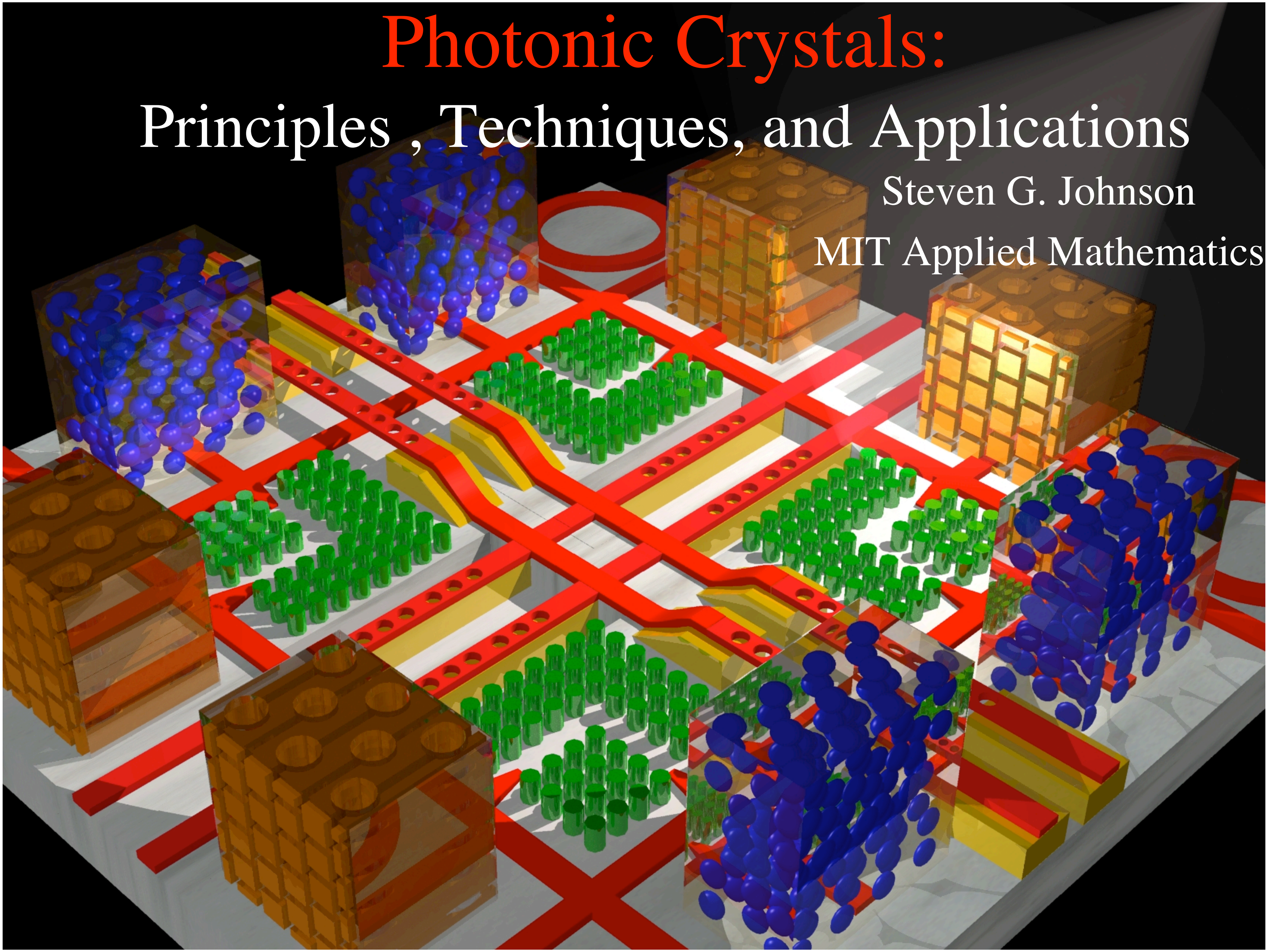
Photonic Crystals:

Principles , Techniques, and Applications

Steven G. Johnson

MIT Applied Mathematics



Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Perturbations, tuning, and disorder

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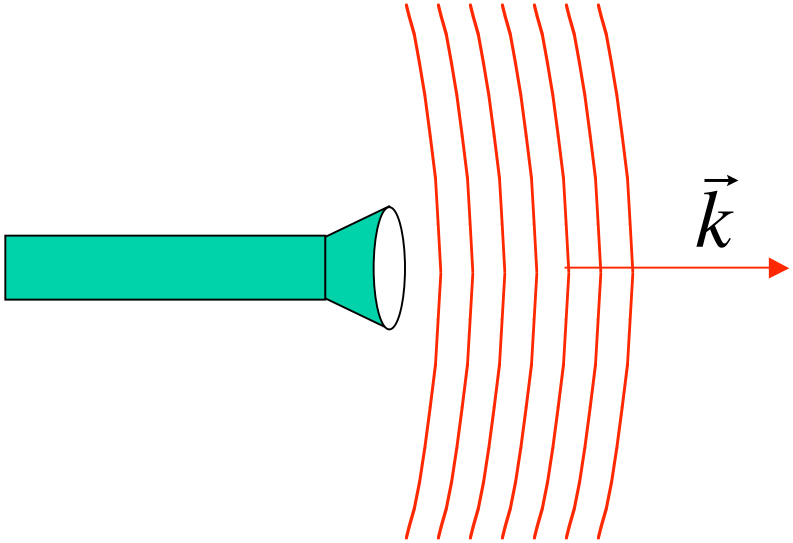
To Begin: A Cartoon in 2d

