Notes for
ECE-606: Spring 2013

L4: Solving the Wave Eq.

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quantum effects

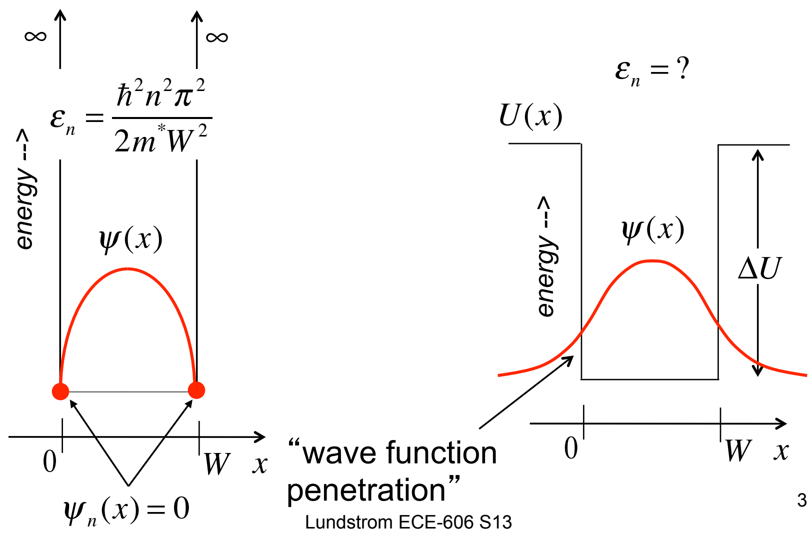
1) Plane waves $\Psi(x,t) = e^{\pm i(kx-\omega t)}$

2) Quantum confinement $\psi(x) = \sin k_n x \quad \varepsilon_n = \frac{\hbar^2 k_n^2}{2m^*} = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$

3) Plane waves **and** quantum confinement

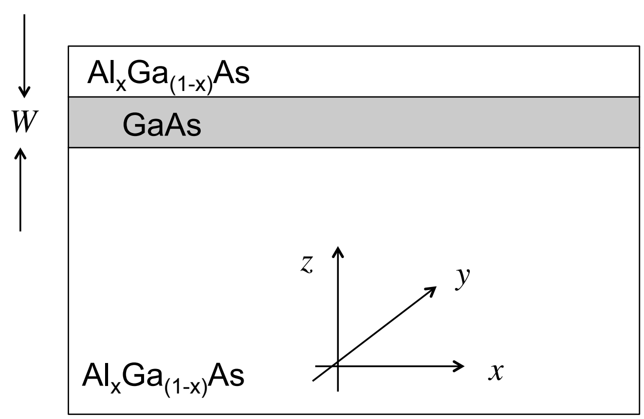
4) Quantum reflection and tunneling

infinite vs. finite quantum well



quantum wells

$$\psi(x,y,z) = ?$$



2D wavefunction

$$\frac{-\hbar^2}{2m_0}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$$

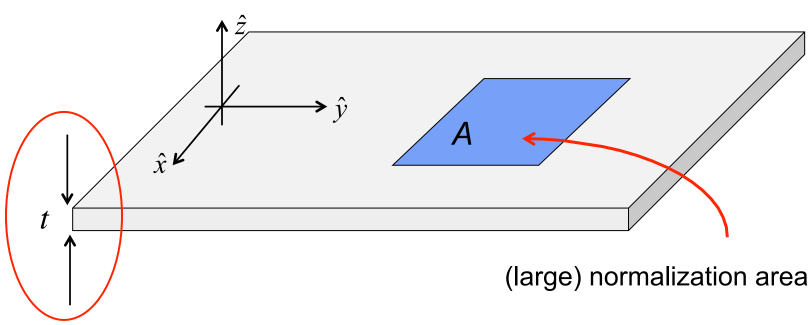
$$\frac{-\hbar^2}{2m_0}\nabla^2\psi(x,y,z)+U(x)\psi(x,y,z)=E\psi(x,y,z)$$

