Pohyb projektilu

```
> eqx := m*diff(vx(t),t) + b*vx(t) = 0;
```

$$eqx := m\left(\frac{d}{dt}vx(t)\right) + b\,vx(t) = 0 \qquad (1)$$

```
> eqy := m*diff(vy(t),t) + b*vy(t) + m*g = 0;
```

$$eqy := m\left(\frac{d}{dt}vy(t)\right) + b\,vy(t) + m\,g = 0 \qquad (2)$$

```
> sol := dsolve({eqx, eqy, vx(0)=v0x, vy(0)=v0y},
                {vx(t), vy(t)});
```

$$sol := \left\{vx(t) = v0x\,e^{-\frac{b\,t}{m}},\ vy(t) = \frac{-m\,g + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g)}{b}\right\} \qquad (3)$$

```
> simplify(sol);
```

$$\left\{vx(t) = v0x\,e^{-\frac{b\,t}{m}},\ vy(t) = \frac{e^{-\frac{b\,t}{m}}\,b\,v0y + e^{-\frac{b\,t}{m}}\,g\,m - m\,g}{b}\right\} \qquad (4)$$

```
> subs(sol, {eqx, eqy});
```

$$\left\{m\left(\frac{v}{vt}\left(\frac{-m\,g + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g)}{b}\right)\right) + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g) = 0,\ m\left(\frac{v}{vt}\left(v0x\,e^{-\frac{b\,t}{m}}\right)\right) + b\,v0x\,e^{-\frac{b\,t}{m}} = 0\right\} \qquad (5)$$

```
> simplify(%);
```

$$\{0 = 0\} \qquad (6)$$

```
> assign(sol);
> vx(t);
```

$$v0x\,e^{-\frac{b\,t}{m}} \qquad (7)$$

```
> vy(t);
```

$$\frac{-m\,g + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g)}{b} \qquad (8)$$

```
> limit(vx(t), t=0);
```

$$v0x \qquad (9)$$

```
> limit(vy(t), t=0);
```

$$v0y \qquad (10)$$

```
> limit(vx(t), b=0);
```

$$v0x \qquad (11)$$

```
> limit(vy(t), b=0);
```

$$-t\,g + v0y \qquad (12)$$

```
> series(vx(t), t, 3);
```

$$v0x - \frac{v0x\,b}{m}\,t + \frac{1}{2}\,\frac{v0x\,b^2}{m^2}\,t^2 + O(t^3) \tag{13}$$

```
> series(vy(t), t, 3);
```

$$v0y - \frac{b\,v0y + m\,g}{m}\,t + \frac{1}{2}\,\frac{b\,(b\,v0y + m\,g)}{m^2}\,t^2 + O(t^3) \tag{14}$$

```
> limit(vy(t), t=infinity);
```

$$\lim_{t \to \infty} \frac{-m\,g + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g)}{b} \tag{15}$$

```
> assume(m>0, b>0);
> limit(vx(t), t=infinity);
```

$$\lim_{t \to \infty} v0x\,e^{-\frac{b\,t}{m}} \tag{16}$$

```
> limit(vy(t), t=infinity);
```

$$\lim_{t \to \infty} \frac{-m\,g + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g)}{b} \tag{17}$$

```
> b := 'b': m := 'm':
> VX := unapply(vx(t),b,t);
```

$$VX := (b, t) \mapsto v0x\,e^{-\frac{b\,t}{m}} \tag{18}$$

```
> VY := unapply(vy(t),b,t);
```

$$VY := (b, t) \mapsto \frac{-m\,g + e^{-\frac{b\,t}{m}}\,(b\,v0y + m\,g)}{b} \tag{19}$$

```
> X := unapply(int(vx(t),t=0..s),b,s);
```

$$X := (b, s) \mapsto -\frac{m\,v0x\left(e^{-\frac{b\,s}{m}} - 1\right)}{b} \tag{20}$$