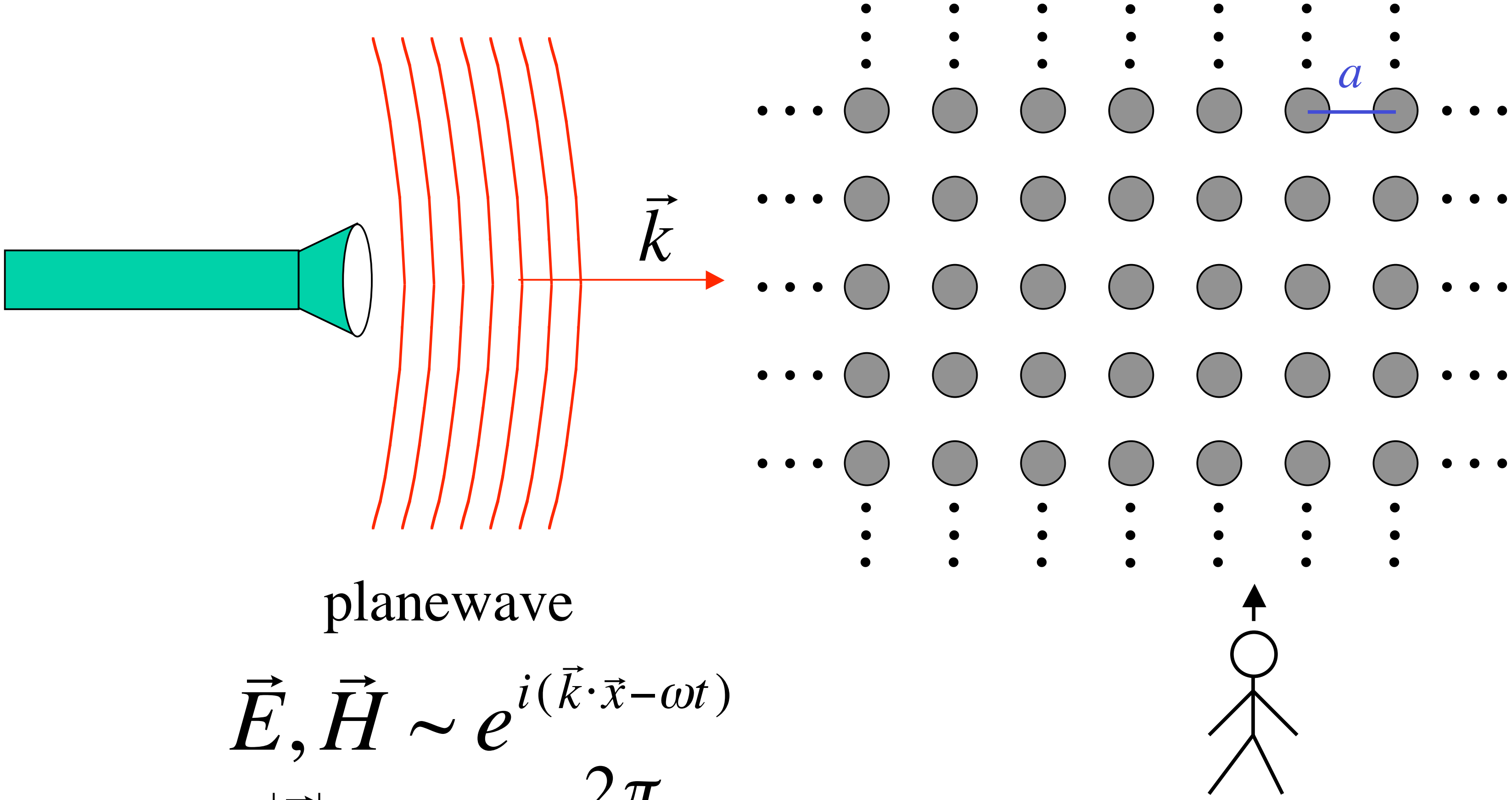


planewave

$$\vec{E},\vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

To Begin: A Cartoon in 2d



$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x} - \omega t)}$$

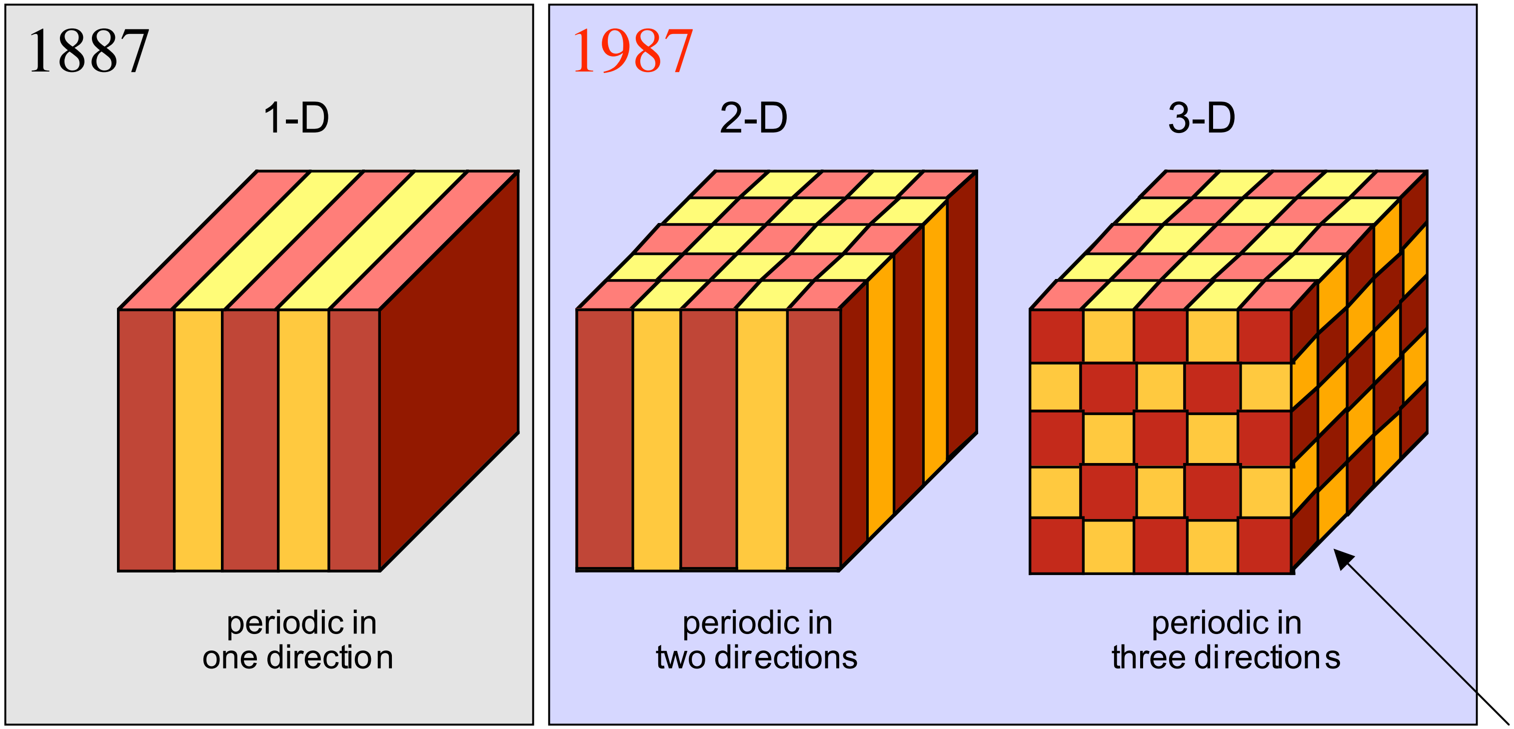
$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

for most λ, beam(s) propagate through crystal without scattering (scattering cancels coherently)

