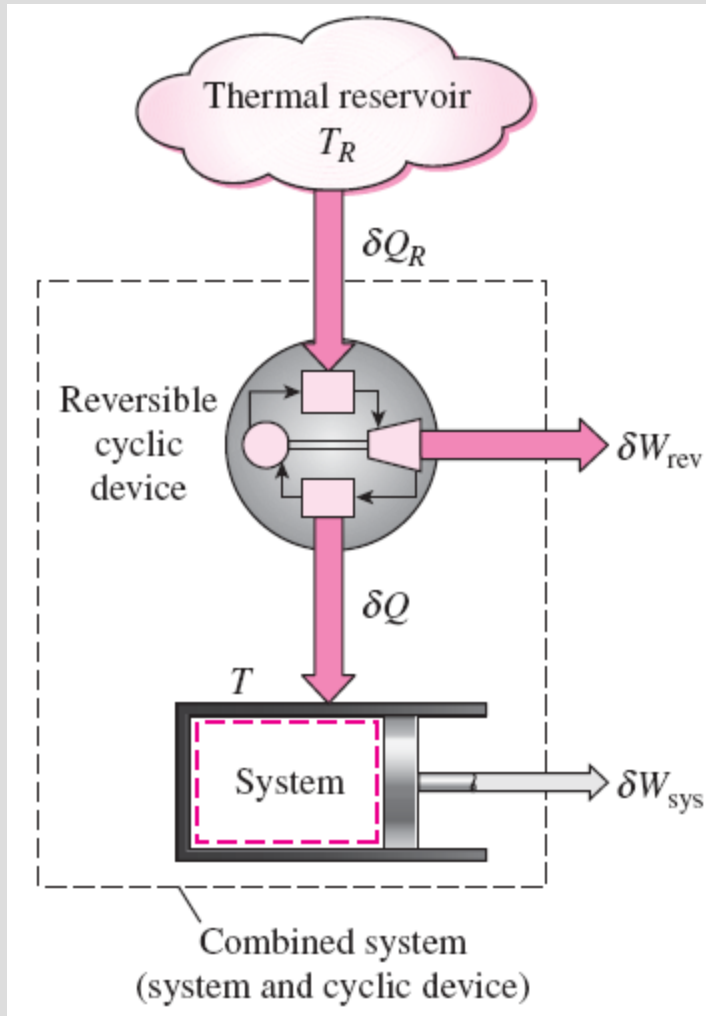
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## Chapter 7

# ENTROPY

# ENTROPY



The system considered in the development of the Clausius inequality.

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\delta W_C = \delta Q_R - dE_C$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

$$\delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

$$W_C = T_R \oint \frac{\delta Q}{T}$$

$$\oint \frac{\delta Q}{T} \leq 0$$

Clausius inequality

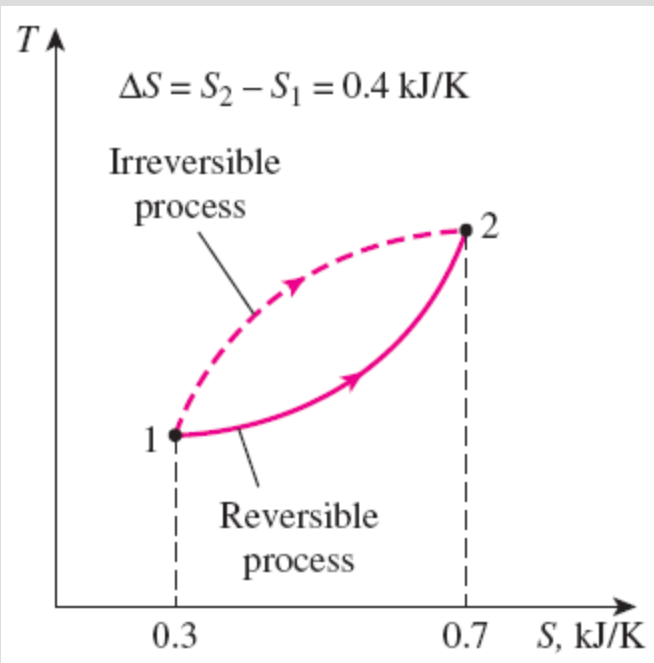
$$\oint \left( \frac{\delta Q}{T} \right)_{int rev} = 0$$

$$dS = \left( \frac{\delta Q}{T} \right)_{int rev} \quad (\text{kJ/K})$$

Formal definition of entropy

$$\Delta S = S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right)_{int rev}$$

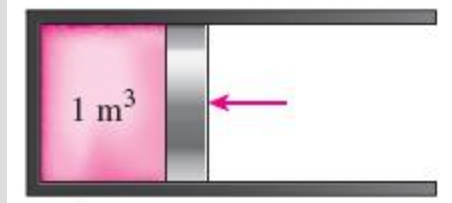
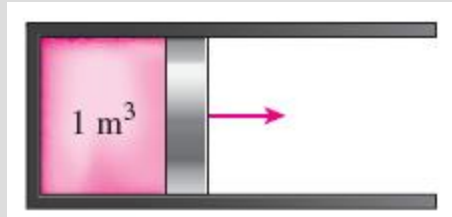
The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.



$$\oint \left( \frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

A quantity whose cyclic integral is zero (i.e., a property like volume)

Entropy is an extensive property of a system.



$$\oint dV = \Delta V_{\text{cycle}} = 0$$

The entropy change between two specified states is the same whether the process is reversible or irreversible.

The net change in volume (a property) during a cycle is always zero.

### A Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left( \frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}}$$

$$\Delta S = \frac{Q}{T_0}$$

This equation is particularly useful for determining the entropy changes of thermal energy reservoirs.

### EXAMPLE 7-1 Entropy Change during an Isothermal Process

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant-pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.

**Solution** Heat is transferred to a liquid–vapor mixture of water in a piston–cylinder device at constant pressure. The entropy change of water is to be determined.

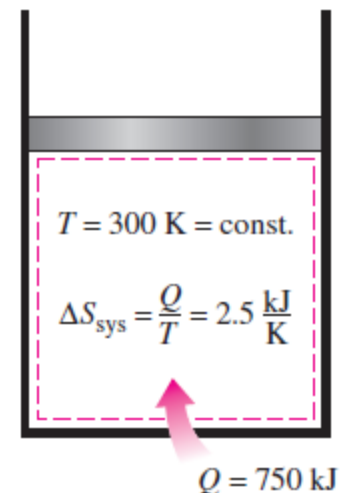
**Assumptions** No irreversibilities occur within the system boundaries during the process.

**Analysis** We take the *entire water* (liquid + vapor) in the cylinder as the system (Fig. 7-4). This is a *closed system* since no mass crosses the system boundary during the process. We note that the temperature of the system remains constant at 300 K during this process since the temperature of a pure substance remains constant at the saturation value during a phase-change process at constant pressure.

The system undergoes an internally reversible, isothermal process, and thus its entropy change can be determined directly from Eq. 7-6 to be

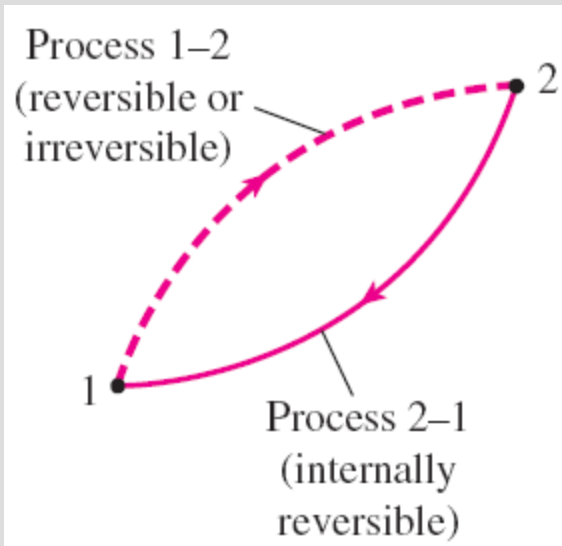
$$\Delta S_{\text{sys, isothermal}} = \frac{Q}{T_{\text{sys}}} = \frac{750 \text{ kJ}}{300 \text{ K}} = 2.5 \text{ kJ/K}$$

**Discussion** Note that the entropy change of the system is positive, as expected, since heat transfer is *to* the system.



**FIGURE 7-4**  
Schematic for Example 7-1.

# THE INCREASE OF ENTROPY PRINCIPLE



A cycle composed of a reversible and an irreversible process.

$$\oint \frac{\delta Q}{T} \leq 0 \quad \int_1^2 \frac{\delta Q}{T} + \int_2^1 \left( \frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0 \quad S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

$$dS \geq \frac{\delta Q}{T}$$

The equality holds for an internally reversible process and the inequality for an irreversible process.

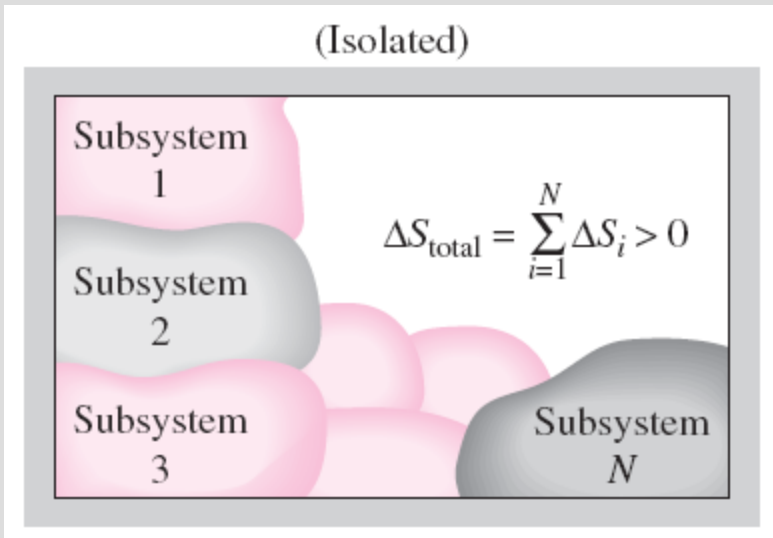
$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation  $S_{\text{gen}}$  is always a *positive* quantity or zero.

Can the entropy of a system during a process decrease?

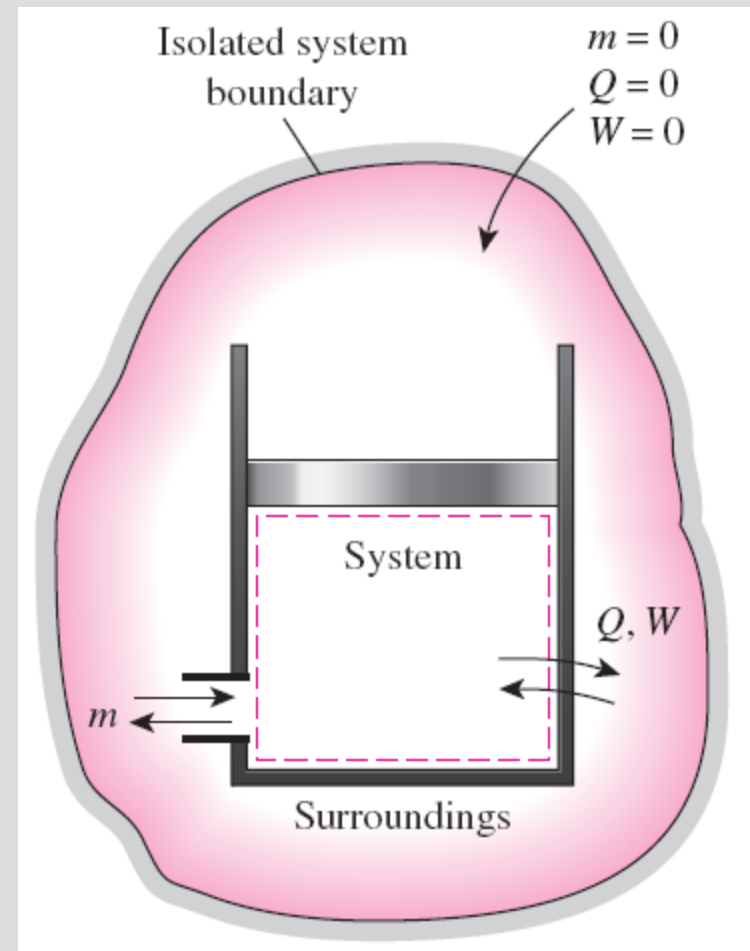


The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{\text{isolated}} \geq 0$$

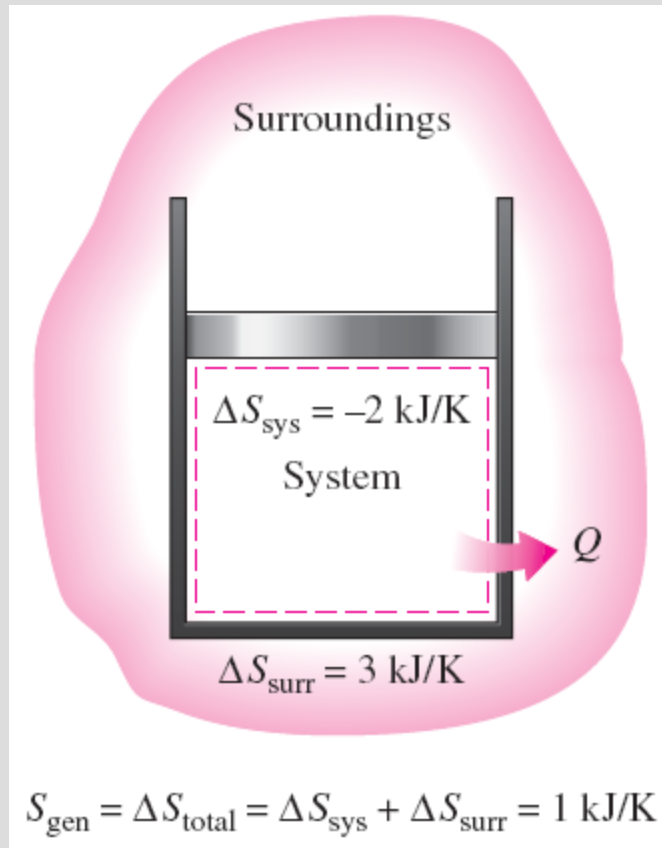
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases} \quad \text{The increase of entropy principle}$$



A system and its surroundings form an isolated system.

# Some Remarks about Entropy

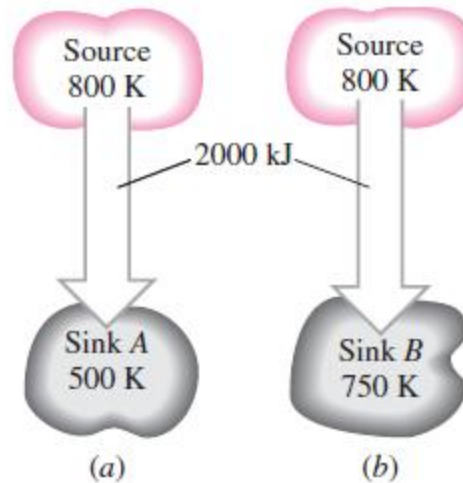


The entropy change of a system can be negative, but the entropy generation cannot.

1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is,  $S_{\text{gen}} \geq 0$ . A process that violates this principle is impossible.
2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

**EXAMPLE 7-2** Entropy Generation during Heat Transfer Processes

A heat source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K and (b) 750 K. Determine which heat transfer process is more irreversible.



**FIGURE 7-9**

Schematic for Example 7-2.



The entropy change for each reservoir can be determined from Eq. 7–6 since each reservoir undergoes an internally reversible, isothermal process.

(a) For the heat transfer process to a sink at 500 K:

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000 \text{ kJ}}{800 \text{ K}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000 \text{ kJ}}{500 \text{ K}} = +4.0 \text{ kJ/K}$$

and

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0) \text{ kJ/K} = \mathbf{1.5 \text{ kJ/K}}$$

Therefore, 1.5 kJ/K of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.