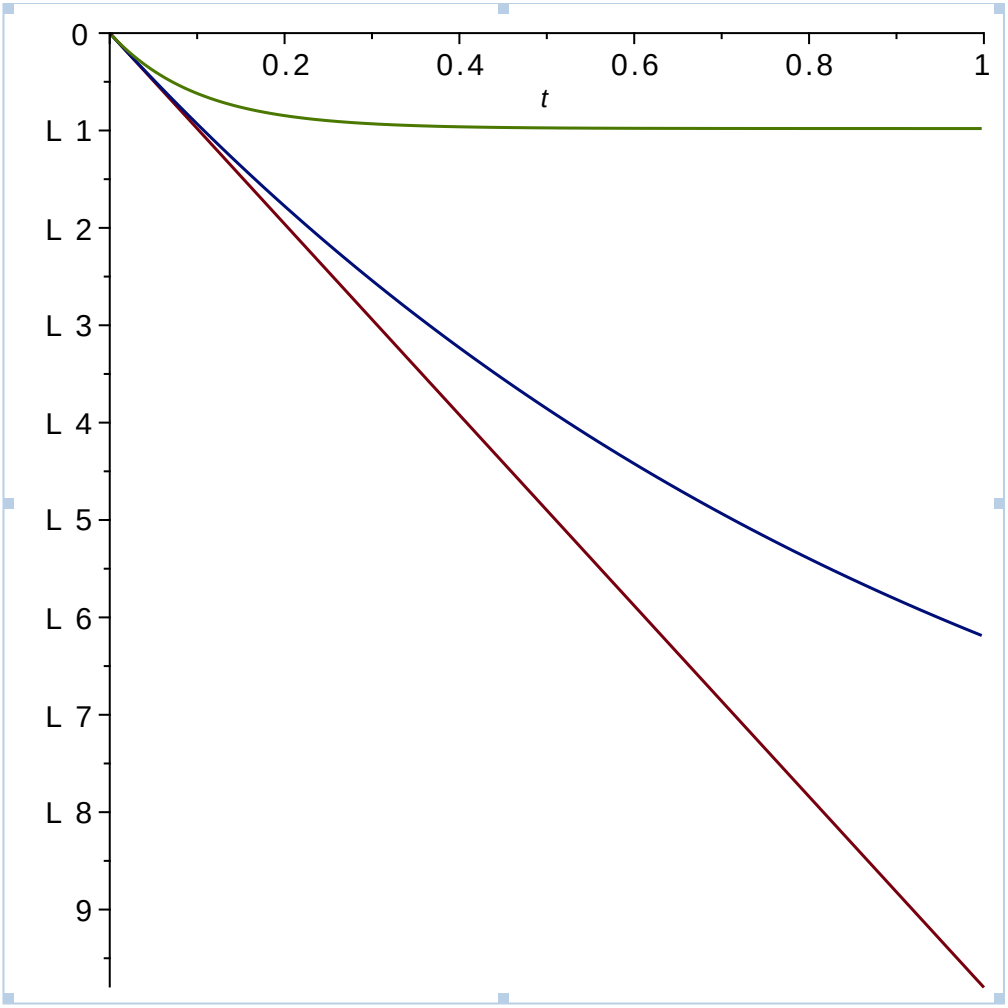
```
> Y := unapply(int(vy(t),t=0..s),b,s);
```

$$Y := (b, s) \mapsto -\frac{m\left(e^{-\frac{b\,s}{m}}\,b\,v0y + e^{-\frac{b\,s}{m}}\,g\,m + g\,s\,b - b\,v0y - m\,g\right)}{b^2} \tag{21}$$