...but for some λ (~ 2a), no light can propagate: a photonic band gap

Photonic Crystals

periodic electromagnetic media



1887

1-D

periodic in
one direction

1987

2-D

periodic in
two directions

3-D

periodic in
three directions

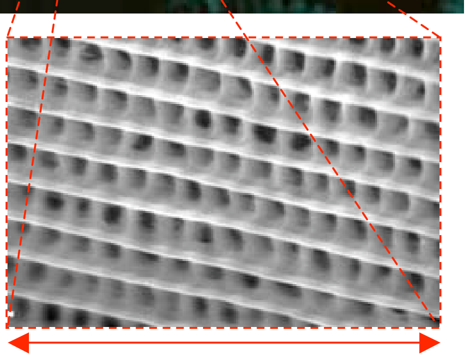
(need a
more
complex
topology)

with photonic band gaps: “optical insulators”

Photonic Crystals in Nature

Morpho butterfly

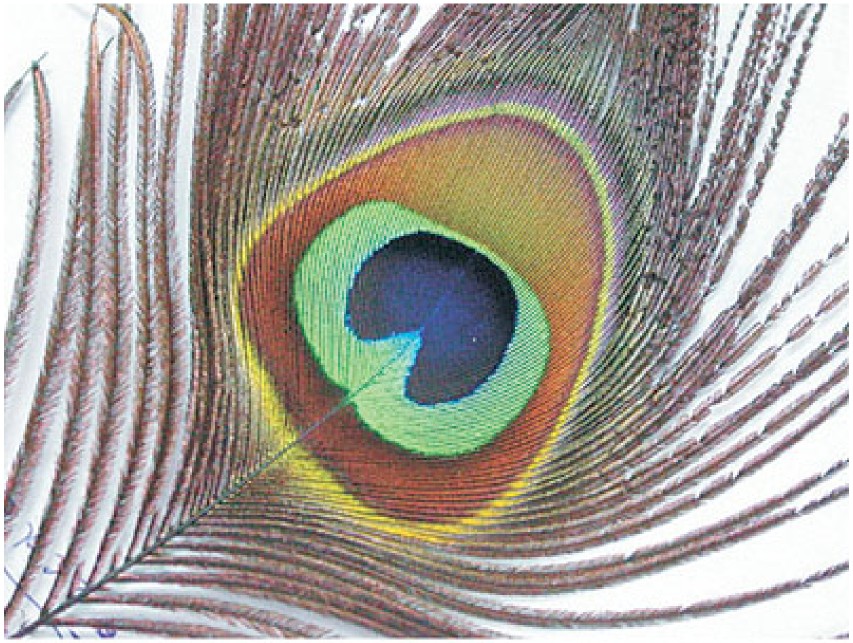
wing scale:

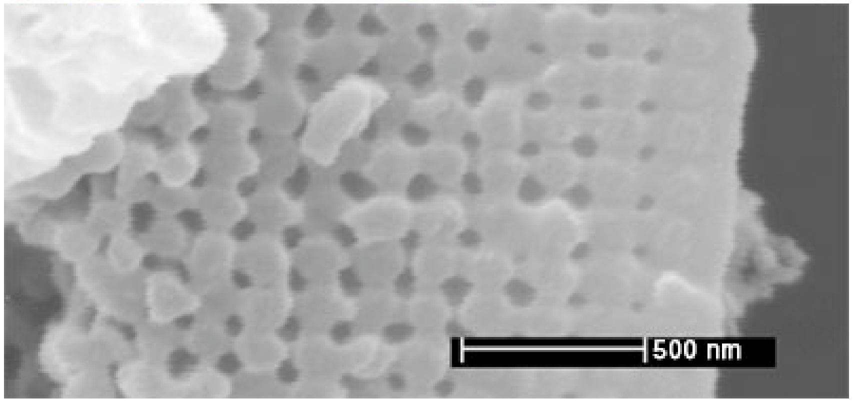


6.21μm

[L. P. Biró *et al.*, *PRE* **67**, 021907 (2003)]

Peacock feather



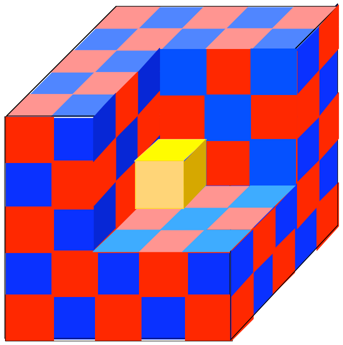


[J. Zi *et al*, *Proc. Nat. Acad. Sci. USA*, **100**, 12576 (2003)]

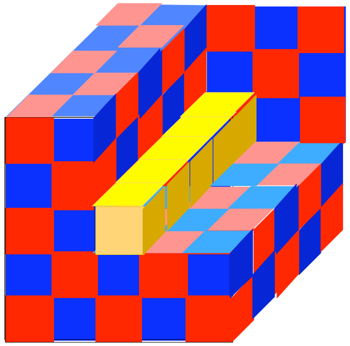
[figs: Blau, *Physics Today* **57**, 18 (2004)]

Photonic Crystals

periodic electromagnetic media



can trap light in cavities

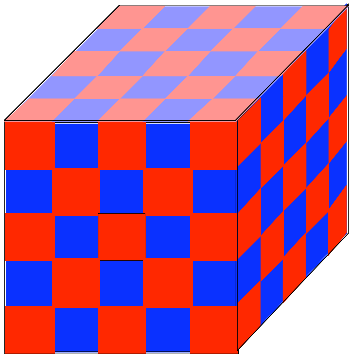


and waveguides (“wires”)