(b) Repeating the calculations in part (a) for a sink temperature of 750 K, we obtain

$$\Delta S_{\text{source}} = -2.5 \text{ kJ/k}$$

$$\Delta S_{\text{sink}} = +2.7 \text{ kJ/K}$$

and

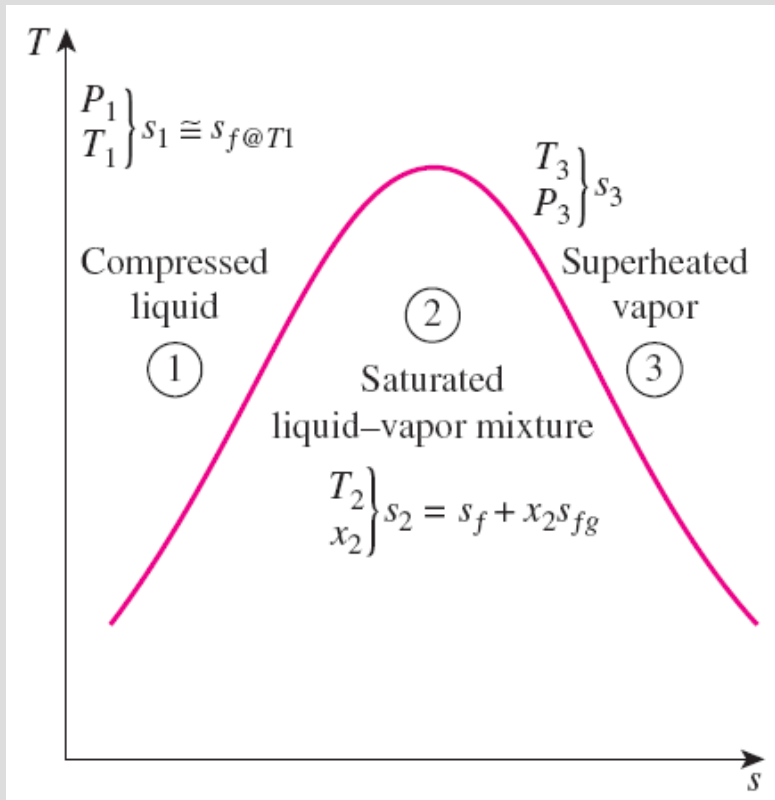
$$S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7) \text{ kJ/K} = \mathbf{0.2 \text{ kJ/K}}$$

The total entropy change for the process in part (b) is smaller, and therefore it is less irreversible. This is expected since the process in (b) involves a smaller temperature difference and thus a smaller irreversibility.

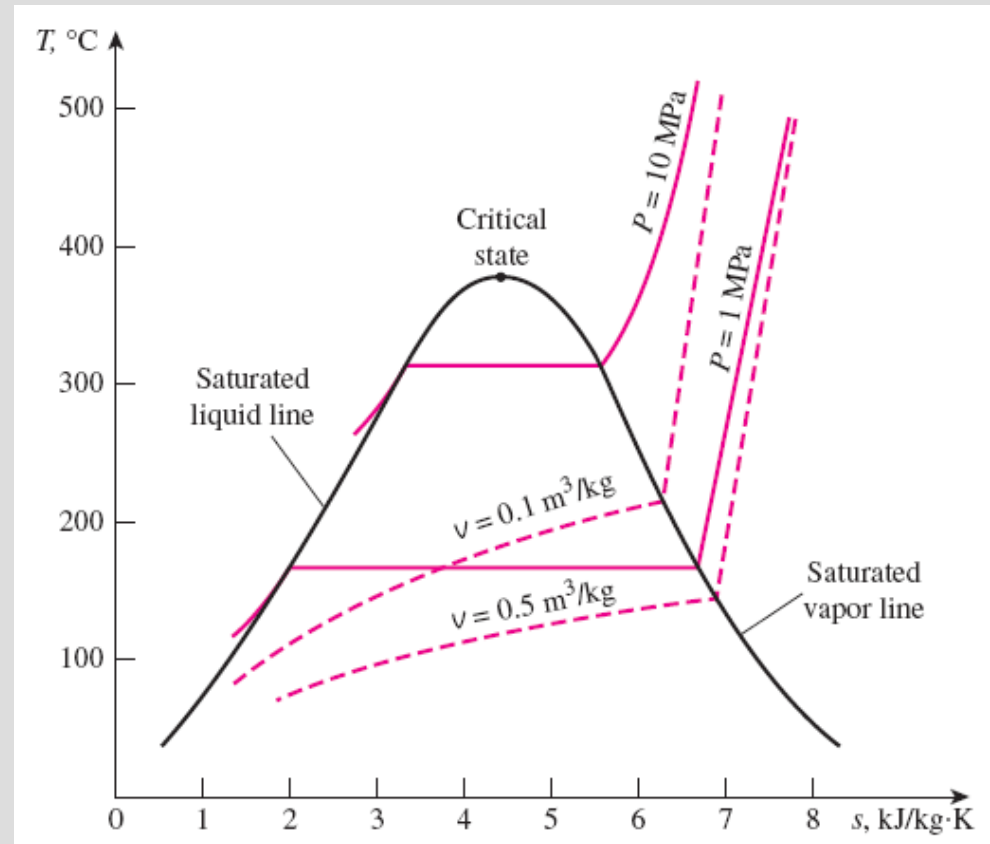
**Discussion** The irreversibilities associated with both processes could be eliminated by operating a Carnot heat engine between the source and the sink. For this case it can be shown that  $\Delta S_{\text{total}} = 0$ .

# ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



The entropy of a pure substance is determined from the tables (like other properties).



Schematic of the  $T-s$  diagram for water.

Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad (\text{kJ/K})$$

### EXAMPLE 7-3 Entropy Change of a Substance in a Tank

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.

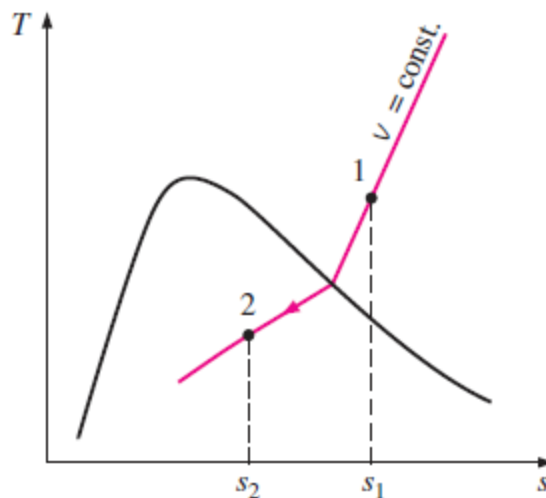
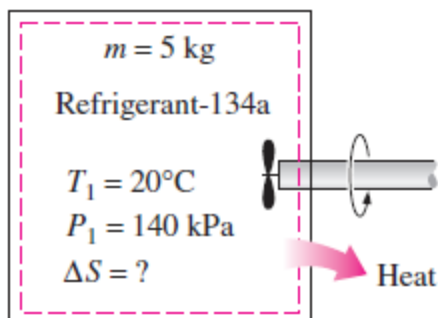


FIGURE 7-12

Schematic and  $T$ - $s$  diagram for Example 7-3.

**Solution** The refrigerant in a rigid tank is cooled while being stirred. The entropy change of the refrigerant is to be determined.

**Assumptions** The volume of the tank is constant and thus  $v_2 = v_1$ .

**Analysis** We take the refrigerant in the tank as the *system* (Fig. 7–12). This is a *closed system* since no mass crosses the system boundary during the process. We note that the change in entropy of a substance during a process is simply the difference between the entropy values at the final and initial states. The initial state of the refrigerant is completely specified.

Recognizing that the specific volume remains constant during this process, the properties of the refrigerant at both states are

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 140 \text{ kPa} \\ T_1 = 20^\circ\text{C} \end{array} \right\} \quad \left. \begin{array}{l} s_1 = 1.0624 \text{ kJ/kg} \cdot \text{K} \\ v_1 = 0.16544 \text{ m}^3/\text{kg} \end{array} \right\}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 100 \text{ kPa} \\ (v_2 = v_1) \end{array} \right\} \quad \left. \begin{array}{l} v_f = 0.0007259 \text{ m}^3/\text{kg} \\ v_g = 0.19254 \text{ m}^3/\text{kg} \end{array} \right\}$$

The refrigerant is a saturated liquid–vapor mixture at the final state since  $v_f < v_2 < v_g$  at 100 kPa pressure. Therefore, we need to determine the quality first:

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.16544 - 0.0007259}{0.19254 - 0.0007259} = 0.859$$

Thus,

$$s_2 = s_f + x_2 s_{fg} = 0.07188 + (0.859)(0.87995) = 0.8278 \text{ kJ/kg} \cdot \text{K}$$

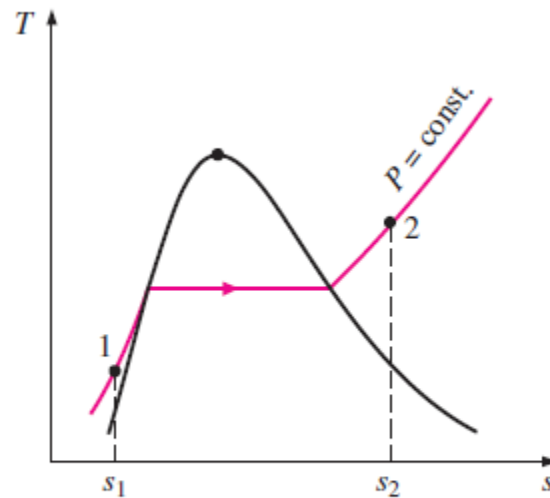
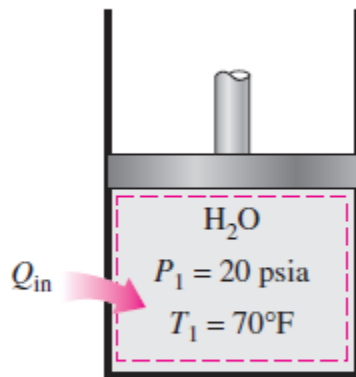
Then the entropy change of the refrigerant during this process is

$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (5 \text{ kg})(0.8278 - 1.0624) \text{ kJ/kg} \cdot \text{K} \\ &= \mathbf{-1.173 \text{ kJ/K}} \end{aligned}$$

**Discussion** The negative sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the *entropy generation*  $S_{\text{gen}}$  that cannot be negative.

### EXAMPLE 7-4 Entropy Change during a Constant-Pressure Process

A piston–cylinder device initially contains 3 lbm of liquid water at 20 psia and 70°F. The water is now heated at constant pressure by the addition of 3450 Btu of heat. Determine the entropy change of the water during this process.



**Solution** Liquid water in a piston–cylinder device is heated at constant pressure. The entropy change of water is to be determined.

**Assumptions** 1 The tank is stationary and thus the kinetic and potential energy changes are zero,  $\Delta KE = \Delta PE = 0$ . 2 The process is quasi-equilibrium. 3 The pressure remains constant during the process and thus  $P_2 = P_1$ .

**Analysis** We take the water in the cylinder as the *system* (Fig. 7–13). This is a *closed system* since no mass crosses the system boundary during the process. We note that a piston–cylinder device typically involves a moving boundary and thus boundary work  $W_b$ . Also, heat is transferred to the system.

Water exists as a compressed liquid at the initial state since its pressure is greater than the saturation pressure of 0.3632 psia at 70°F. By approximating the compressed liquid as a saturated liquid at the given temperature, the properties at the initial state are

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 20 \text{ psia} \\ T_1 = 70^\circ\text{F} \end{array} \right\} \quad \begin{array}{l} s_1 \cong s_{f@70^\circ\text{F}} = 0.07459 \text{ Btu/lbm} \cdot \text{R} \\ h_1 \cong h_{f@70^\circ\text{F}} = 38.08 \text{ Btu/lbm} \end{array}$$

At the final state, the pressure is still 20 psia, but we need one more property to fix the state. This property is determined from the energy balance,

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{\text{system}}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}}$$

$$Q_{\text{in}} - W_b = \Delta U$$

$$Q_{\text{in}} = \Delta H = m(h_2 - h_1)$$

$$3450 \text{ Btu} = (3 \text{ lbm})(h_2 - 38.08 \text{ Btu/lbm})$$

$$h_2 = 1188.1 \text{ Btu/lbm}$$

since  $\Delta U + W_b = \Delta H$  for a constant-pressure quasi-equilibrium process. Then,

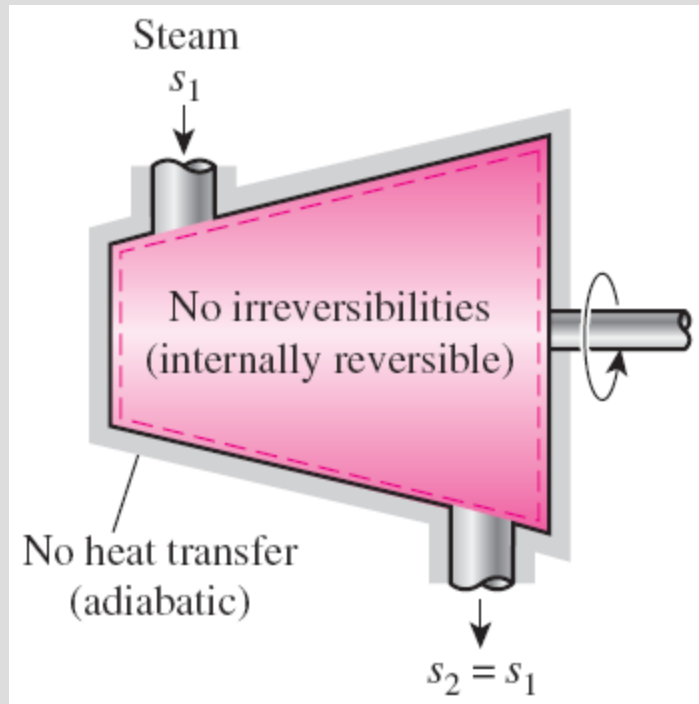
$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 20 \text{ psia} \\ h_2 = 1188.1 \text{ Btu/lbm} \end{array} \right\} \quad \begin{array}{l} s_2 = 1.7761 \text{ Btu/lbm} \cdot \text{R} \\ \text{(Table A-6E, interpolation)} \end{array}$$

Therefore, the entropy change of water during this process is

$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (3 \text{ lbm})(1.7761 - 0.07459) \text{ Btu/lbm} \cdot \text{R} \\ &= \mathbf{5.105 \text{ Btu/R}} \end{aligned}$$

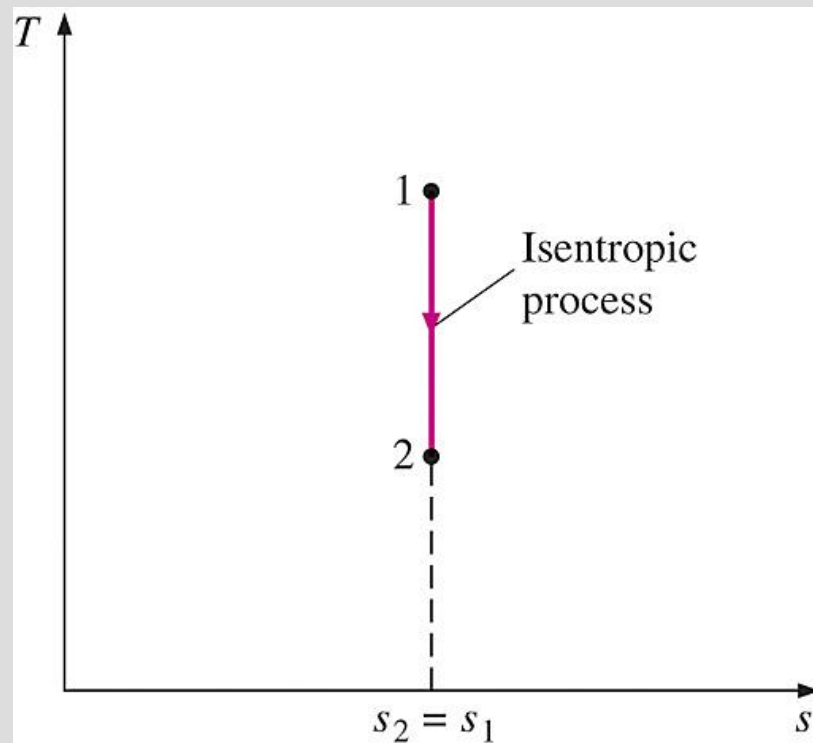
# ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.