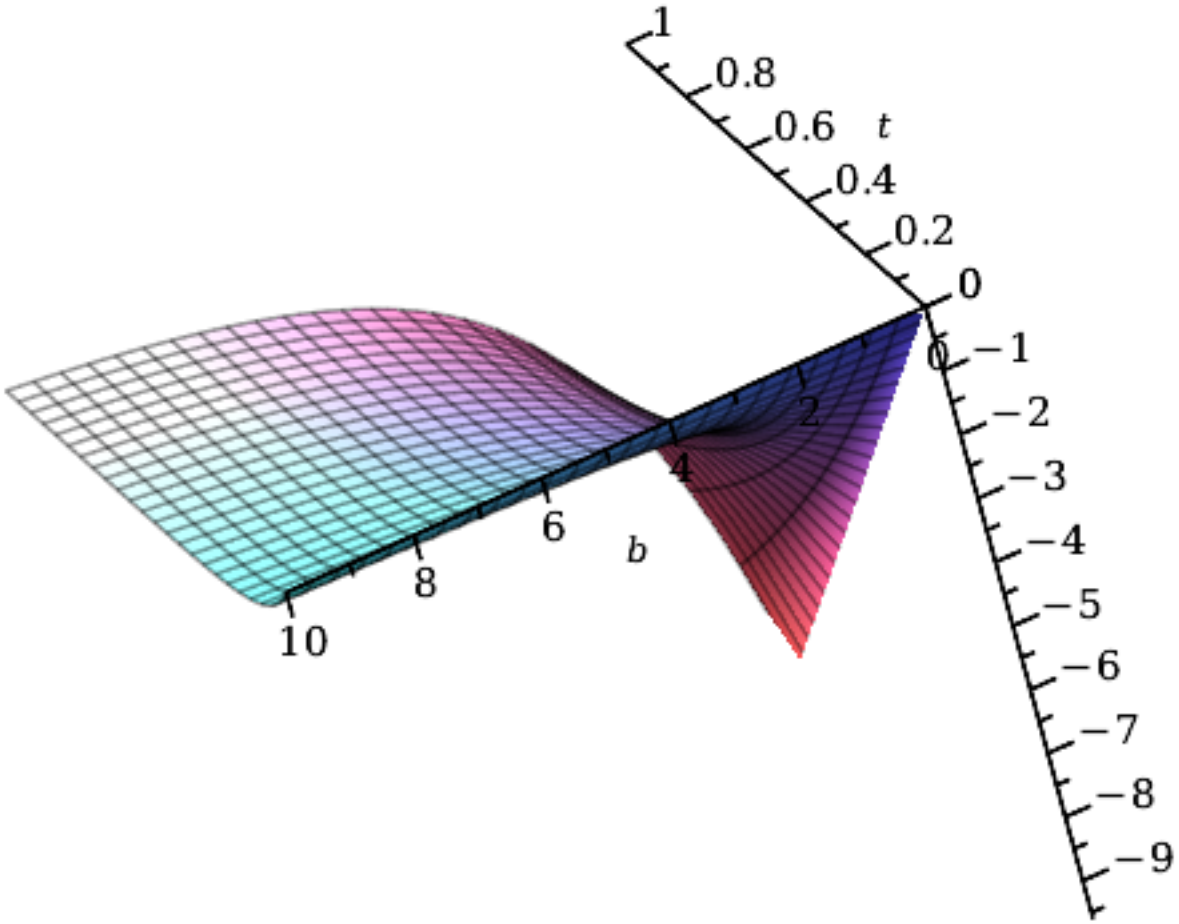
```
> m:=1: g:=9.8: v0x:=0: v0y:=0;
```

$$v0y := 0 \tag{22}$$

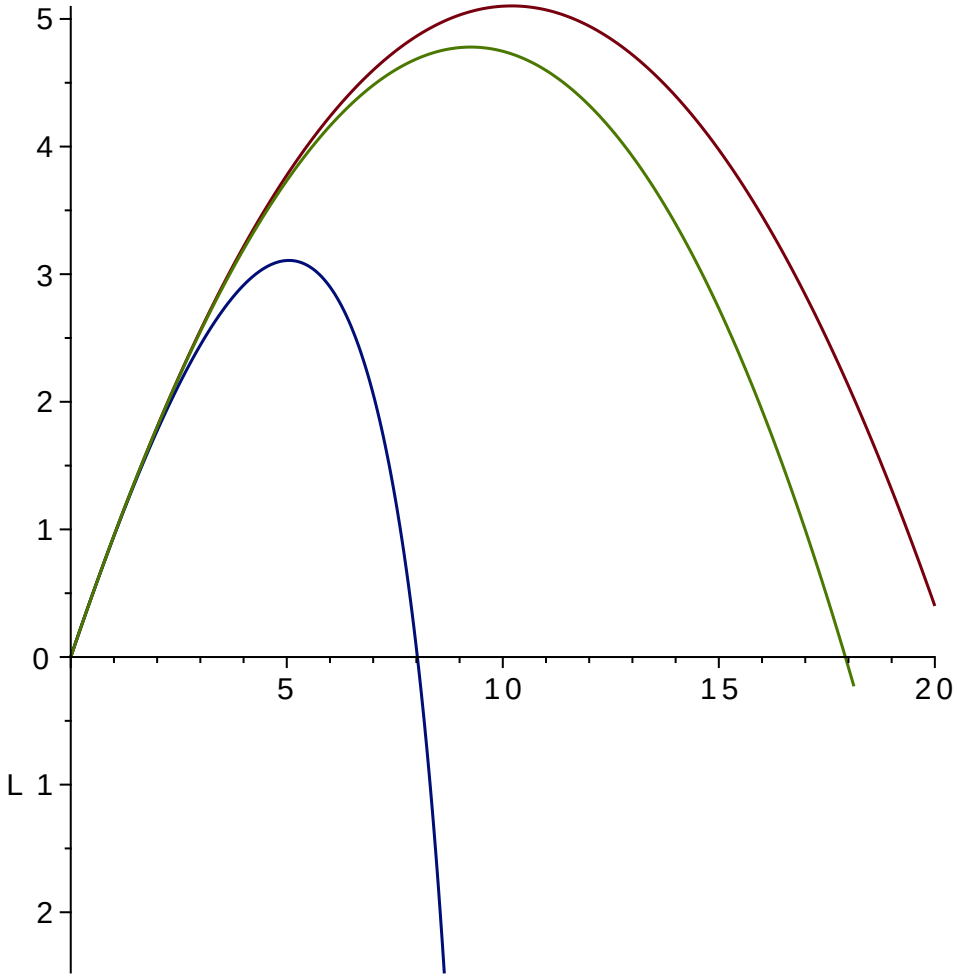
```
> plot({limit(VY(b,t),b=0), VY(1,t), VY(10,t)}, t=0..1);
```




```
> plot3d(VY(b,t), t=0..1, b=0..10,orientation=[-60,75],
        axes=NORMAL);
```




```
> v0x:=10: v0y:=10:
> plot({[limit(X(b,t),b=0), limit(Y(b,t),b=0),t=0..2], [X(.1,t), Y(.1,
  t), t=0..2], [X(1,t), Y(1,t), t=0..2]});
```




```
> Pgrav := (b,t) -> m*g*abs(VY(b,t));
```