with photonic band gaps:
“optical insulators”
for holding and controlling light

Photonic Crystals

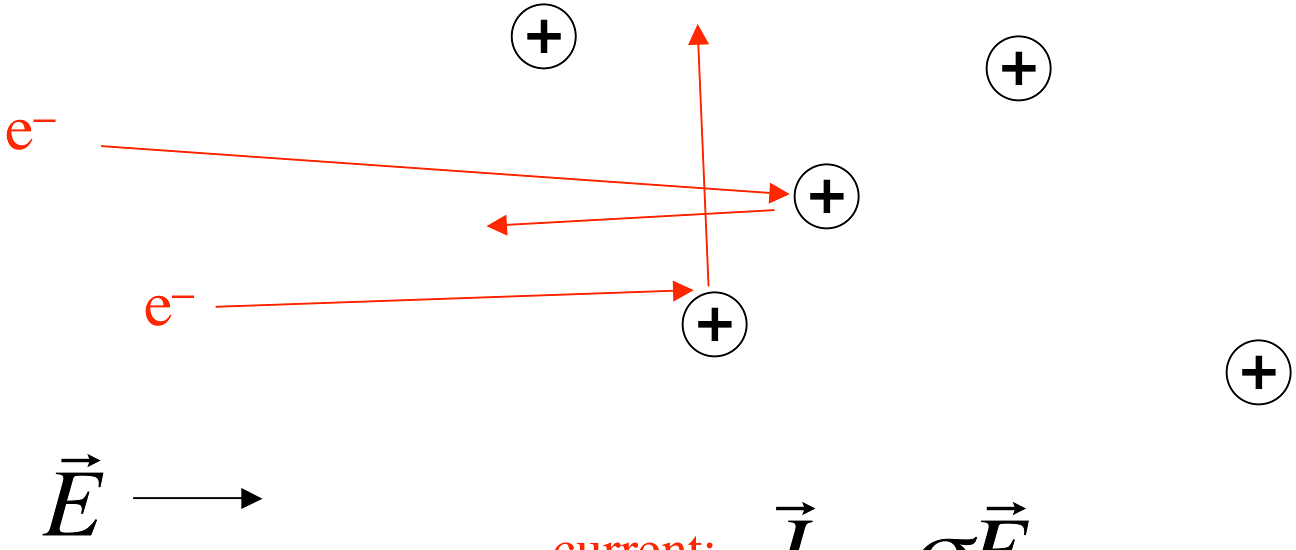
periodic electromagnetic media



But how can we understand such complex systems?
Add up the infinite sum of scattering? Ugh!

A mystery from the 19th century

conductive material



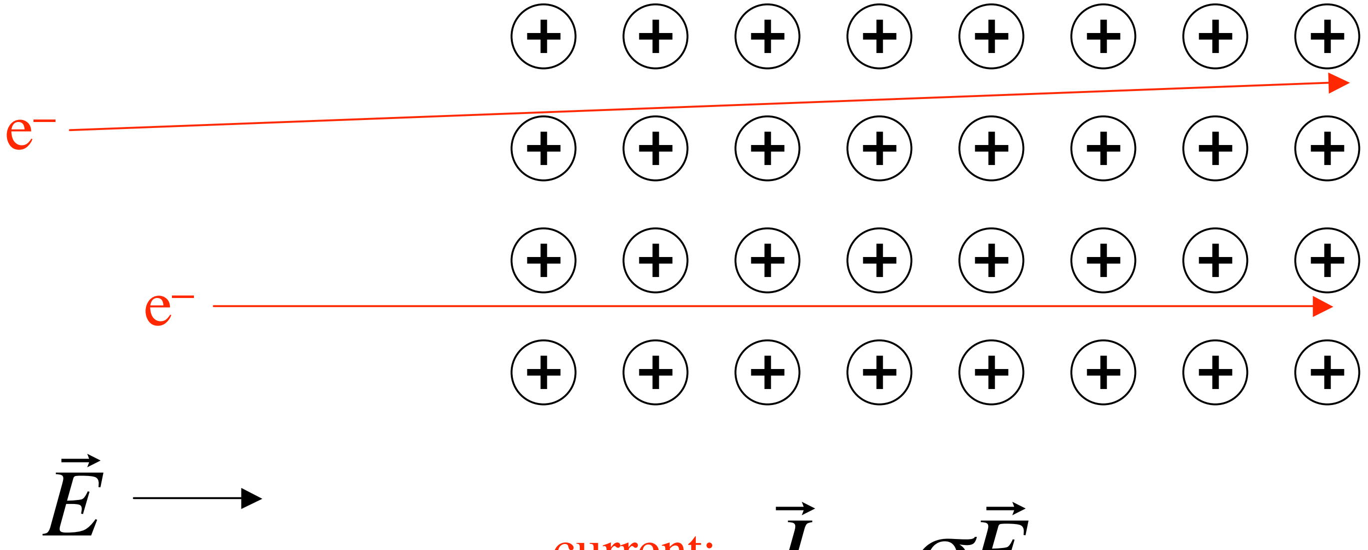
current: $\vec{J} = \sigma\vec{E}$

conductivity (measured)

mean free path (distance) of electrons

A mystery from the 19th century

crystalline conductor (e.g. copper)



10's of periods!

current: $\vec{J} = \sigma\vec{E}$

conductivity (measured)

↓

mean free path (distance) of electrons

A mystery solved…

① electrons are waves (quantum mechanics)

② waves in a periodic medium can propagate without scattering:

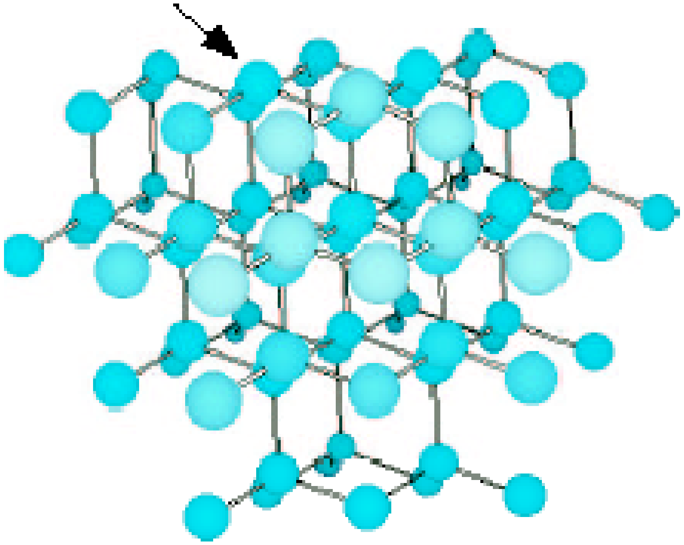
Bloch's Theorem (1d: Floquet's)

The foundations do not depend on the specific wave equation.