During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

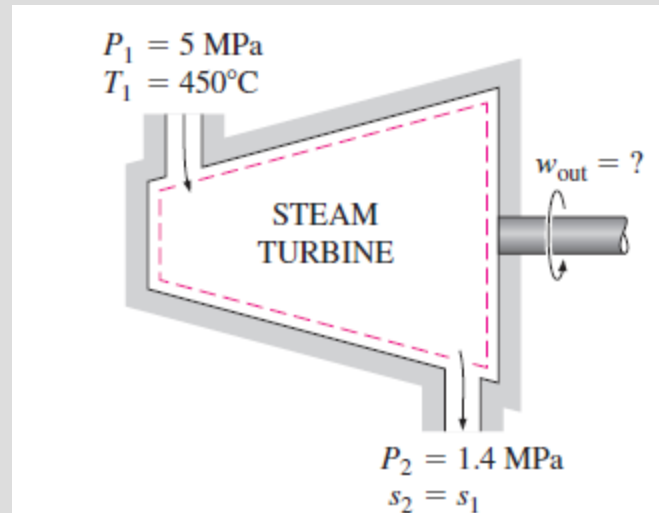
$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad (\text{kJ/kg} \cdot \text{K})$$



The isentropic process appears as a *vertical* line segment on a  $T$ - $s$  diagram.

### EXAMPLE 7-5 Isentropic Expansion of Steam in a Turbine

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.



**FIGURE 7-15**

Schematic and  $T$ - $s$  diagram for Example 7-5.



The power output of the turbine is determined from the rate form of the energy balance,

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\frac{dE_{\text{system}}}{dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0, \text{ ke} \cong \text{pe} \cong 0)$$

$$\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)$$

The inlet state is completely specified since two properties are given. But only one property (pressure) is given at the final state, and we need one more property to fix it. The second property comes from the observation that the process is reversible and adiabatic, and thus isentropic. Therefore,  $s_2 = s_1$ , and

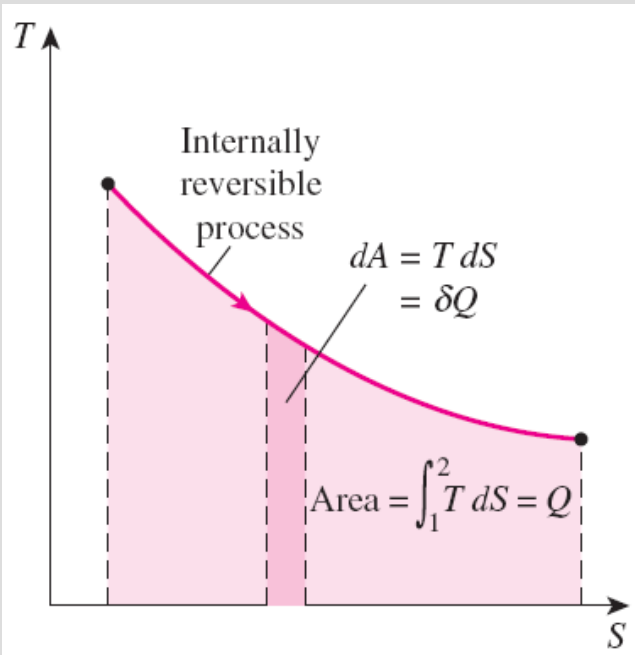
$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array} \right\} \quad \left. \begin{array}{l} h_1 = 3317.2 \text{ kJ/kg} \\ s_1 = 6.8210 \text{ kJ/kg} \cdot \text{K} \end{array} \right\}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} \quad \left. \begin{array}{l} h_2 = 2967.4 \text{ kJ/kg} \end{array} \right\}$$

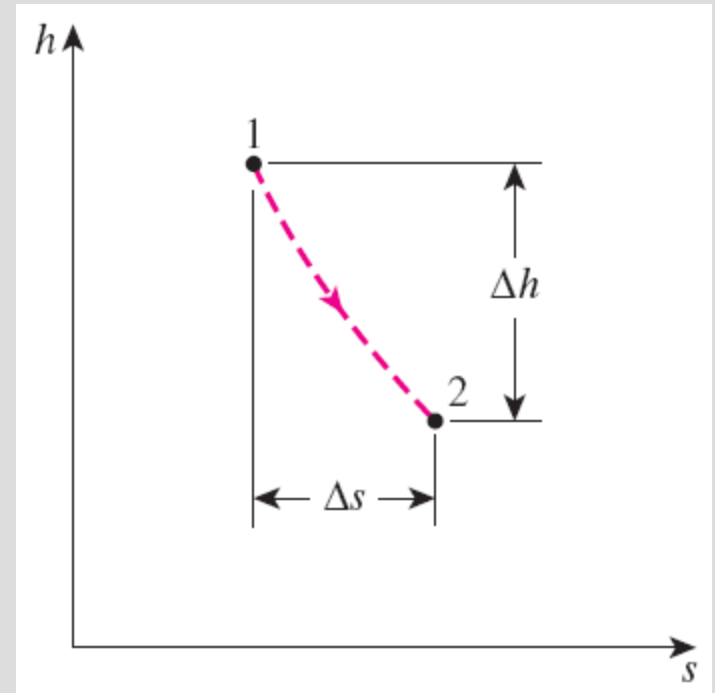
Then the work output of the turbine per unit mass of the steam becomes

$$w_{\text{out}} = h_1 - h_2 = 3317.2 - 2967.4 = \mathbf{349.8 \text{ kJ/kg}}$$

# PROPERTY DIAGRAMS INVOLVING ENTROPY



On a  $T$ - $S$  diagram, the area under the process curve represents the heat transfer for internally reversible processes.



For adiabatic steady-flow devices, the vertical distance  $\Delta h$  on an  $h$ - $s$  diagram is a measure of work, and the horizontal distance  $\Delta s$  is a measure of irreversibilities.

$$\delta Q_{\text{int rev}} = T dS$$

$$Q_{\text{int rev}} = \int_1^2 T dS$$

$$\delta q_{\text{int rev}} = T ds$$

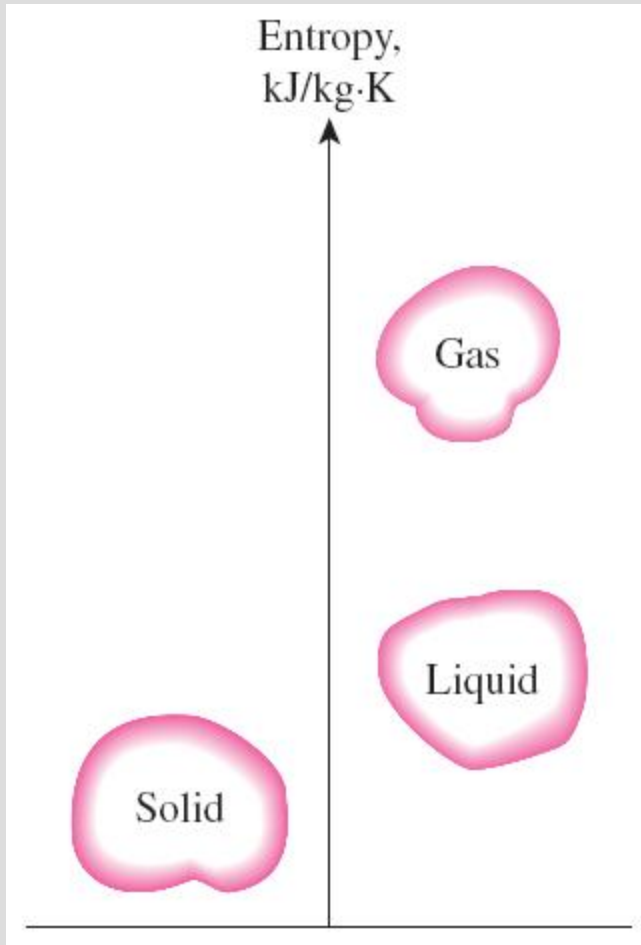
$$q_{\text{int rev}} = \int_1^2 T ds$$

$$Q_{\text{int rev}} = T_0 \Delta S$$

$$q_{\text{int rev}} = T_0 \Delta s$$

**Mollier diagram:** The  $h$ - $s$  diagram

# WHAT IS ENTROPY?



The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

Boltzmann relation

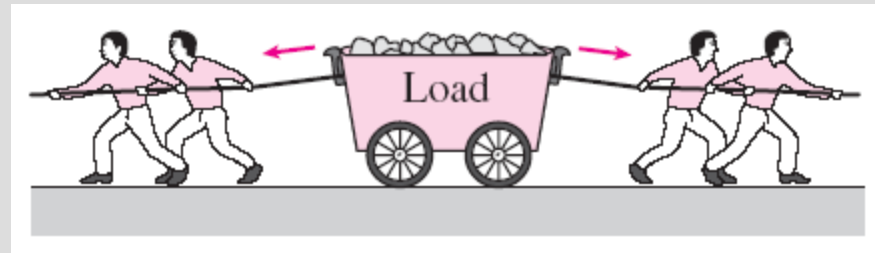
$$S = k \ln p$$

$$k = 1.3806 \times 10^{-23} \text{ J/K}$$

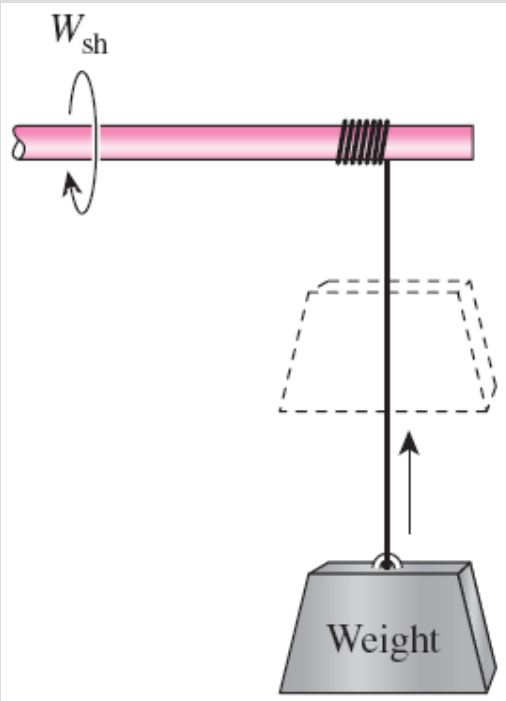
Pure crystal  
 $T = 0 \text{ K}$   
Entropy = 0



A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (**the third law of thermodynamics**).



Disorganized energy does not create much useful effect, no matter how large it is.

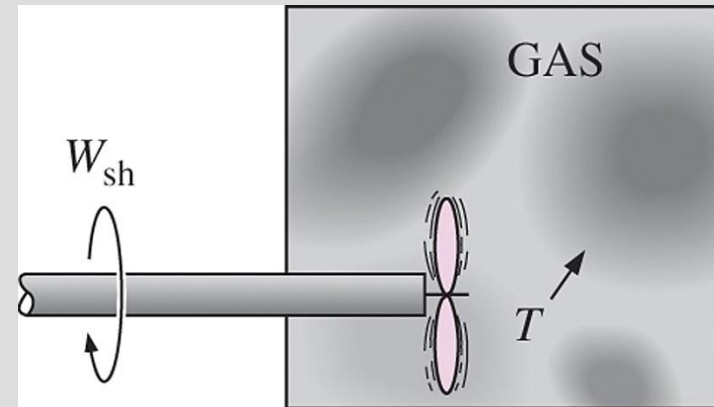


In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.

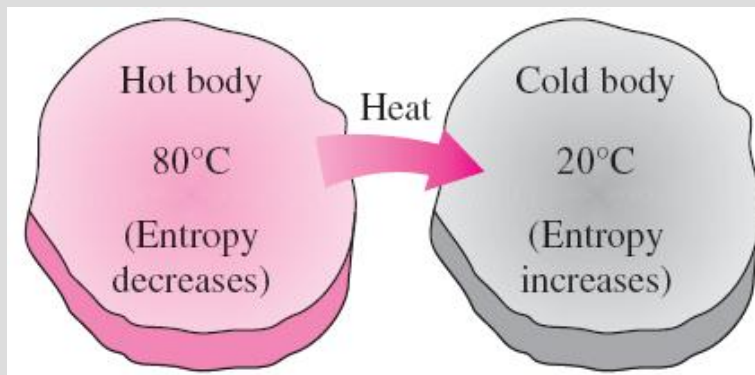


**FIGURE 7-26**

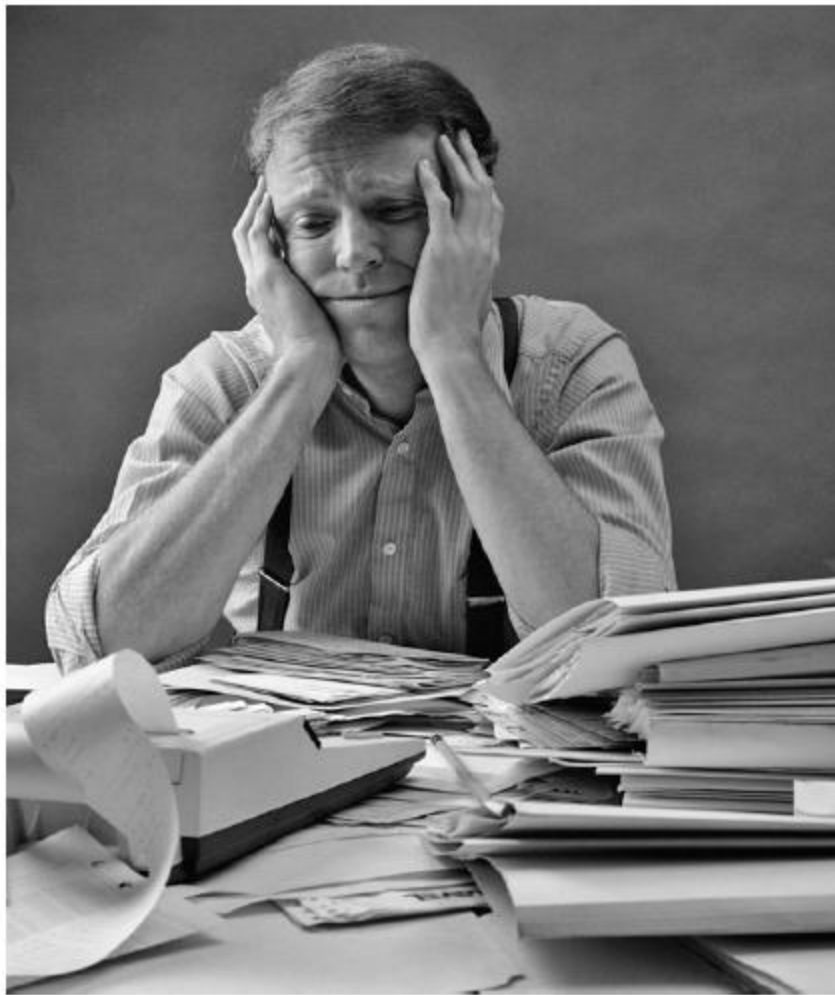
The use of entropy (disorganization, uncertainty) is not limited to thermodynamics.



The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.