$$\psi(x,y,z)=X(x)Y(y)Z(z)$$

$$\psi(x,y,z)\propto e^{\pm ik_x x}e^{\pm ik_y y}\phi(z)$$

$$\psi(x,y,z)\propto e^{\pm i\vec{k}_\parallel\bullet\vec{\rho}}\phi(z)$$

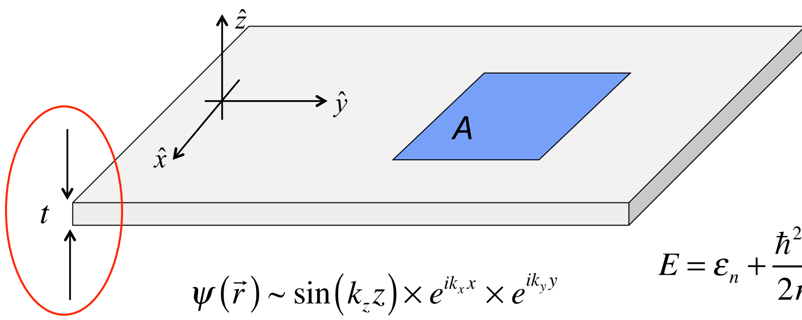
2D electrons



Semi-infinite in the x-y plane, but very thin in the z-direction.

$$\psi(\vec{r})\sim e^{i\vec{k}\bullet\vec{r}}\rightarrow\sin(k_z z)\times e^{ik_x x}\times e^{ik_y y}$$

2D electrons: subbands



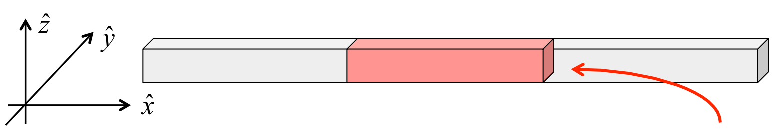
$$\psi(\vec{r}) \sim \sin(k_z z) \times e^{ik_x x} \times e^{ik_y y}$$

$$E = \varepsilon_n + \frac{\hbar^2 k_\parallel^2}{2m_0}$$

$$\psi(z=0) = \psi(z=t) = 0$$

$$k_z t = n\pi \qquad k_z = \frac{n\pi}{t} \qquad \varepsilon_n = \frac{\hbar^2 n^2 \pi^2}{2m_0 t^2}$$

1D electrons

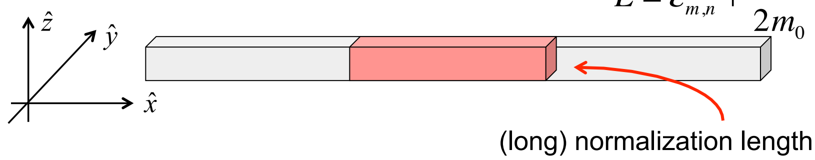


semi-infinite in along the x-direction, but very small in the y- and z-directions.

$$\psi(\vec{r}) \sim e^{i\vec{k}\bullet\vec{r}} \rightarrow \sin(k_y y)\sin(k_z z) \times e^{ik_x x}$$

1D electrons

$$E = \varepsilon_{m,n} + \frac{\hbar^2 k_x^2}{2m_0}$$



$$\psi(\vec{r}) \sim e^{i\vec{k}\bullet\vec{r}} \rightarrow \sin(k_y y)\sin(k_z z) \times e^{ik_x x}$$

$$\psi(y=0) = \psi(y=t_y) = 0 \qquad \psi(z=0) = \psi(z=t_z) = 0$$

$$k_y t_y = m\pi \qquad k_y = \frac{m\pi}{t_y} \qquad k_z t_z = n\pi \qquad k_z = \frac{n\pi}{t_z}$$

$$\varepsilon_{m,n} = \frac{\hbar^2 m^2 \pi^2}{2m_0 t_y^2} + \frac{\hbar^2 n^2 \pi^2}{2m_0 t_z^2}$$

quantum effects

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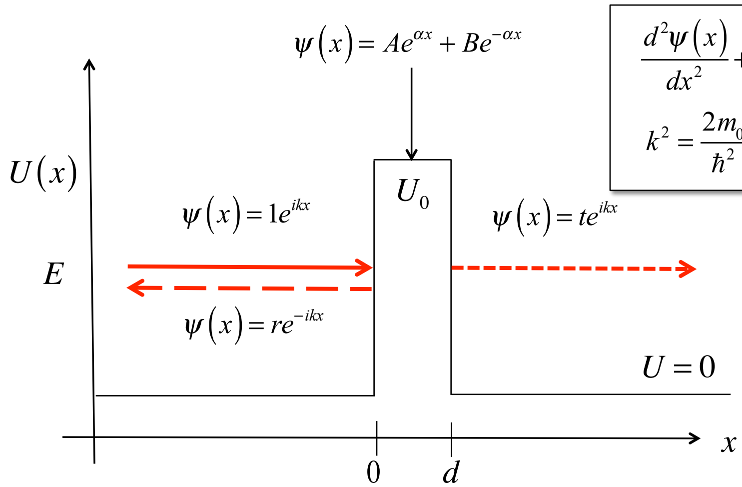
tunneling