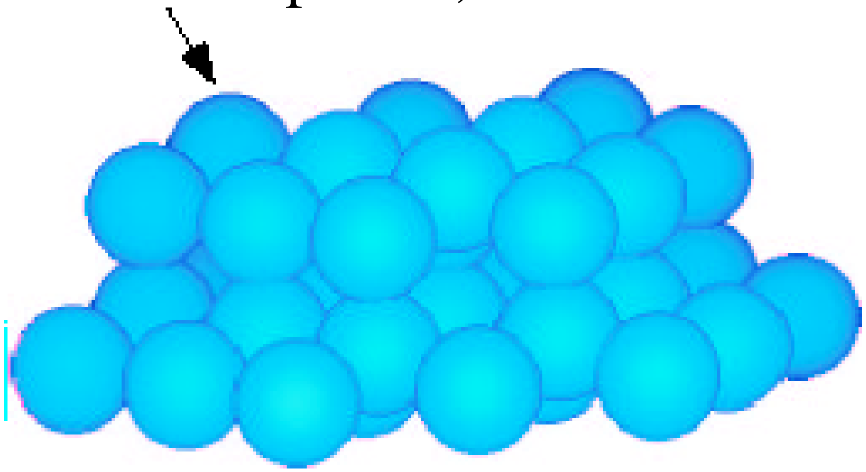
Electronic and Photonic Crystals

Periodic Medium

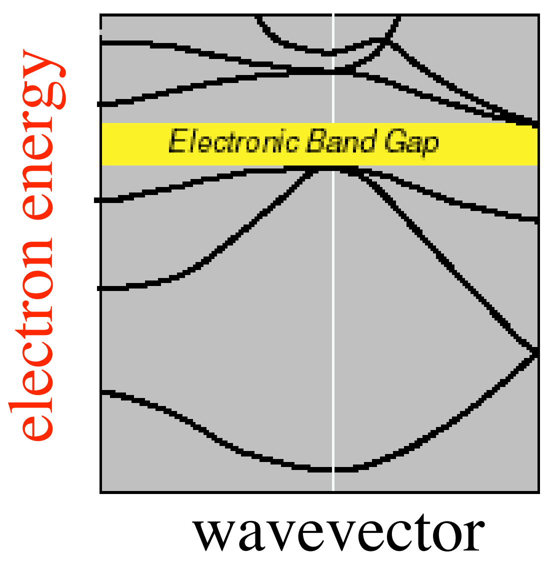
atoms in diamond structure

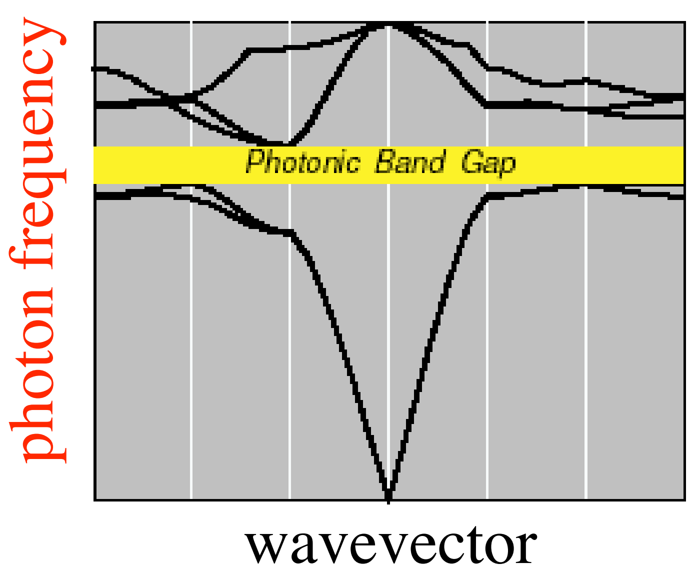


dielectric spheres, diamond lattice



Bloch waves: **Band Diagram**

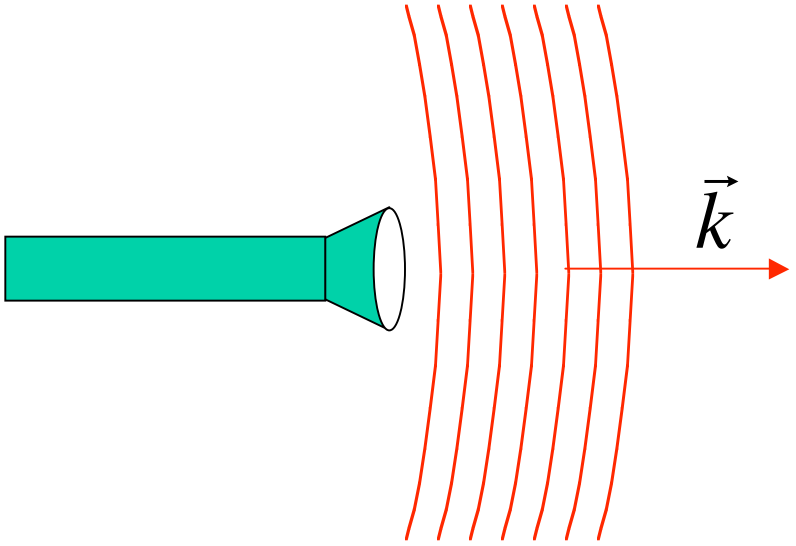


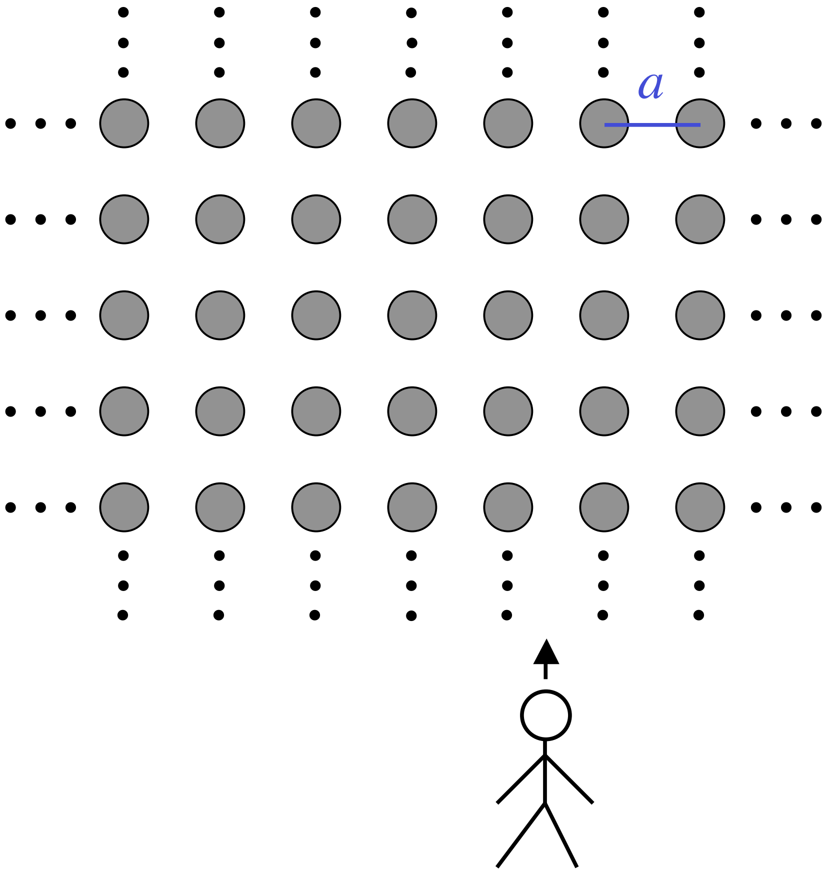


interacting: hard problem

non-interacting: “easy” problem

Time to Analyze the Cartoon





planewave

$$\vec{E}, \vec{H} \sim e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

$$\left|\vec{k}\right| = \omega / c = \frac{2\pi}{\lambda}$$

for most λ, beam(s) propagate through crystal without scattering (scattering cancels coherently)