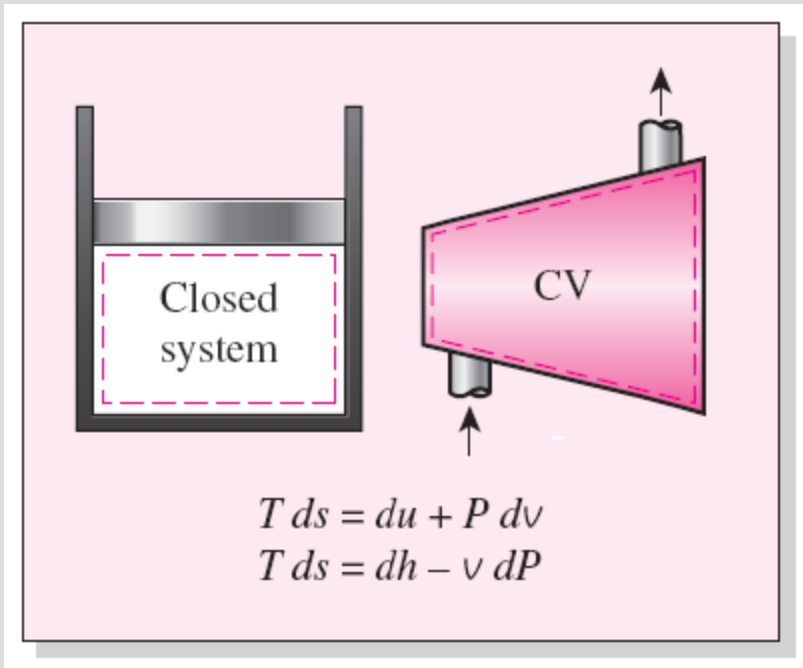
During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)



**FIGURE 7-27**

As in mechanical systems, friction in the workplace is bound to generate entropy and reduce performance.

# THE $T ds$ RELATIONS



The  $T ds$  relations are valid for both reversible and irreversible processes and for both closed and open systems.

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = T dS$$

$$\delta W_{\text{int rev,out}} = P dV$$

$$T dS = dU + P dV \quad (\text{kJ})$$

$$T ds = du + P dv \quad (\text{kJ/kg})$$

the first  $T ds$ , or *Gibbs equation*

$$h = u + Pv$$

$$\left. \begin{array}{l} dh = du + P dv + v dP \\ T ds = du + P dv \end{array} \right\} T ds = dh - v dP$$

the second  $T ds$  equation

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

Differential changes in entropy in terms of other properties

# ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

Since  $dv \cong 0$  for liquids and solids

$$ds = \frac{du}{T} = \frac{c dT}{T}$$

since  $c_p = c_v = c$  and  $du = c dT$

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

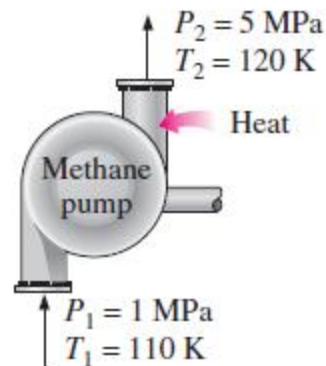
$$\text{Liquids, solids:} \quad s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{\text{avg}} \ln \frac{T_2}{T_1} \quad (\text{kJ/kg} \cdot \text{K})$$

For an isentropic process of an incompressible substance

$$\text{Isentropic:} \quad s_2 - s_1 = c_{\text{avg}} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

### EXAMPLE 7-7 Effect of Density of a Liquid on Entropy

Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or  $-82^{\circ}\text{C}$ ), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7-1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (*a*) using tabulated properties and (*b*) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?



**FIGURE 7-29**

Schematic for Example 7-7.



**Solution** Liquid methane undergoes a process between two specified states. The entropy change of methane is to be determined by using actual data and by assuming methane to be incompressible.

**Analysis** (a) We consider a unit mass of liquid methane (Fig. 7–29). The properties of the methane at the initial and final states are

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 1 \text{ MPa} \\ T_1 = 110 \text{ K} \end{array} \right\} \quad \left. \begin{array}{l} s_1 = 4.875 \text{ kJ/kg} \cdot \text{K} \\ c_{p1} = 3.471 \text{ kJ/kg} \cdot \text{K} \end{array} \right\}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 5 \text{ MPa} \\ T_2 = 120 \text{ K} \end{array} \right\} \quad \left. \begin{array}{l} s_2 = 5.145 \text{ kJ/kg} \cdot \text{K} \\ c_{p2} = 3.486 \text{ kJ/kg} \cdot \text{K} \end{array} \right\}$$

Therefore,

$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = \mathbf{0.270 \text{ kJ/kg} \cdot \text{K}}$$

(b) Approximating liquid methane as an incompressible substance, its entropy change is determined to be

$$\Delta s = c_{\text{avg}} \ln \frac{T_2}{T_1} = (3.4785 \text{ kJ/kg} \cdot \text{K}) \ln \frac{120 \text{ K}}{110 \text{ K}} = \mathbf{0.303 \text{ kJ/kg} \cdot \text{K}}$$

since

$$c_{\text{avg}} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785 \text{ kJ/kg} \cdot \text{K}$$

Therefore, the error involved in approximating liquid methane as an incompressible substance is

$$\text{Error} = \frac{|\Delta s_{\text{actual}} - \Delta s_{\text{ideal}}|}{\Delta s_{\text{actual}}} = \frac{|0.270 - 0.303|}{0.270} = \mathbf{0.122 \text{ (or 12.2\%)}}$$

**Discussion** This result is not surprising since the density of liquid methane changes during this process from 425.8 to 415.2 kg/m<sup>3</sup> (about 3 percent), which makes us question the validity of the incompressible substance assumption. Still, this assumption enables us to obtain reasonably accurate results with less effort, which proves to be very convenient in the absence of compressed liquid data.

### EXAMPLE 7–8 Economics of Replacing a Valve by a Turbine

A cryogenic manufacturing facility handles liquid methane at 115 K and 5 MPa at a rate of 0.280 m<sup>3</sup>/s . A process requires dropping the pressure of liquid methane to 1 MPa, which is done by throttling the liquid methane by passing it through a flow resistance such as a valve. A recently hired engineer proposes to replace the throttling valve by a turbine in order to produce power while dropping the pressure to 1 MPa. Using data from Table 7–1, determine the maximum amount of power that can be produced by such a turbine. Also, determine how much this turbine will save the facility from electricity usage costs per year if the turbine operates continuously (8760 h/yr) and the facility pays \$0.075/kWh for electricity.

**TABLE 7–1**

Properties of liquid methane

Temp., $T$ , K	Pressure, $P$ , MPa	Density, $\rho$ , kg/m <sup>3</sup>	Enthalpy, $h$ , kJ/kg	Entropy, $s$ , kJ/kg · K	Specific heat, $c_p$ , kJ/kg · K
110	0.5	425.3	208.3	4.878	3.476
	1.0	425.8	209.0	4.875	3.471
	2.0	426.6	210.5	4.867	3.460
	5.0	429.1	215.0	4.844	3.432
120	0.5	410.4	243.4	5.185	3.551
	1.0	411.0	244.1	5.180	3.543
	2.0	412.0	245.4	5.171	3.528
	5.0	415.2	249.6	5.145	3.486

**Solution** Liquid methane is expanded in a turbine to a specified pressure at a specified rate. The maximum power that this turbine can produce and the amount of money it can save per year are to be determined.

**Assumptions** 1 This is a steady-flow process since there is no change with time at any point and thus  $\Delta m_{CV} = 0$ ,  $\Delta E_{CV} = 0$ , and  $\Delta S_{CV} = 0$ . 2 The turbine is adiabatic and thus there is no heat transfer. 3 The process is reversible. 4 Kinetic and potential energies are negligible.

**Analysis** We take the *turbine* as the system (Fig. 7–30). This is a *control volume* since mass crosses the system boundary during the process. We note that there is only one inlet and one exit and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ .

The assumptions above are reasonable since a turbine is normally well insulated and it must involve no irreversibilities for best performance and thus *maximum* power production. Therefore, the process through the turbine must be *reversible adiabatic* or *isentropic*. Then,  $s_2 = s_1$  and

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 115 \text{ K} \end{array} \right\} \begin{array}{l} h_1 = 232.3 \text{ kJ/kg} \\ s_1 = 4.9945 \text{ kJ/kg} \cdot \text{K} \\ \rho_1 = 422.15 \text{ kg/s} \end{array}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} h_2 = 222.8 \text{ kJ/kg}$$

Also, the mass flow rate of liquid methane is

$$\dot{m} = \rho_1 \dot{V}_1 = (422.15 \text{ kg/m}^3)(0.280 \text{ m}^3/\text{s}) = 118.2 \text{ kg/s}$$

Then the power output of the turbine is determined from the rate form of the energy balance to be

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{dE_{\text{system}}/dt}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{\text{out}} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0, \text{ ke} \cong \text{pe} \cong 0)$$

$$\begin{aligned} \dot{W}_{\text{out}} &= \dot{m}(h_1 - h_2) \\ &= (118.2 \text{ kg/s})(232.3 - 222.8) \text{ kJ/kg} \\ &= \mathbf{1123 \text{ kW}} \end{aligned}$$

For continuous operation ( $365 \times 24 = 8760$  h), the amount of power produced per year is

$$\begin{aligned} \text{Annual power production} &= \dot{W}_{\text{out}} \times \Delta t = (1123 \text{ kW})(8760 \text{ h/yr}) \\ &= 0.9837 \times 10^7 \text{ kWh/yr} \end{aligned}$$

At \$0.075/kWh, the amount of money this turbine can save the facility is

$$\begin{aligned} \text{Annual power savings} &= (\text{Annual power production})(\text{Unit cost of power}) \\ &= (0.9837 \times 10^7 \text{ kWh/yr})(\$0.075/\text{kWh}) \\ &= \mathbf{\$737,800/\text{yr}} \end{aligned}$$

That is, this turbine can save the facility \$737,800 a year by simply taking advantage of the potential that is currently being wasted by a throttling valve, and the engineer who made this observation should be rewarded.

**Discussion** This example shows the importance of the property entropy since it enabled us to quantify the work potential that is being wasted. In practice, the turbine will not be isentropic, and thus the power produced will be less. The analysis above gave us the upper limit. An actual turbine-generator assembly can utilize about 80 percent of the potential and produce more than 900 kW of power while saving the facility more than \$600,000 a year.

It can also be shown that the temperature of methane drops to 113.9 K (a drop of 1.1 K) during the isentropic expansion process in the turbine instead of remaining constant at 115 K as would be the case if methane were assumed to be an incompressible substance. The temperature of methane would rise to 116.6 K (a rise of 1.6 K) during the throttling process.

# THE ENTROPY CHANGE OF IDEAL GASES

From the first  $T ds$  relation

$$ds = \frac{du}{T} + \frac{P dv}{T} \quad \begin{matrix} du = c_v dT \\ P = RT/v \end{matrix}$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second  $T ds$  relation

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

$$dh = c_p dT \quad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} P v &= RT \\ du &= C_v dT \\ dh &= C_p dT \end{aligned}$$

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# Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$



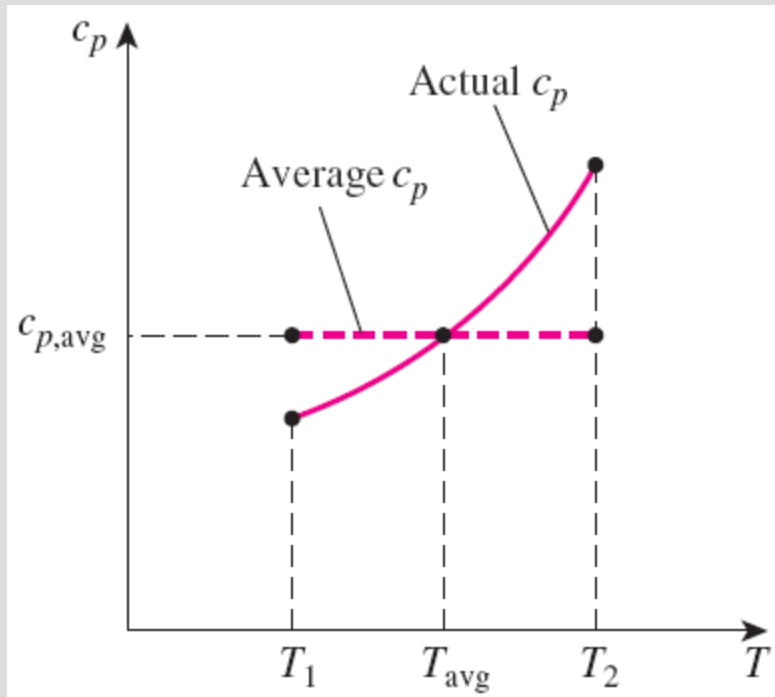
$$s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$



$$s_2 - s_1 = c_{p,avg} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

(kJ/kg · K)



Entropy change of an ideal gas on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,avg} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,avg} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

# Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function  $s^\circ$  as

$$s^\circ = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_1^2 c_p(T) \frac{dT}{T} = s_2^\circ - s_1^\circ$$

On a unit-mass basis

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$

On a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

<u>T, K</u>	<u>s<sup>°</sup>, kJ/kg·K</u>
·	·
·	·
·	·
300	1.70203
310	1.73498
320	1.76690
·	·
·	·
·	·
(Table A-17)	

The entropy of an ideal gas depends on both  $T$  and  $P$ . The function  $s^\circ$  represents only the temperature-dependent part of entropy.

Air

$$T_1 = 290 \text{ K}$$
$$T_2 = 330 \text{ K}$$
$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$
$$= -0.3844 \text{ kJ/kg}\cdot\text{K}$$
  
$$s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$
$$= -0.3842 \text{ kJ/kg}\cdot\text{K}$$

**FIGURE 7–35**

For small temperature differences, the exact and approximate relations for entropy changes of ideal gases give almost identical results.



# Isentropic Processes of Ideal Gases

## Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,avg} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

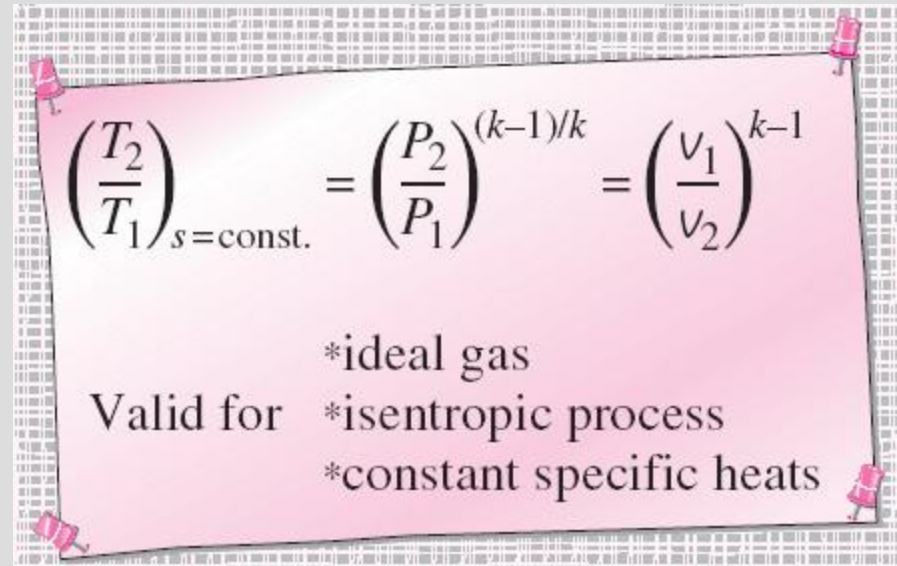
$$\ln \frac{T_2}{T_1} = \ln \left( \frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, k = c_p/c_v$$

and thus  $R/c_v = k - 1$

$$\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{v_1}{v_2} \right)^{k-1}$$

$$\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k} \quad \left( \frac{P_2}{P_1} \right)_{s=\text{const.}} = \left( \frac{v_1}{v_2} \right)^k$$


$$\left( \frac{T_2}{T_1} \right)_{s=\text{const.}} = \left( \frac{P_2}{P_1} \right)^{(k-1)/k} = \left( \frac{v_1}{v_2} \right)^{k-1}$$

\*ideal gas  
Valid for \*isentropic process  
\*constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

$$TV^{k-1} = \text{constant}$$
$$TP^{(1-k)/k} = \text{constant}$$
$$PV^k = \text{constant}$$

# Isentropic Processes of Ideal Gases

## Variable Specific Heats (Exact Analysis)

$$0 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \longrightarrow s_2^\circ = s_1^\circ + R \ln \frac{P_2}{P_1}$$

## Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^\circ - s_1^\circ}{R} \quad \text{exp}(s^\circ/R) \text{ is the relative pressure } P_r$$

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ/R)}{\exp(s_1^\circ/R)}$$

$$\left( \frac{P_2}{P_1} \right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \longrightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

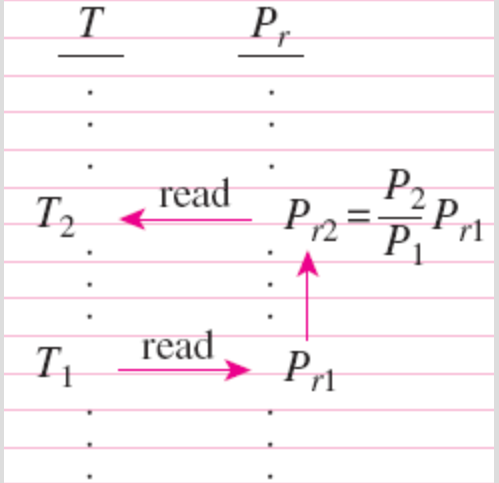
$$\left( \frac{v_2}{v_1} \right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

$T/P_r$  is the relative specific volume  $v_r$

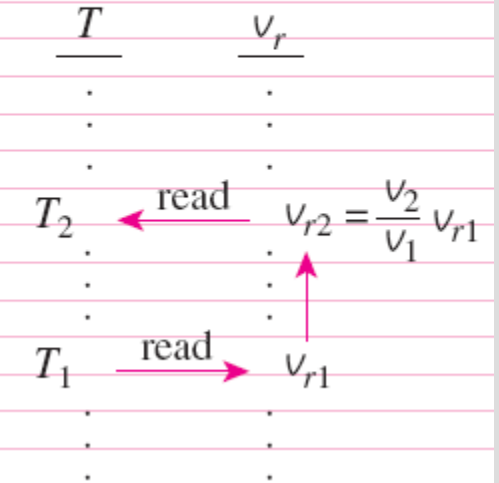
The use of  $P_r$  data for calculating the final temperature during an isentropic process.