$$\frac{d^2\psi(x)}{dx^2}+k^2\psi(x)=0$$

$$k^2=\frac{2m_0}{\hbar^2}\left[E-U(x)\right]$$



tunneling

$$\frac{d^2\psi(x)}{dx^2}+k^2\psi(x)=0$$

$$k^2=\frac{2m_0}{\hbar^2}\left[E-U(x)\right]$$

$x < 0$:

$$\psi(x)=1e^{ikx}+re^{-ikx}$$

$0 < x < d$:

$$\psi(x)=Ae^{\alpha x}+Be^{-\alpha x}$$

$x > d$

$$\psi(x)=te^{ikx}$$

$x = 0$:

$$1+r=A+B$$

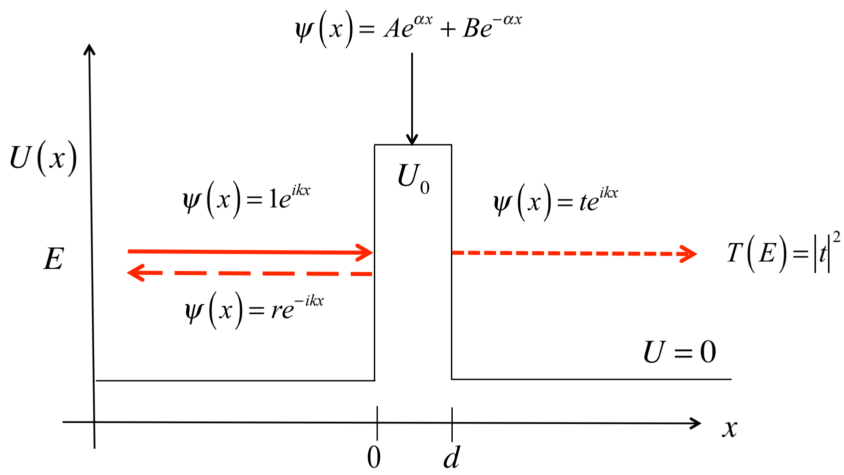
$$ik(1-r)=\alpha(A-B)$$

$x = d$:

$$Ae^{\alpha d}+Be^{-\alpha d}=re^{ikd}$$

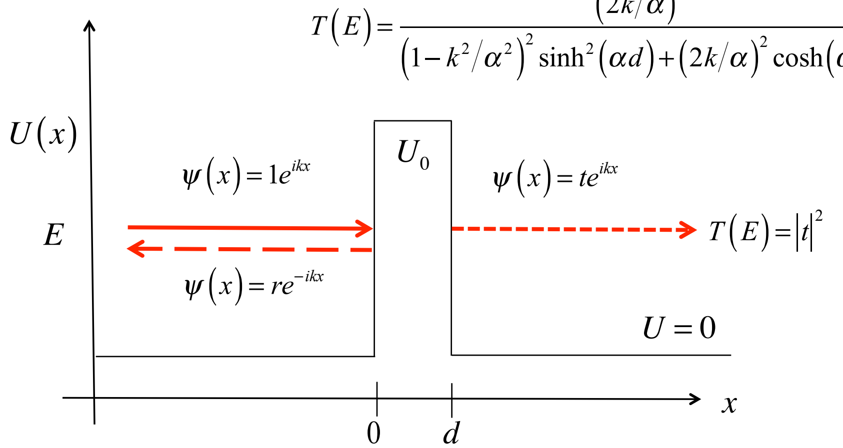
$$\alpha\left(Ae^{\alpha d}-Be^{-\alpha d}\right)=ikte^{ikd}$$

tunneling



tunneling

$$T(E) = \frac{(2k/\alpha)^2}{(1 - k^2/\alpha^2)^2 \sinh^2(\alpha d) + (2k/\alpha)^2 \cosh(\alpha d)}$$



tunneling

$$T(E) = \frac{(2k/\alpha)^2}{(1-k^2/\alpha^2)^2 \sinh^2(\alpha d) + (2k/\alpha)^2 \cosh(\alpha d)}$$

$$T(E) \approx 16\left(\frac{E}{U_0}\right)\left(1-\frac{E}{U_0}\right)\exp\left(-2d\sqrt{2m_0(U_0-E)/\hbar^2}\right)$$

$$T(E) \approx \exp\left(-2d\sqrt{2m_0(U_0-E)/\hbar^2}\right)$$