...but for some λ (~ 2*a*), no light can propagate: a photonic band gap

Fun with Math

$$\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{H} = i\frac{\omega}{c}\vec{H}$$

First task:

get rid of this mess

$$\vec{\nabla}\times\vec{H} = \varepsilon\frac{1}{c}\frac{\partial}{\partial t}\vec{E} + \cancelto{0}{\vec{J}} = i\frac{\omega}{c}\varepsilon\vec{E}$$

dielectric function $\varepsilon(\mathbf{x}) = n^2(\mathbf{x})$

$$\underset{\text{eigen-operator}}{\underline{\nabla\times\frac{1}{\varepsilon}\nabla\times}}\,\vec{H} = \underset{\text{eigen-value}}{\underline{\left(\frac{\omega}{c}\right)^2}}\,\underset{\text{eigen-state}}{\vec{H}}$$

+ constraint

$$\nabla\cdot\vec{H} = 0$$

Hermitian Eigenproblems

$$\underset{\text{eigen-operator}}{\underline{\nabla\times\frac{1}{\varepsilon}\nabla\times}}\,\underset{\text{eigen-state}}{\vec{H}} = \underset{\text{eigen-value}}{\underline{\left(\frac{\omega}{c}\right)^{2}}}\,\vec{H}$$

\+ constraint

$$\nabla\cdot\vec{H} = 0$$

Hermitian for real (lossless) ε

➡ well-known properties from linear algebra:

ω are real (lossless)

eigen-states are orthogonal

eigen-states are complete (give all solutions)

Periodic Hermitian Eigenproblems

[G. Floquet, “Sur les équations différentielles linéaries à coefficients périodiques,” *Ann. École Norm. Sup.* **12**, 47–88 (1883).]
[F. Bloch, “Über die quantenmechanik der electronen in kristallgittern,” *Z. Physik* **52**, 555–600 (1928).]

if eigen-operator is periodic, then Bloch-Floquet theorem applies:

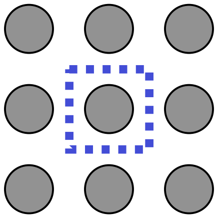
can choose: $$\vec{H}(\vec{x},t) = e^{i\left(\vec{k}\cdot\vec{x}-\omega t\right)}\vec{H}_{\vec{k}}(\vec{x})$$

planewave

periodic “envelope”

Corollary 1: **k** is conserved, *i.e.* no scattering of Bloch wave

Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
so ω are discrete $\omega_n(\mathbf{k})$



Periodic Hermitian Eigenproblems

Corollary 2: $\vec{H}_{\vec{k}}$ given by finite unit cell,
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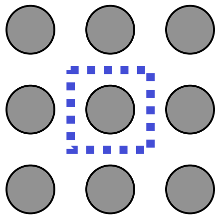
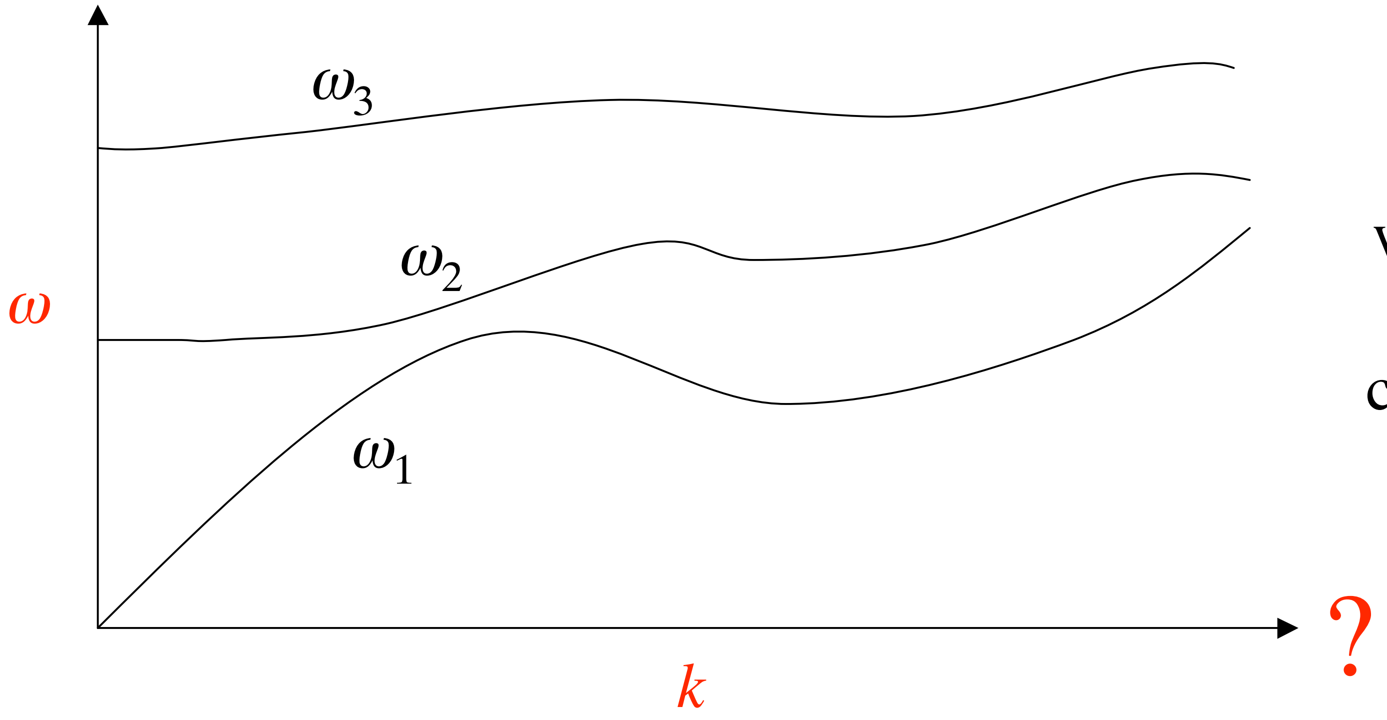


band diagram (dispersion relation)