The use of  $v_r$  data for calculating the final temperature during an isentropic process

Process: isentropic  
Given:  $P_1$ ,  $T_1$ , and  $P_2$   
Find:  $T_2$

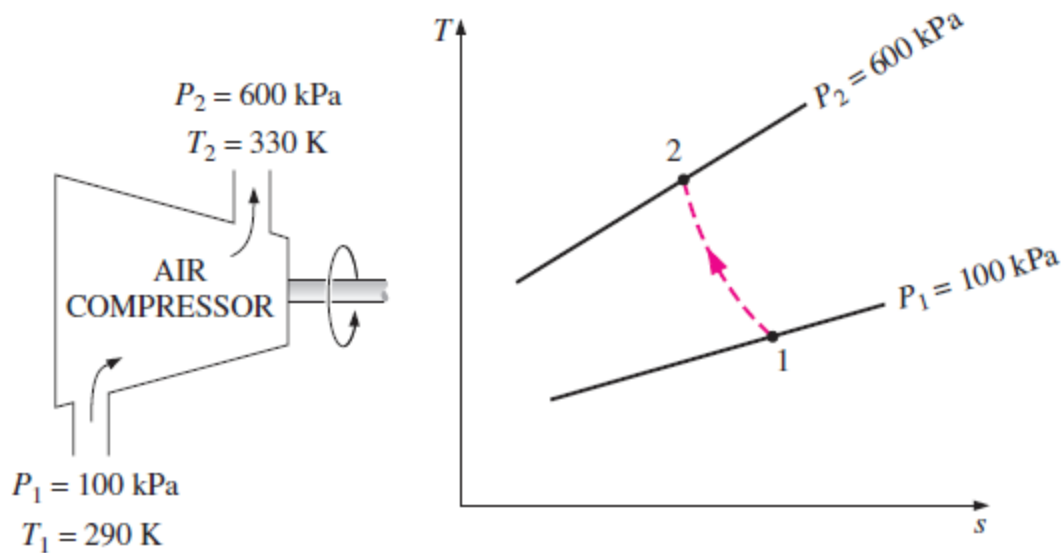


Process: isentropic  
Given:  $v_1$ ,  $T_1$ , and  $v_2$   
Find:  $T_2$



### EXAMPLE 7-9 Entropy Change of an Ideal Gas

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using (a) property values from the air table and (b) average specific heats.



**FIGURE 7-34**

Schematic and  $T$ - $s$  diagram for Example 7-9.

**Solution** Air is compressed between two specified states. The entropy change of air is to be determined by using tabulated property values and also by using average specific heats.

**Assumptions** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. Therefore, entropy change relations developed under the ideal-gas assumption are applicable.

**Analysis** A sketch of the system and the  $T$ - $s$  diagram for the process are given in Fig. 7–34. We note that both the initial and the final states of air are completely specified.

(a) The properties of air are given in the air table (Table A–17). Reading  $s^\circ$  values at given temperatures and substituting, we find

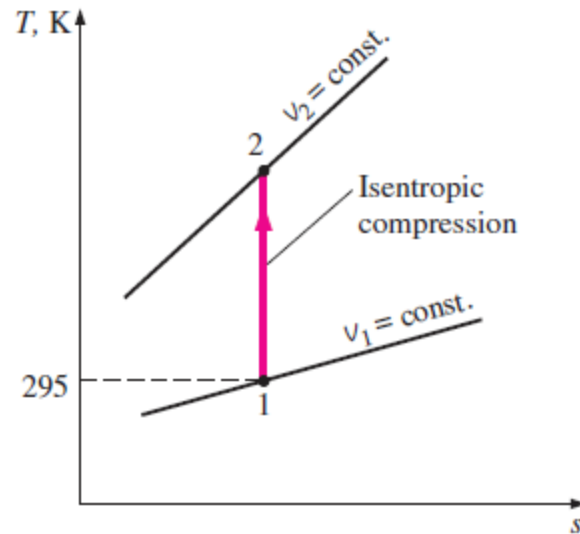
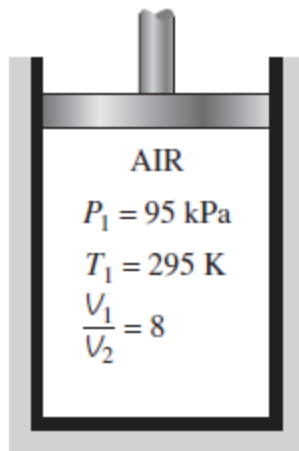
$$\begin{aligned} s_2 - s_1 &= s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \\ &= [(1.79783 - 1.66802) \text{ kJ/kg} \cdot \text{K}] - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\ &= \mathbf{-0.3844 \text{ kJ/kg} \cdot \text{K}} \end{aligned}$$

(b) The entropy change of air during this process can also be determined approximately from Eq. 7–34 by using a  $c_p$  value at the average temperature of 37°C (Table A–2b) and treating it as a constant:

$$\begin{aligned} s_2 - s_1 &= c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\ &= (1.006 \text{ kJ/kg} \cdot \text{K}) \ln \frac{330 \text{ K}}{290 \text{ K}} - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\ &= \mathbf{-0.3842 \text{ kJ/kg} \cdot \text{K}} \end{aligned}$$

### EXAMPLE 7-10 Isentropic Compression of Air in a Car Engine

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio  $V_1/V_2$  of this engine is 8, determine the final temperature of the air.



**Solution** Air is compressed in a car engine isentropically. For a given compression ratio, the final air temperature is to be determined.

**Assumptions** At specified conditions, air can be treated as an ideal gas. Therefore, the isentropic relations for ideal gases are applicable.

**Analysis** A sketch of the system and the  $T$ - $s$  diagram for the process are given in Fig. 7–38.

This process is easily recognized as being isentropic since it is both reversible and adiabatic. The final temperature for this isentropic process can be determined from Eq. 7–50 with the help of relative specific volume data (Table A–17), as illustrated in Fig. 7–39.

For closed systems: 
$$\frac{V_2}{V_1} = \frac{v_2}{v_1}$$

At  $T_1 = 295 \text{ K}$ : 
$$v_{r1} = 647.9$$

From Eq. 7–50: 
$$v_{r2} = v_{r1} \left( \frac{v_2}{v_1} \right) = (647.9) \left( \frac{1}{8} \right) = 80.99 \rightarrow T_2 = \mathbf{662.7 \text{ K}}$$

Therefore, the temperature of air will increase by  $367.7^\circ\text{C}$  during this process.

# REVERSIBLE STEADY-FLOW WORK

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe$$

$$\left. \begin{aligned} \delta q_{\text{rev}} &= T ds && \text{(Eq. 7-16)} \\ T ds &= dh - v dP && \text{(Eq. 7-24)} \end{aligned} \right\} \delta q_{\text{rev}} = dh - v dP$$

$$-\delta w_{\text{rev}} = v dP + dke + dpe$$

$$w_{\text{rev}} = - \int_1^2 v dP - \Delta ke - \Delta pe$$

$$w_{\text{rev}} = - \int_1^2 v dP$$

When kinetic and potential energies are negligible

$$w_{\text{rev,in}} = \int_1^2 v dP + \Delta ke + \Delta pe$$

$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta ke - \Delta pe$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (**Bernoulli equation**):

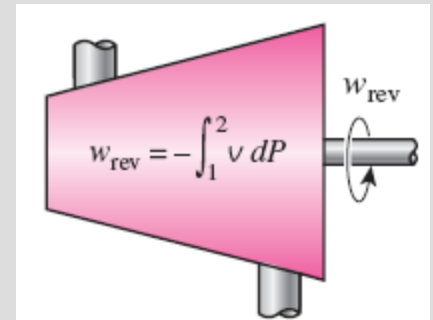
$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

$$w = - \int_1^2 v dP$$

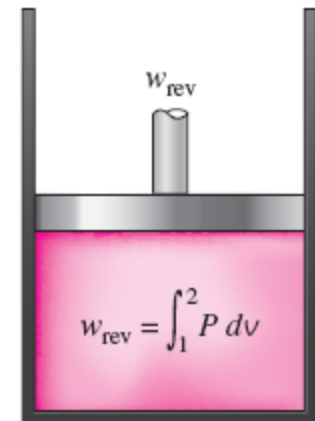
$$W = - \int_1^2 V dP$$

$$W = - \int_1^2 V dP$$

The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.



(a) Steady-flow system



(b) Closed system

Reversible work relations for steady-flow and closed systems.

# Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe \quad \text{Actual}$$

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe \quad \text{Reversible}$$

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

$$\delta w_{\text{rev}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta q_{\text{act}}$$

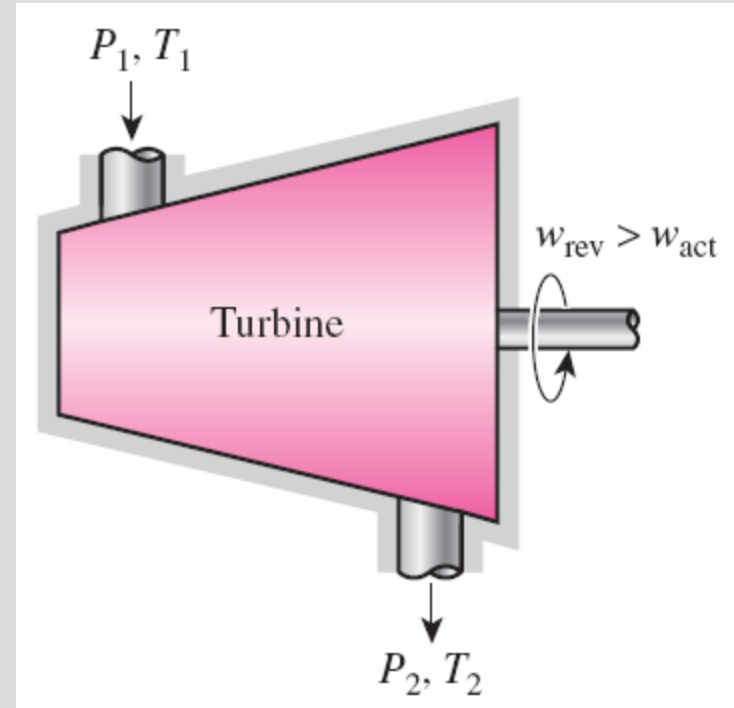
$$\delta q_{\text{rev}} = T ds \quad ds \geq \frac{\delta q_{\text{act}}}{T}$$

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \geq 0$$

$$\delta w_{\text{rev}} \geq \delta w_{\text{act}}$$

$$w_{\text{rev}} \geq w_{\text{act}}$$

Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.



A reversible turbine delivers more work than an irreversible one if both operate between the same end states.



# MINIMIZING THE COMPRESSOR WORK

$$w_{\text{rev,in}} = \int_1^2 v \, dP$$

When kinetic and potential energies are negligible

Isentropic ( $Pv^k = \text{constant}$ ):

$$w_{\text{comp,in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

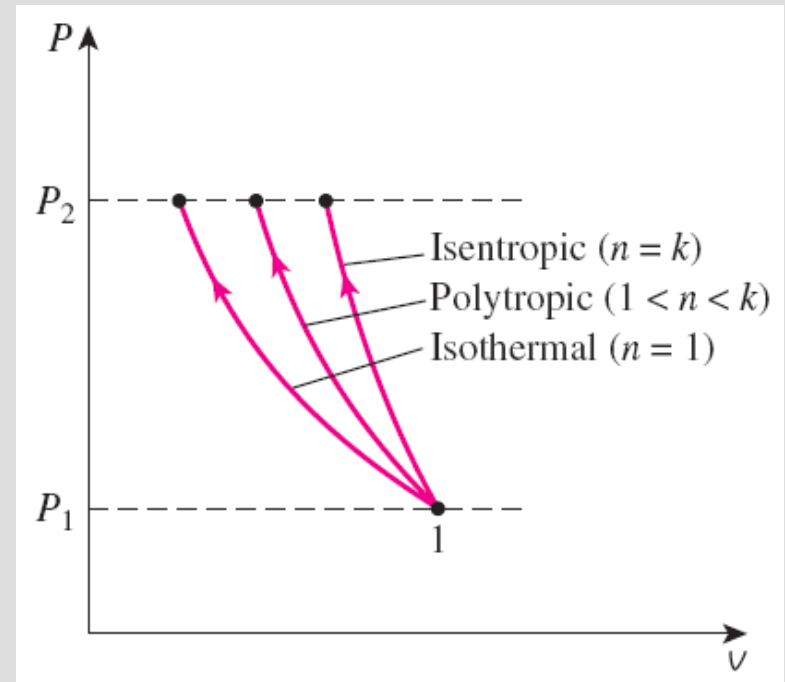
Polytropic ( $Pv^n = \text{constant}$ ):

$$w_{\text{comp,in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

Isothermal ( $Pv = \text{constant}$ ):

$$w_{\text{comp,in}} = RT \ln \frac{P_2}{P_1}$$

The adiabatic compression ( $Pv^k = \text{constant}$ ) requires the maximum work and the isothermal compression ( $T = \text{constant}$ ) requires the minimum. **Why?**

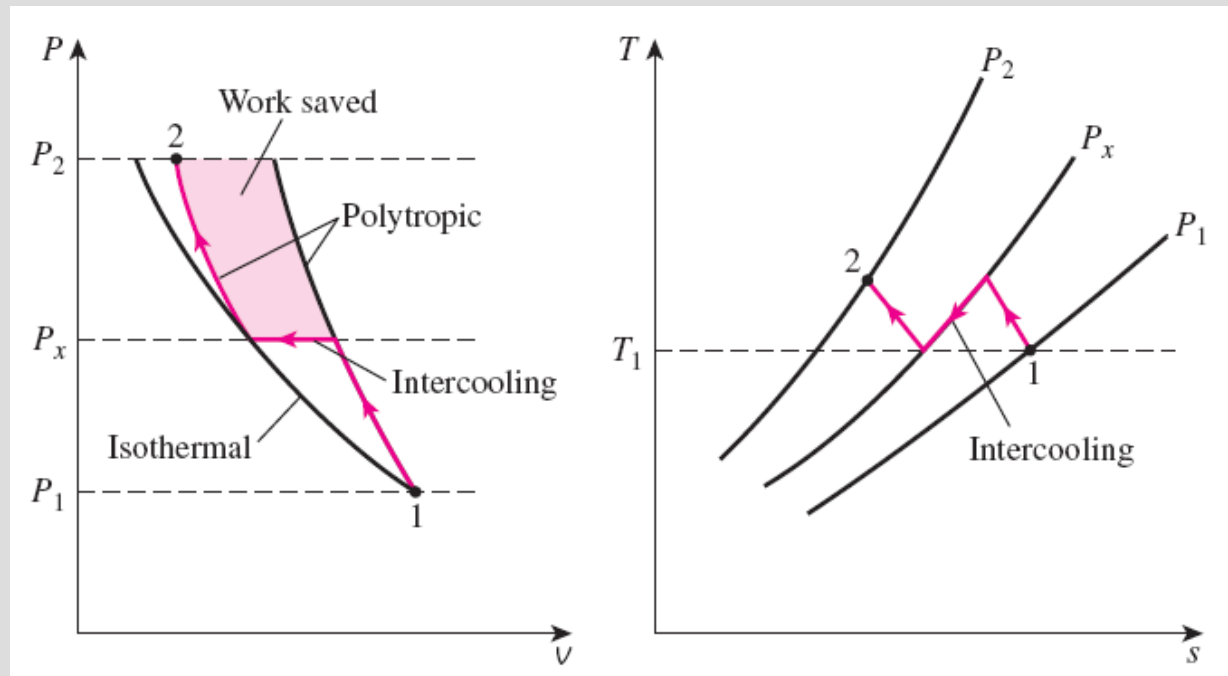


$P$ - $v$  diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

# Multistage Compression with Intercooling

The gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an *intercooler*.