$$Pgrav := (b,t) \rightarrow m\$g\$|VY(b,t)| \quad (23)$$

```
> Pdrag := (b,t) -> -b*VY(b,t)^2;
```

$$Pdrag := (b,t) \rightarrow -b\$VY(b,t)^2 \quad (24)$$

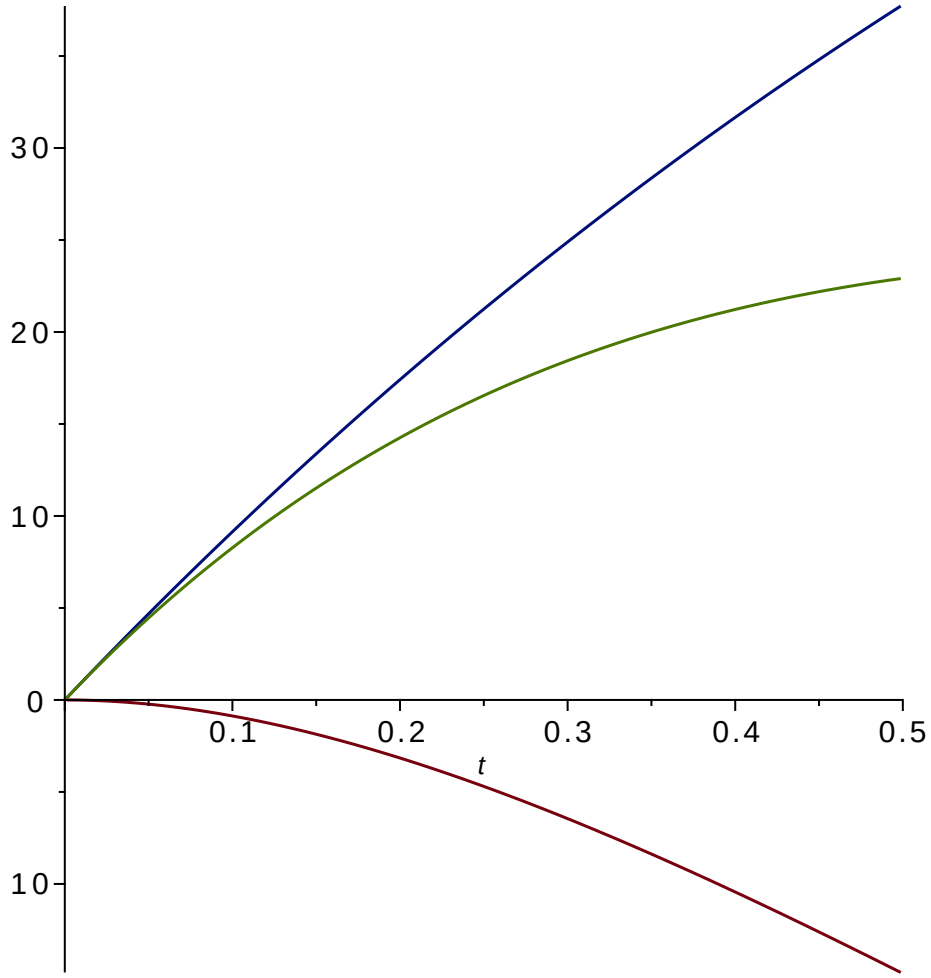
```
> Pnet := (b,t) -> Pgrav(b,t) + Pdrag(b,t);
```