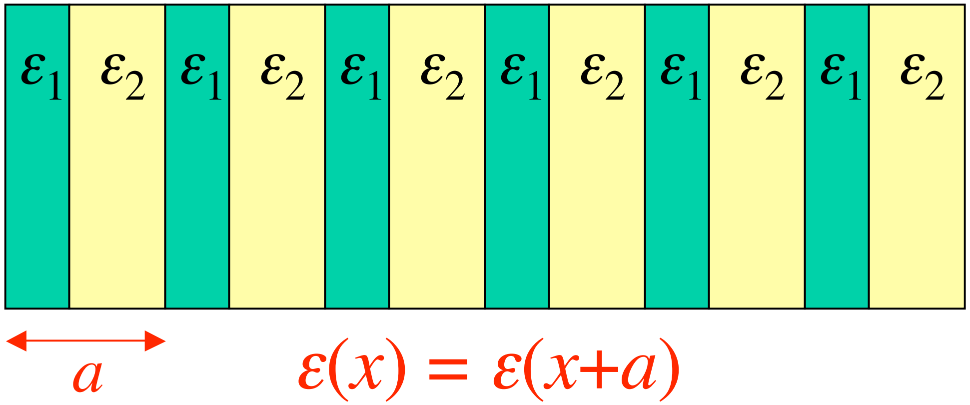


map of
what states
exist &
can interact

range of k?

Periodic Hermitian Eigenproblems in 1d

$$H(x) = e^{ikx} H_k(x)$$



$\varepsilon(x) = \varepsilon(x+a)$

Consider $k+2\pi/a$: $$e^{i(k+\frac{2\pi}{a})x} H_{k+\frac{2\pi}{a}}(x) = e^{ikx} \underline{\left[e^{i\frac{2\pi}{a}x} H_{k+\frac{2\pi}{a}}(x) \right]}$$

periodic!
satisfies same
equation as H_k
$= H_k$

k is periodic:

$k + 2\pi/a$ equivalent to k

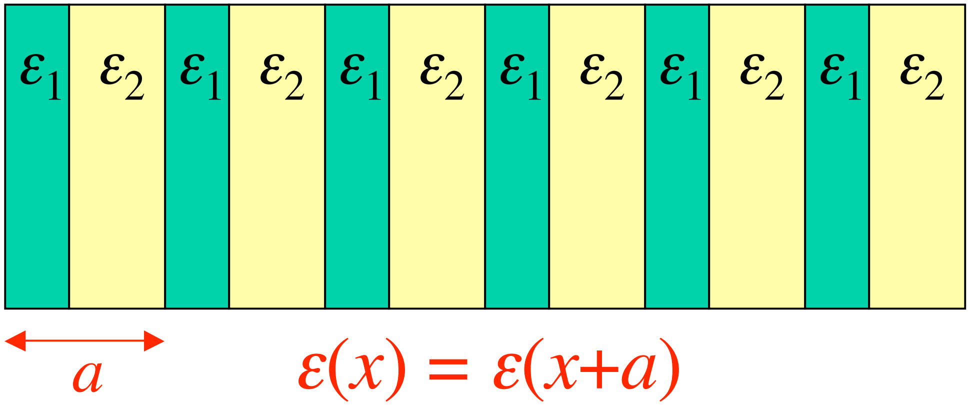
“quasi-phase-matching”

Periodic Hermitian Eigenproblems in 1d

k is periodic:

k + 2π/*a* equivalent to *k*

“quasi-phase-matching”



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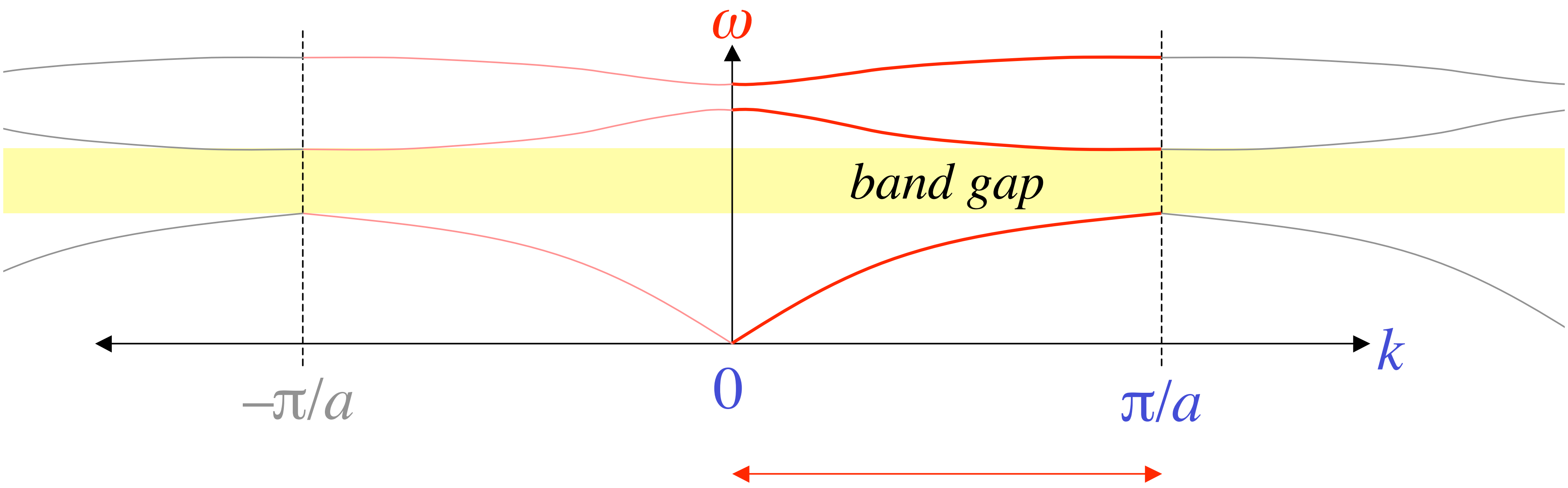
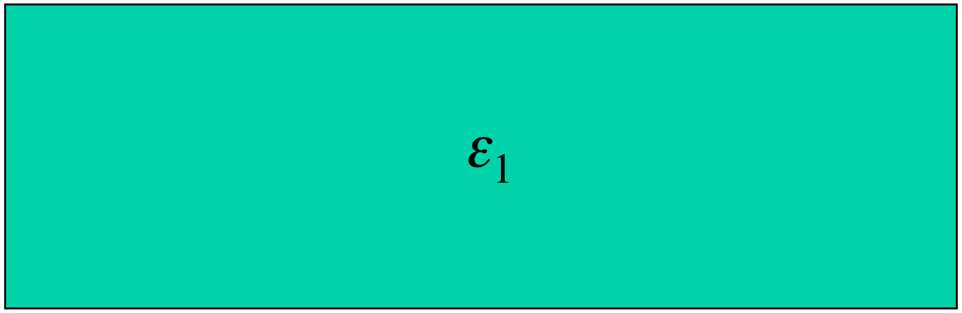