*P-v* and *T-s* diagrams for a two-stage steady-flow compression process.



$$W_{\text{comp, in}} = W_{\text{comp I, in}} + W_{\text{comp II, in}}$$

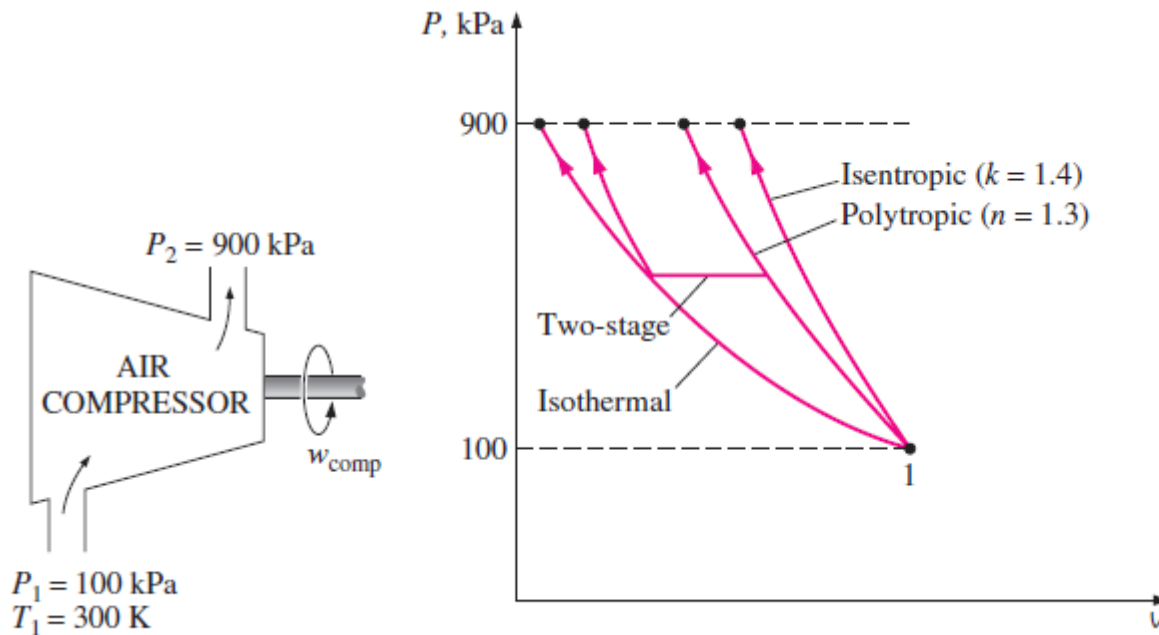
$$= \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$

$$P_x = (P_1 P_2)^{1/2} \quad \text{or} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

To minimize compression work during two-stage compression, the pressure ratio across each stage of the compressor must be the same.

### EXAMPLE 7–13 Work Input for Various Compression Processes

Air is compressed steadily by a reversible compressor from an inlet state of 100 kPa and 300 K to an exit pressure of 900 kPa. Determine the compressor work per unit mass for (a) isentropic compression with  $k = 1.4$ , (b) polytropic compression with  $n = 1.3$ , (c) isothermal compression, and (d) ideal two-stage compression with intercooling with a polytropic exponent of 1.3.



**FIGURE 7–47**

Schematic and  $P$ - $v$  diagram for Example 7–13.

**Assumptions** 1 Steady operating conditions exist. 2 At specified conditions, air can be treated as an ideal gas. 3 Kinetic and potential energy changes are negligible.

**Analysis** We take the compressor to be the system. This is a control volume since mass crosses the boundary. A sketch of the system and the  $T$ - $s$  diagram for the process are given in Fig. 7–47.

The steady-flow compression work for all these four cases is determined by using the relations developed earlier in this section:

(a) Isentropic compression with  $k = 1.4$ :

$$\begin{aligned} w_{\text{comp,in}} &= \frac{kRT_1}{k-1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right] \\ &= \frac{(1.4)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.4-1} \left[ \left( \frac{900 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.4-1)/1.4} - 1 \right] \\ &= \mathbf{263.2 \text{ kJ/kg}} \end{aligned}$$

(b) Polytropic compression with  $n = 1.3$ :

$$\begin{aligned} w_{\text{comp,in}} &= \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right] \\ &= \frac{(1.3)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.3-1} \left[ \left( \frac{900 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.3-1)/1.3} - 1 \right] \\ &= \mathbf{246.4 \text{ kJ/kg}} \end{aligned}$$

(c) Isothermal compression:

$$\begin{aligned} w_{\text{comp,in}} &= RT \ln \frac{P_2}{P_1} = (0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K}) \ln \frac{900 \text{ kPa}}{100 \text{ kPa}} \\ &= \mathbf{189.2 \text{ kJ/kg}} \end{aligned}$$

(d) Ideal two-stage compression with intercooling ( $n = 1.3$ ): In this case, the pressure ratio across each stage is the same, and its value is

$$P_x = (P_1 P_2)^{1/2} = [(100 \text{ kPa})(900 \text{ kPa})]^{1/2} = 300 \text{ kPa}$$

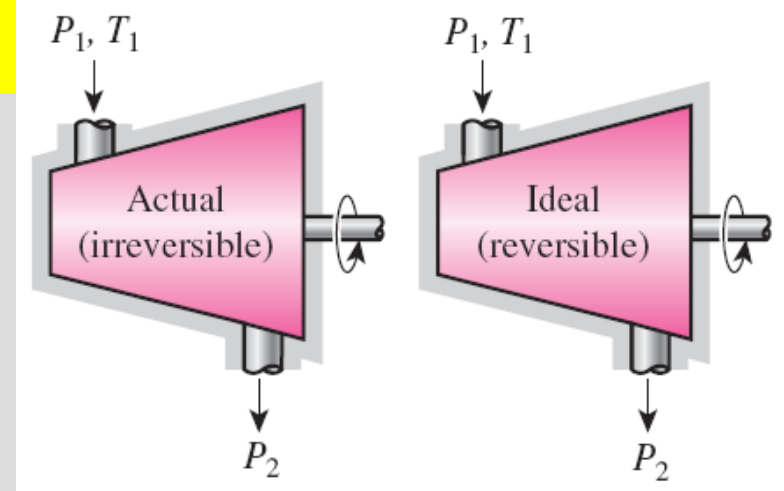
The compressor work across each stage is also the same. Thus the total compressor work is twice the compression work for a single stage:

$$\begin{aligned}w_{\text{comp, in}} &= 2w_{\text{comp 1, in}} = 2 \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] \\&= \frac{2(1.3)(0.287 \text{ kJ/kg} \cdot \text{K})(300 \text{ K})}{1.3-1} \left[ \left( \frac{300 \text{ kPa}}{100 \text{ kPa}} \right)^{(1.3-1)/1.3} - 1 \right] \\&= \mathbf{215.3 \text{ kJ/kg}}\end{aligned}$$

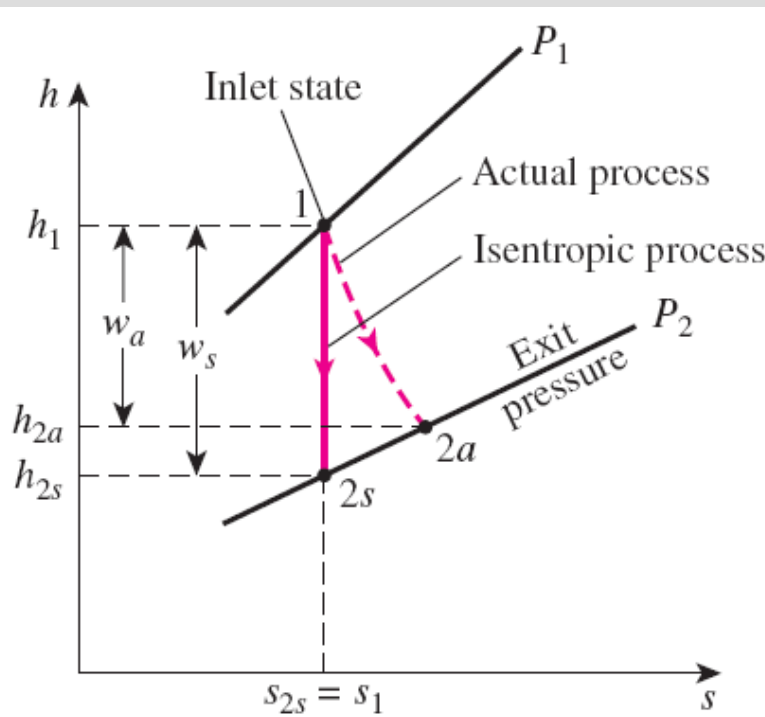
**Discussion** Of all four cases considered, the isothermal compression requires the minimum work and the isentropic compression the maximum. The compressor work is decreased when two stages of polytropic compression are utilized instead of just one. As the number of compressor stages is increased, the compressor work approaches the value obtained for the isothermal case.

# ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.



## Iisentropic Efficiency of Turbines



$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

The  $h-s$  diagram for the actual and isentropic processes of an adiabatic turbine.

# Isentropic Efficiencies of Compressors and Pumps

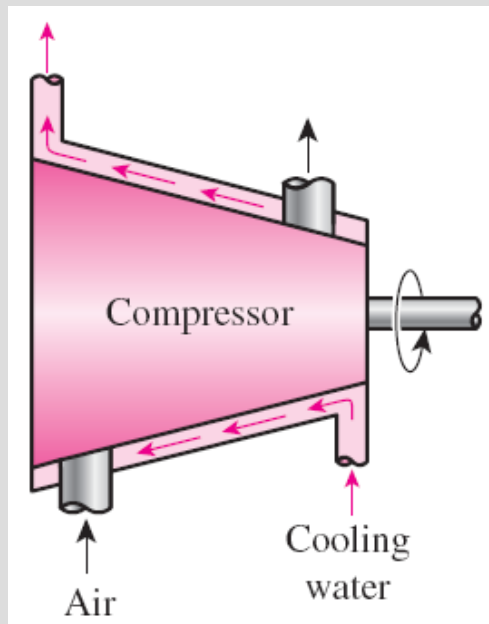
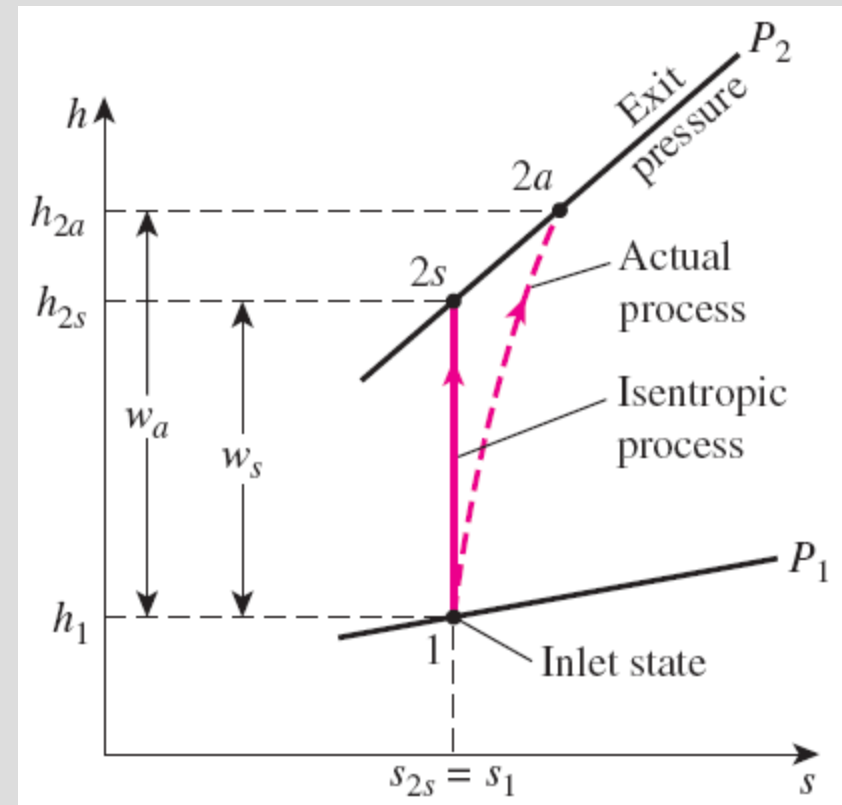
$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \text{When kinetic and potential energies are negligible}$$

$$\eta_p = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \quad \text{For a pump}$$

$$\eta_c = \frac{w_t}{w_a} \quad \text{Isothermal efficiency}$$

The  $h$ - $s$  diagram of the actual and isentropic processes of an adiabatic compressor.



Compressors are sometimes intentionally cooled to minimize the work input.

Can you use isentropic efficiency for a non-adiabatic compressor?

Can you use isothermal efficiency for an adiabatic compressor?

