

Numerical  
Techniques in  

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**Electromagnetics**

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Second Edition

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Matthew N. O. Sadiku, Ph.D.



CRC Press

Boca Raton London New York Washington, D.C.

### Library of Congress Cataloging-in-Publication Data

Sadiku, Matthew N. O.  
Numerical techniques in electromagnetics / Matthew N.O. Sadiku.—[2nd ed.].  
p. cm.  
Includes bibliographical references and index.  
ISBN 0-8493-1395-3 (alk. paper)  
1. Electromagnetism. 2. Numerical analysis. I. Title.  
QC760 .S24 2000  
537'.01'515—dc21  
00-026823  
CIP

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International Standard Book Number 0-8493-1395-3  
Library of Congress Card Number 00-026823  
Printed in the United States of America 1 2 3 4 5 6 7 8 9 0  
Printed on acid-free paper

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## *Preface*

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The art of computation of electromagnetic (EM) problems has grown exponentially for three decades due to the availability of powerful computer resources. In spite of this, the EM community has suffered without a suitable text on computational techniques commonly used in solving EM-related problems. Although there have been monographs on one particular technique or the other, the monographs are written for the experts rather than students. Only a few texts cover the major techniques and do that in a manner suitable for classroom use. It seems experts in this area are familiar with one or few techniques and not many experts seem to be familiar with all the common techniques. This text attempts to fill the gap.

The text is intended for seniors or graduate students and may be used for a one-semester or two-semester course. The main requirements for students taking a course based on this text are introductory EM courses and a knowledge of a high-level computer language, preferably FORTRAN or C. Software packages such as Matlab and Mathcad may be helpful tools. Although familiarity with linear algebra and numerical analysis is useful, it is not required.

In writing this book, three major objectives were borne in mind. First, the book is intended to teach students how to pose, numerically analyze, and solve EM problems. Second, it is designed to give them the ability to expand their problem solving skills using a variety of available numerical methods. Third, it is meant to prepare graduate students for research in EM. The aim throughout has been simplicity of presentation so that the text can be useful for both teaching and self-study. In striving after simplicity, however, the reader is referred to the references for more information. Toward the end of each chapter, the techniques covered in the chapter are applied to real life problems. Since the application of the technique is as vast as EM and author's experience is limited, the choice of application is selective.

Chapter 1 covers some fundamental concepts in EM. Chapter 2 is intended to put numerical methods in a proper perspective. Analytical methods such as separation of variables and series expansion are covered. Chapter 3 discusses the finite difference methods and begins with the derivation of difference equation from a partial differential equation (PDE) using forward, backward, and central differences. The finite-difference time-domain (FDTD) technique involving Yee's algorithm is pre-

sented and applied to scattering problems. Numerical integration is covered using trapezoidal, Simpson's, Newton-Cotes rules, and Gaussian quadratures.

Chapter 4 on variational methods serves as a preparatory ground for the next two major topics: moment methods and finite element methods. Basic concepts such as inner product, self-adjoint operator, functionals, and Euler equation are covered. Chapter 5 on moment methods focuses on the solution of integral equations. Chapter 6 on finite element method covers the basic steps involved in using the finite element method. Solutions of Laplace's, Poisson's, and wave equations using the finite element method are covered.

Chapter 7 is devoted to transmission-line matrix or modeling (TLM). The method is applied to diffusion and scattering problems. Chapter 8 is on Monte Carlo methods, while Chapter 9 is on the method of lines.

Since the publication of the first edition, there has been an increased awareness and utilization of numerical techniques. Many graduate curricula now include courses in numerical analysis of EM problems. However, not much has changed in computational electromagnetics. A major noticeable change is in the FDTD method. The method seems to have attracted much attention and many improvements are being made to the standard algorithm. This edition adds the noticeable change in incorporating absorbing boundary conditions in FDTD, FEM, and TLM. Chapter 9 is a new chapter on the method of lines.

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## Acknowledgements

I am greatly indebted to Temple University for granting me a sabbatical in Fall 1998 during which I was able to do most of the revision. I specifically would like to thank my dean, Dr. Keya Sadeghipour, and my chairman, Dr. John Helferty, for their support. Special thanks are due to Raymond Garcia of Georgia Tech for writing Appendices C and D in C++. I am deeply grateful to Dr. Arthur D. Snider of the University of South Florida and Mohammad R. Zunoubi of Mississippi State University for taking the time to send me the list of errors in the first edition. I thank Dr. Reinhold Pregla for helping in clarifying concepts in Chapter 9 on the method of lines. I express my deepest gratitude to my wife, Chris, and our daughters, Ann and Joyce, for their patience, sacrifices, and prayers.

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## A Note to Students

Before you embark on writing your own computer program or using the ones in this text, you should try to understand all relevant theoretical backgrounds. A computer

is no more than a tool used in the analysis of a program. For this reason, you should be as clear as possible what the machine is really being asked to do before setting it off on several hours of expensive computations.

It has been well said by A.C. Doyle that “It is a capital mistake to theorize before you have all the evidence. It biases the judgment.” Therefore, you should never trust the results of a numerical computation unless they are validated, at least in part. You validate the results by comparing them with those obtained by previous investigators or with similar results obtained using a different approach which may be analytical or numerical. For this reason, it is advisable that you become familiar with as many numerical techniques as possible.

The references provided at the end of each chapter are by no means exhaustive but are meant to serve as the starting point for further reading.

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*To my teacher*

*Carl A. Ventrice*

*and my parents*

*Ayisat and Solomon Sadiku*