$$Pnet := (b,t) \rightarrow Pgrav(b,t) + Pdrag(b,t) \quad (25)$$

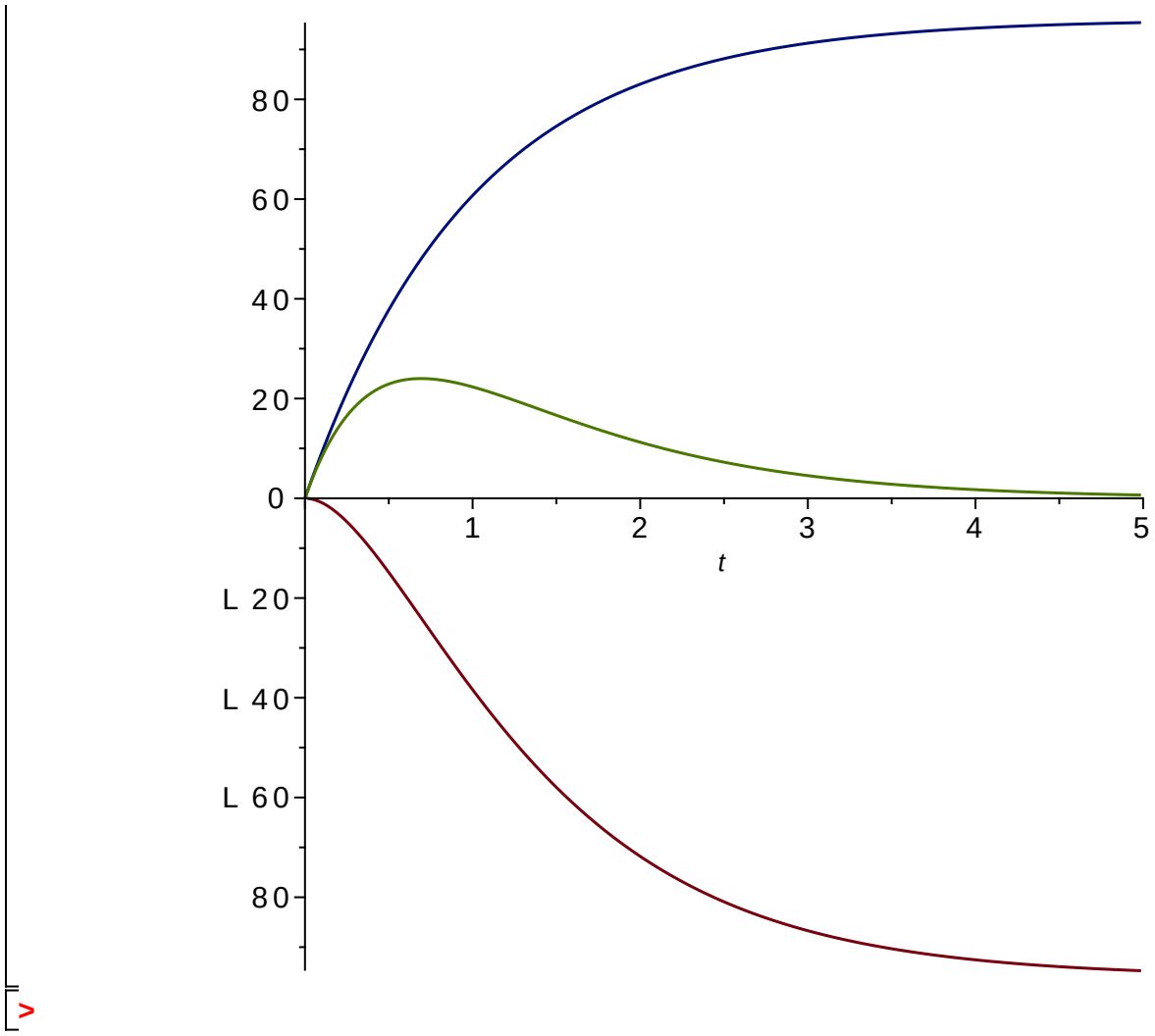
```
> PL := (b,T) -> plot({Pgrav(b,t), Pdrag(b,t), Pnet(b,t)}, t=0..T);
```

$$PL := (b,T) \rightarrow plot(\{Pdrag(b,t), Pgrav(b,t), Pnet(b,t)\}, t = 0..T) \quad (26)$$

```
> m :=1: g := 9.8: v0y := 0:
> PL(1,0.5);
```




```
> PL(1,5);
```


80
60
40
20
0
1
2
3
4
5
t
L 20
L 40
L 60
80