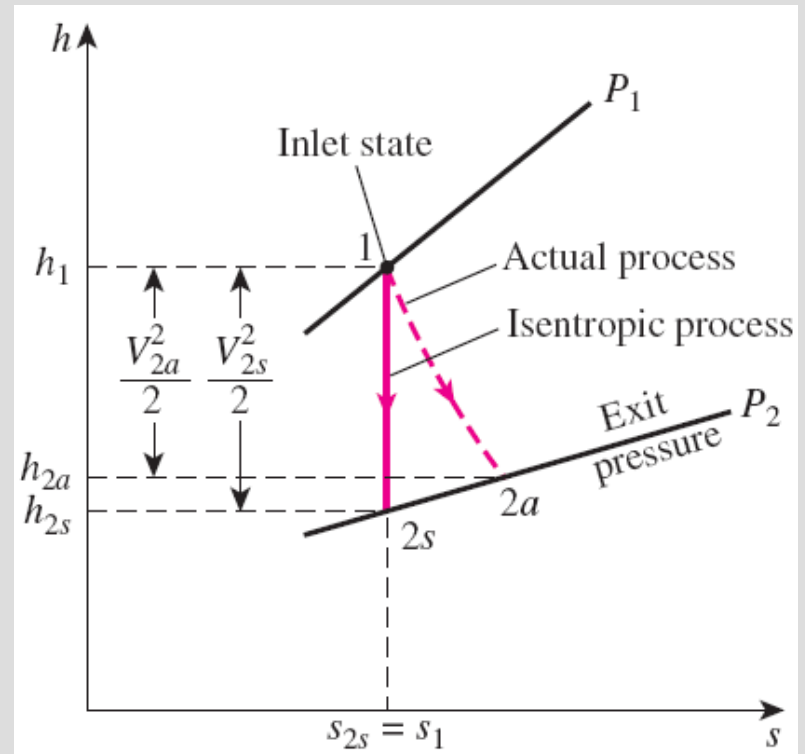
# Isentropic Efficiency of Nozzles

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

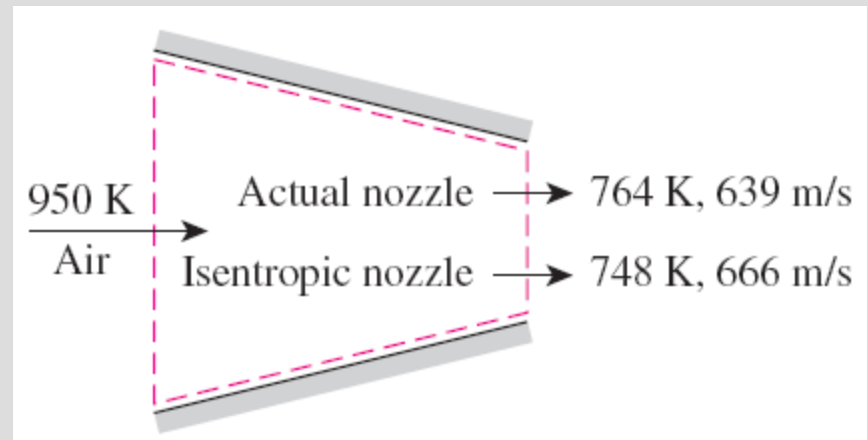
The  $h$ - $s$  diagram of the actual and isentropic processes of an adiabatic nozzle.



Then,

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.





# ENTROPY BALANCE

$$\left( \begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{entering} \end{array} \right) - \left( \begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{array} \right) + \left( \begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{generated} \end{array} \right) = \left( \begin{array}{c} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{array} \right)$$

$$S_{\text{in}} - S_{\text{out}} + S_{\text{gen}} = \Delta S_{\text{system}}$$

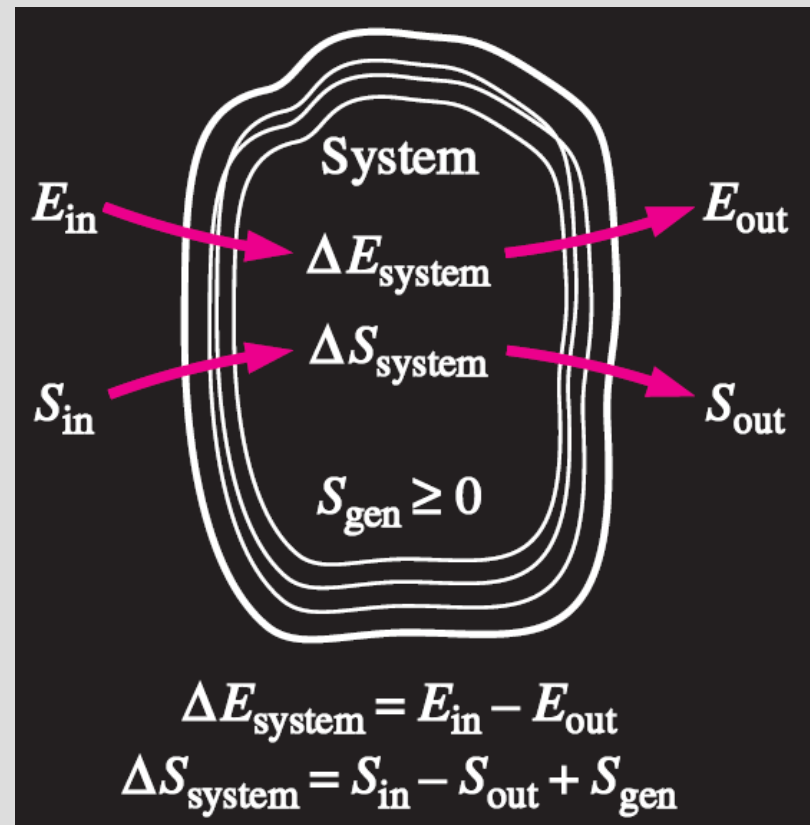
## Entropy Change of a System, $\Delta S_{\text{system}}$

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

When the properties of the system are not uniform

$$S_{\text{system}} = \int s \delta m = \int_V s \rho dV$$

Energy and entropy balances for a system.



# Mechanisms of Entropy Transfer, $S_{in}$ and $S_{out}$

## 1 Heat Transfer

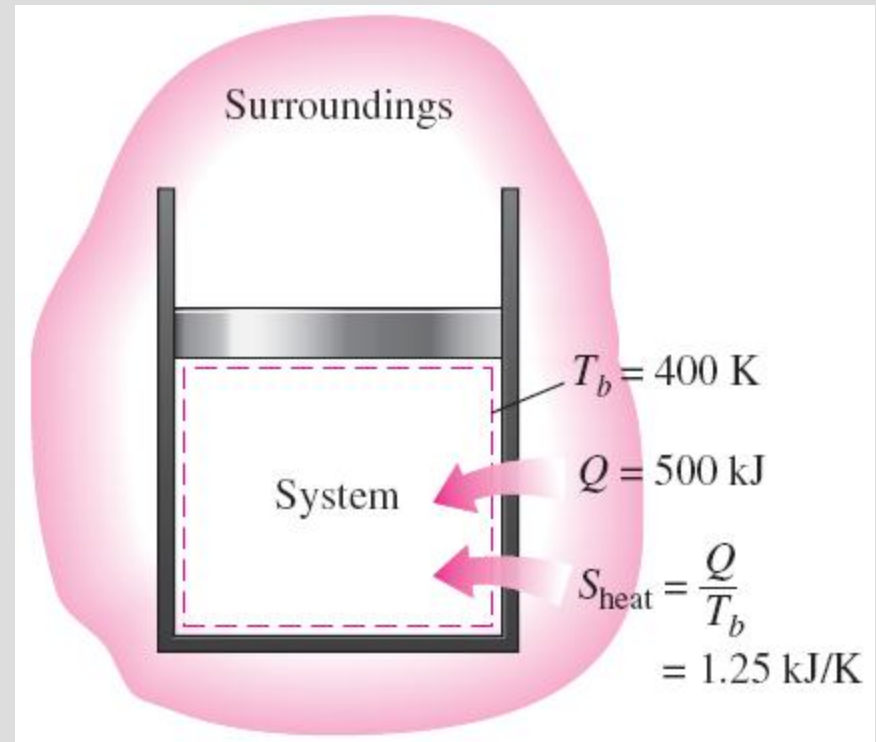
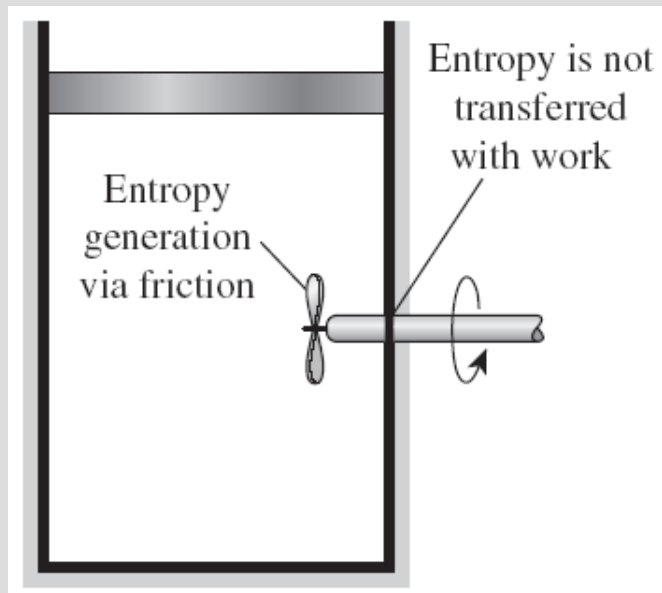
Entropy transfer by heat transfer:

$$S_{\text{heat}} = \frac{Q}{T} \quad (T = \text{constant})$$

$$S_{\text{heat}} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$

Entropy transfer by work:

$$S_{\text{work}} = 0$$



Heat transfer is always accompanied by entropy transfer in the amount of  $Q/T$ , where  $T$  is the boundary temperature.

No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

## 2 Mass Flow

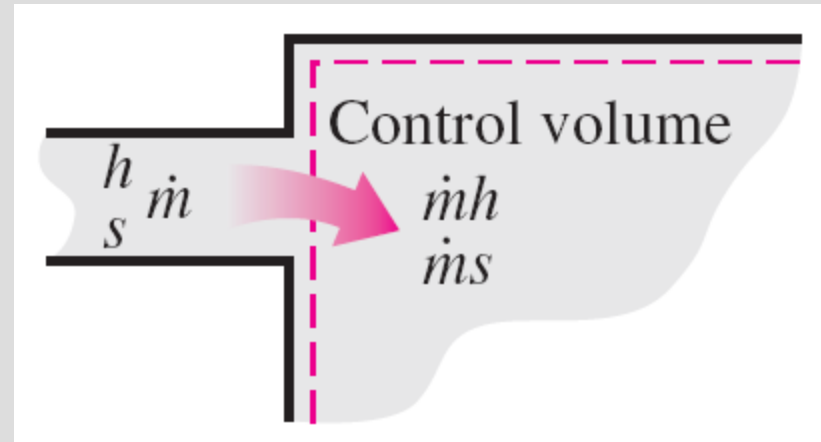
Entropy transfer by mass:

$$S_{\text{mass}} = ms$$

When the properties of the mass change during the process

$$\dot{S}_{\text{mass}} = \int_{A_c} s \rho V_n dA_c$$

$$S_{\text{mass}} = \int s \delta m = \int_{\Delta t} \dot{S}_{\text{mass}} dt$$



Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

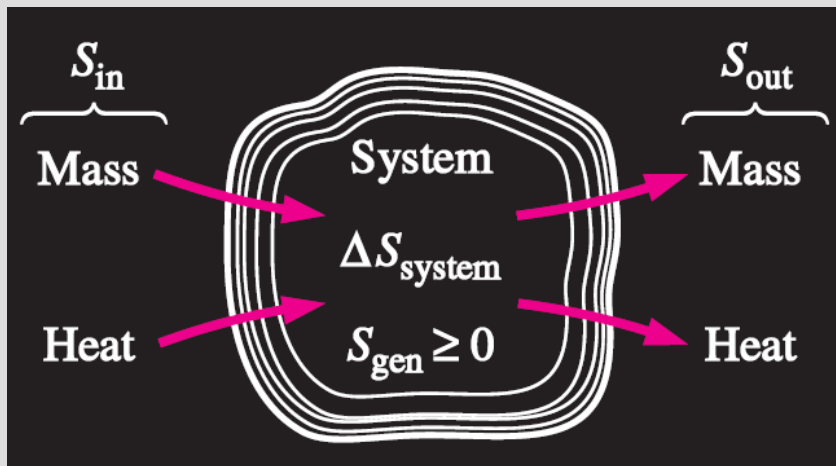
# Entropy Generation, $S_{gen}$

$$\underbrace{S_{in} - S_{out}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{gen}}_{\text{Entropy generation}} = \underbrace{\Delta S_{system}}_{\text{Change in entropy}} \quad (\text{kJ/K})$$

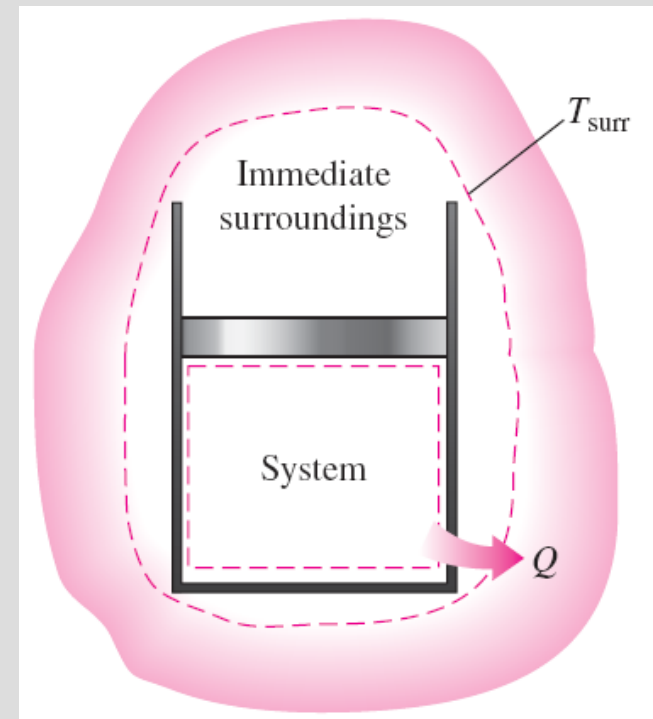
$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{dS_{system}/dt}_{\text{Rate of change in entropy}} \quad (\text{kW/K})$$

$$(S_{in} - S_{out}) + S_{gen} = \Delta S_{system} \quad (\text{kJ/kg} \cdot \text{K})$$

Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.



Mechanisms of entropy transfer for a general system.



# Closed Systems

$$\text{Closed system: } \sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \quad (\text{kJ/K})$$

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

$$\text{Adiabatic closed system: } S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$$

$$\text{System + Surroundings: } S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$$

$$\Delta S_{\text{system}} = m(s_2 - s_1) \quad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$$

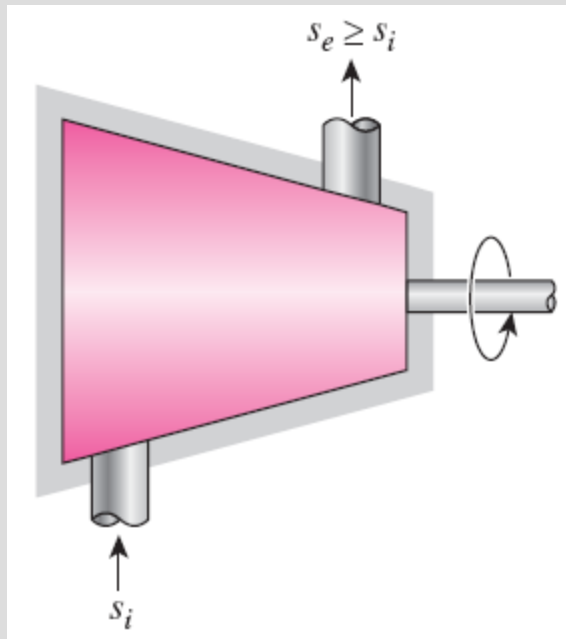
$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}} \quad (\text{kJ/K})$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = dS_{\text{CV}}/dt \quad (\text{kW/K})$$

*Steady-flow:* 
$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

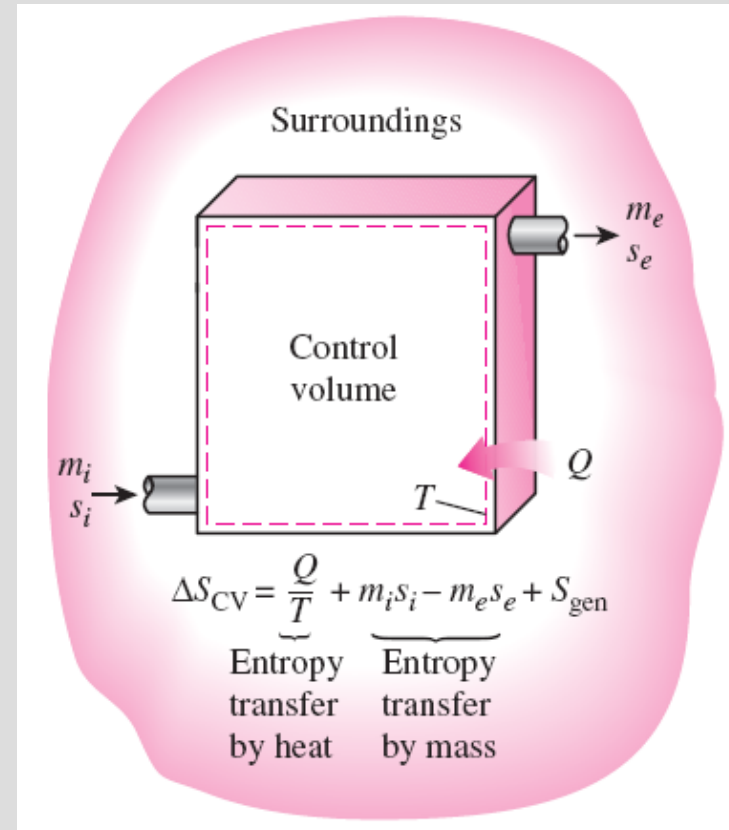
*Steady-flow, single-stream:* 
$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

*Steady-flow, single-stream, adiabatic:* 
$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i)$$



The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.

## Control Volumes



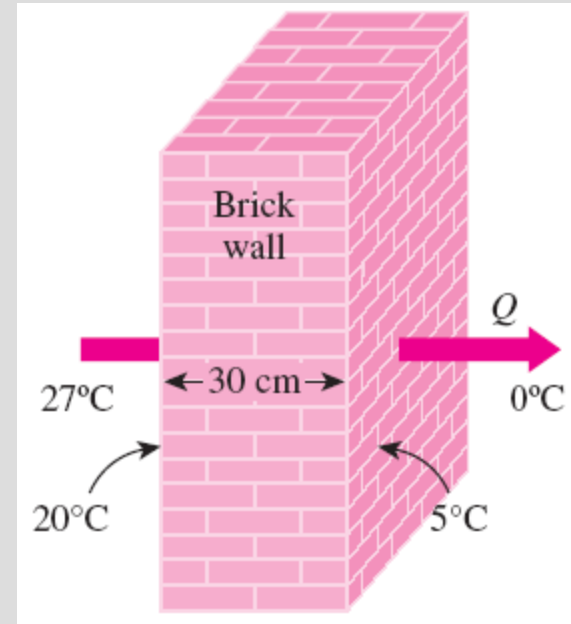
The entropy of a control volume changes as a result of mass flow as well as heat transfer.

# EXAMPLES

## Entropy balance for heat transfer through a wall

$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{system}}{dt}}_{\text{Rate of change in entropy}} \xrightarrow{0 \text{ (steady)}}$$

$$\left(\frac{\dot{Q}}{T}\right)_{in} - \left(\frac{\dot{Q}}{T}\right)_{out} + \dot{S}_{gen} = 0$$

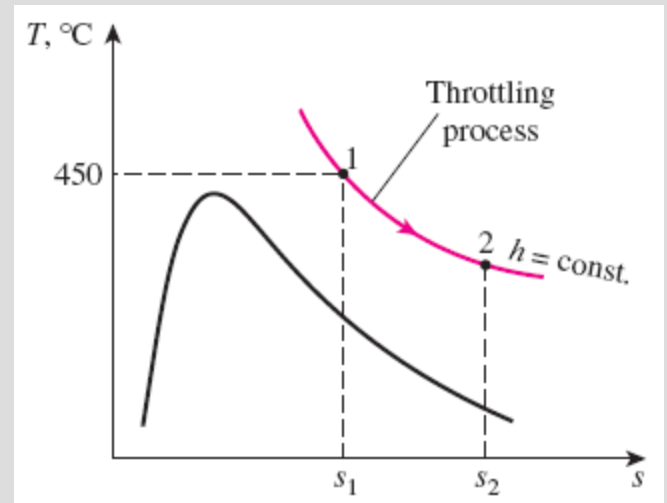
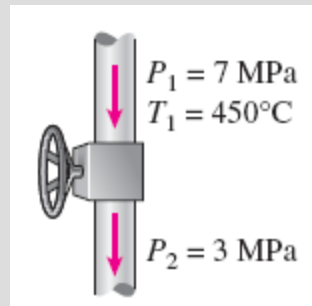


## Entropy balance for a throttling process

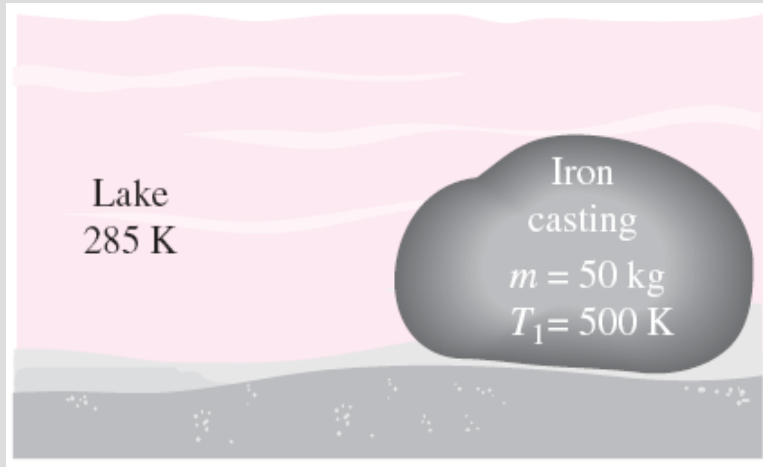
$$\underbrace{\dot{S}_{in} - \dot{S}_{out}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{gen}}_{\text{Rate of entropy generation}} = \underbrace{\frac{dS_{system}}{dt}}_{\text{Rate of change in entropy}} \xrightarrow{0 \text{ (steady)}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen} = 0$$

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1)$$



## Entropy Generated when a Hot Block Is Dropped in a Lake

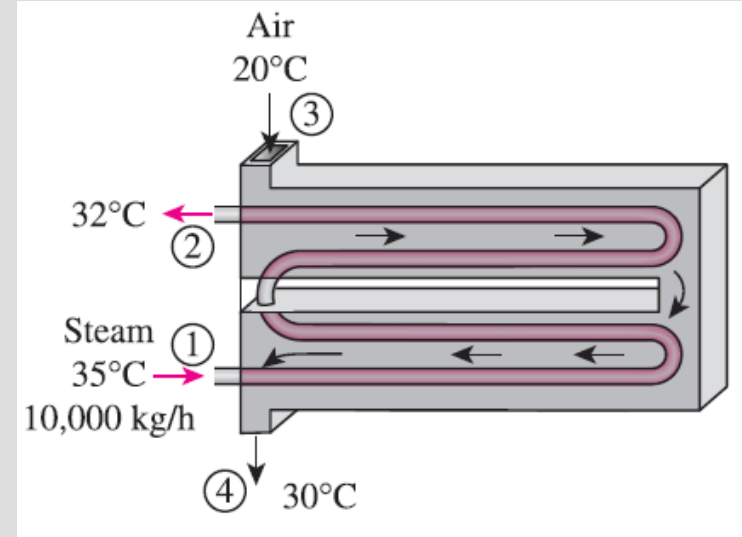


or  $S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{lake}}$

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$-\frac{Q_{\text{out}}}{T_b} + S_{\text{gen}} = \Delta S_{\text{system}}$$

## Entropy Generation in a Heat Exchanger



$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \overset{0 \text{ (steady)}}{\rightarrow}$$

$$\dot{m}_{\text{steam}} s_1 + \dot{m}_{\text{air}} s_3 - \dot{m}_{\text{steam}} s_2 - \dot{m}_{\text{air}} s_4 + \dot{S}_{\text{gen}} = 0$$

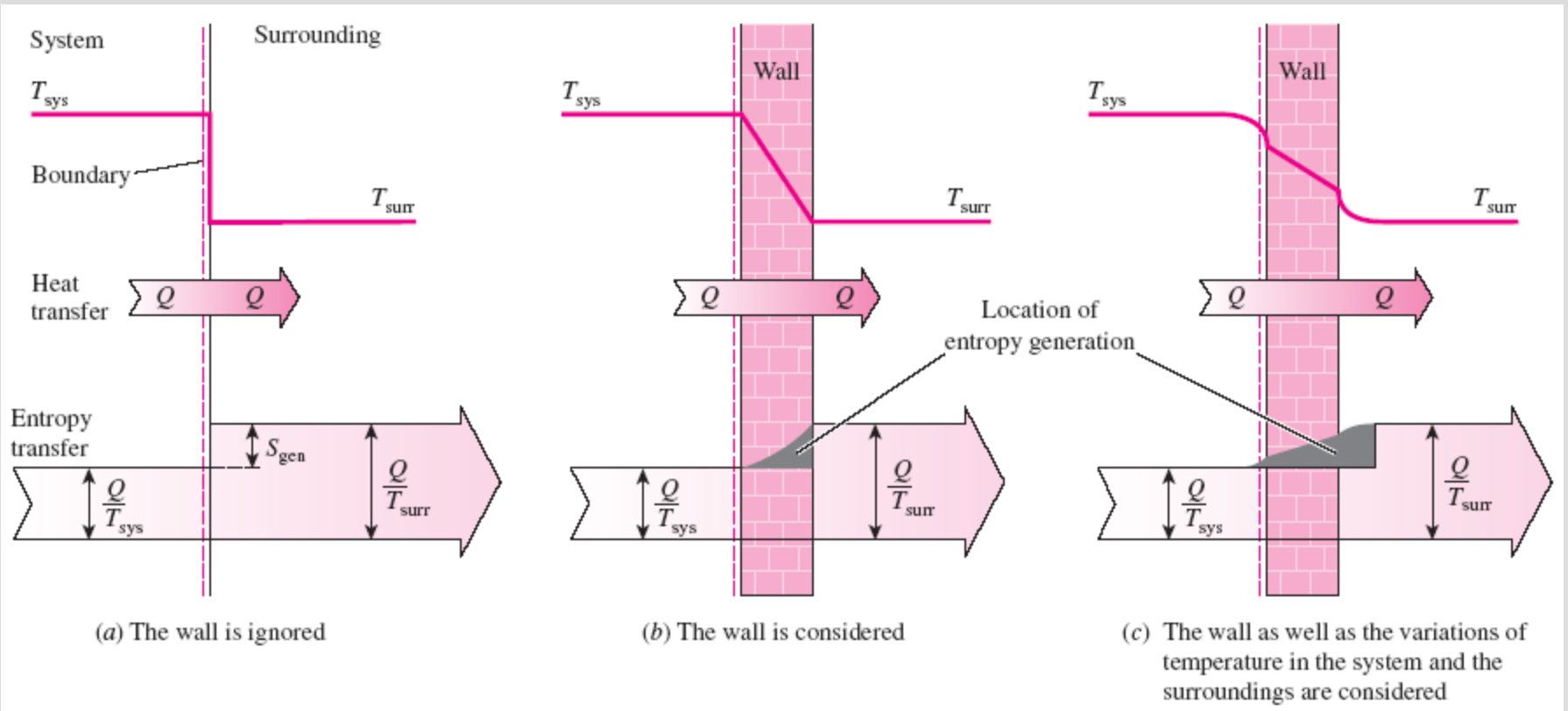
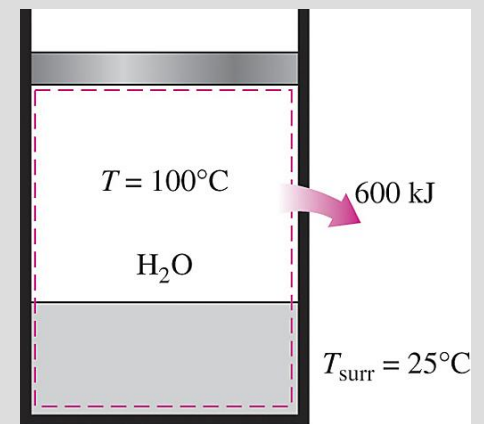
$$\dot{S}_{\text{gen}} = \dot{m}_{\text{steam}} (s_2 - s_1) + \dot{m}_{\text{air}} (s_4 - s_3)$$



# Entropy generation associated with a heat transfer process

$$\underbrace{S_{in} - S_{out}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{gen}}_{\text{Entropy generation}} = \underbrace{\Delta S_{system}}_{\text{Change in entropy}}$$

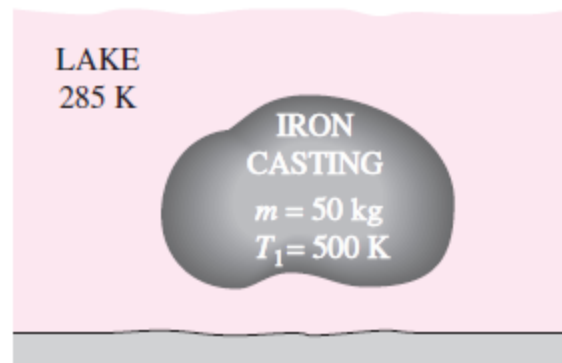
$$-\frac{Q_{out}}{T_b} + S_{gen} = \Delta S_{system}$$



Graphical representation of entropy generation during a heat transfer process through a finite temperature difference.

**EXAMPLE 7–19** Entropy Generated when a Hot Block Is Dropped in a Lake

A 50-kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K. The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of 0.45 kJ/kg · K for the iron, determine (a) the entropy change of the iron block, (b) the entropy change of the lake water, and (c) the entropy generated during this process.



**FIGURE 7–67**

Schematic for Example 7–19.

**Solution** A hot iron block is thrown into a lake, and cools to the lake temperature. The entropy changes of the iron and of the lake as well as the entropy generated during this process are to be determined.

**Assumptions** 1 Both the water and the iron block are incompressible substances. 2 Constant specific heats can be used for the water and the iron. 3 The kinetic and potential energy changes of the iron are negligible,  $\Delta KE = \Delta PE = 0$  and thus  $\Delta E = \Delta U$ .

**Properties** The specific heat of the iron is  $0.45 \text{ kJ/kg} \cdot \text{K}$  (Table A–3).

**Analysis** We take the *iron casting* as the system (Fig. 7–67). This is a *closed system* since no mass crosses the system boundary during the process.

To determine the entropy change for the iron block and for the lake, first we need to know the final equilibrium temperature. Given that the thermal energy capacity of the lake is very large relative to that of the iron block, the lake will absorb all the heat rejected by the iron block without experiencing any change in its temperature. Therefore, the iron block will cool to  $285 \text{ K}$  during this process while the lake temperature remains constant at  $285 \text{ K}$ .

(a) The entropy change of the iron block can be determined from

$$\begin{aligned}\Delta S_{\text{iron}} &= m(s_2 - s_1) = mc_{\text{avg}} \ln \frac{T_2}{T_1} \\ &= (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \ln \frac{285 \text{ K}}{500 \text{ K}} \\ &= -12.65 \text{ kJ/K}\end{aligned}$$

(b) The temperature of the lake water remains constant during this process at 285 K. Also, the amount of heat transfer from the iron block to the lake is determined from an energy balance on the iron block to be

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc., energies}}$$

$$-Q_{\text{out}} = \Delta U = mc_{\text{avg}}(T_2 - T_1)$$

or

$$Q_{\text{out}} = mc_{\text{avg}}(T_1 - T_2) = (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K})(500 - 285) \text{ K} = 4838 \text{ kJ}$$

Then the entropy change of the lake becomes

$$\Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = \frac{+4838 \text{ kJ}}{285 \text{ K}} = \mathbf{16.97 \text{ kJ/K}}$$

(c) The entropy generated during this process can be determined by applying an entropy balance on an *extended system* that includes the iron block and its immediate surroundings so that the boundary temperature of the extended system is at 285 K at all times:

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$-\frac{Q_{\text{out}}}{T_b} + S_{\text{gen}} = \Delta S_{\text{system}}$$

or

$$S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}} = \frac{4838 \text{ kJ}}{285 \text{ K}} - (12.65 \text{ kJ/K}) = \mathbf{4.32 \text{ kJ/K}}$$