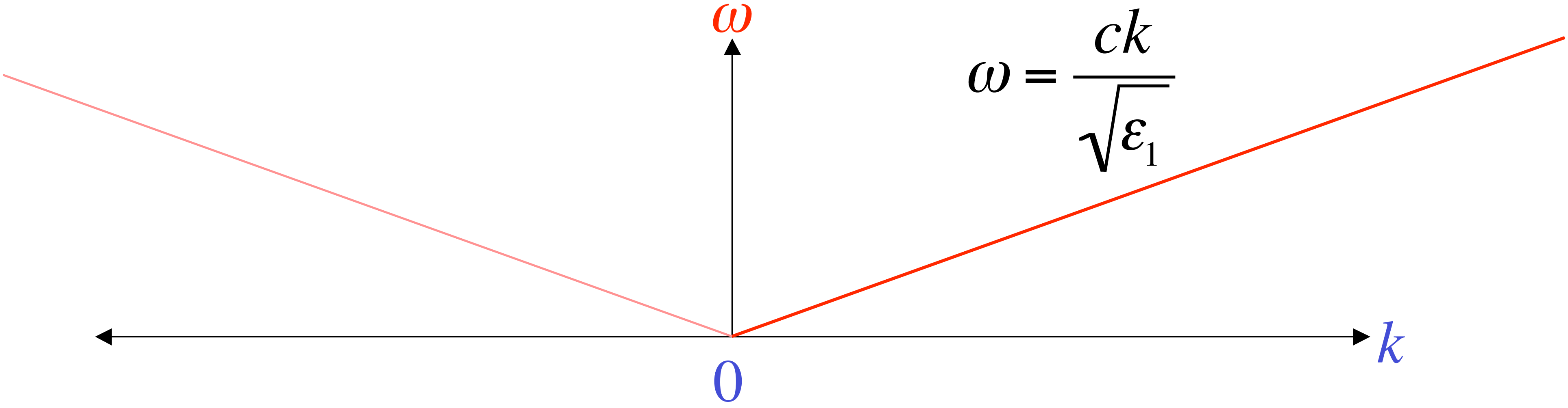
irreducible Brillouin zone

Any 1d Periodic System has a Gap

[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

Start with
a uniform (1d) medium:



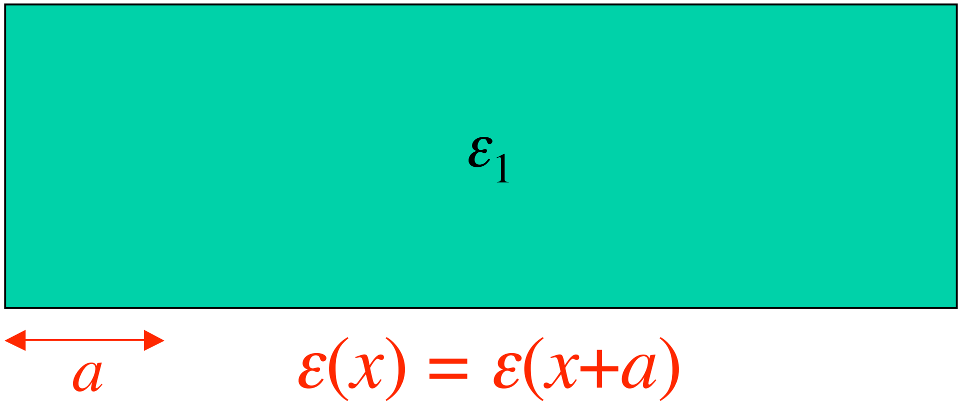


$$\omega = \frac{ck}{\sqrt{\varepsilon_1}}$$

Any 1d Periodic System has a Gap

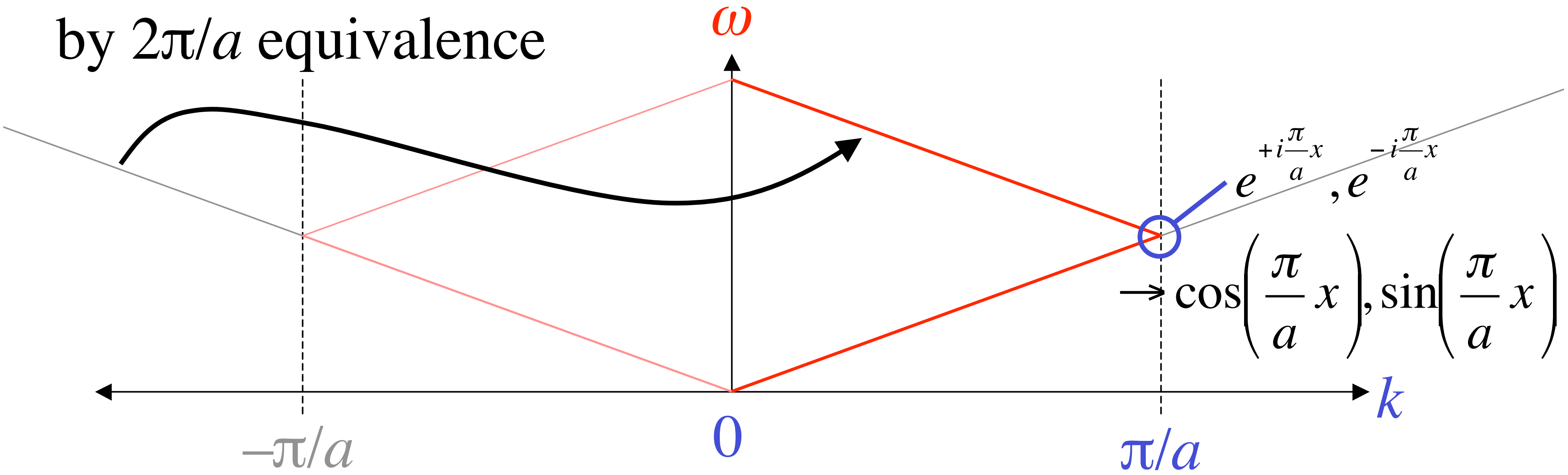
[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

Treat it as
“artificially” periodic



$\varepsilon(x) = \varepsilon(x+a)$

bands are “folded”
by 2π/*a* equivalence



$e^{+i\frac{\pi}{a}x}, e^{-i\frac{\pi}{a}x}$

$\rightarrow \cos\left(\frac{\pi}{a}x\right), \sin\left(\frac{\pi}{a}x\right)$

Any 1d Periodic System has a Gap

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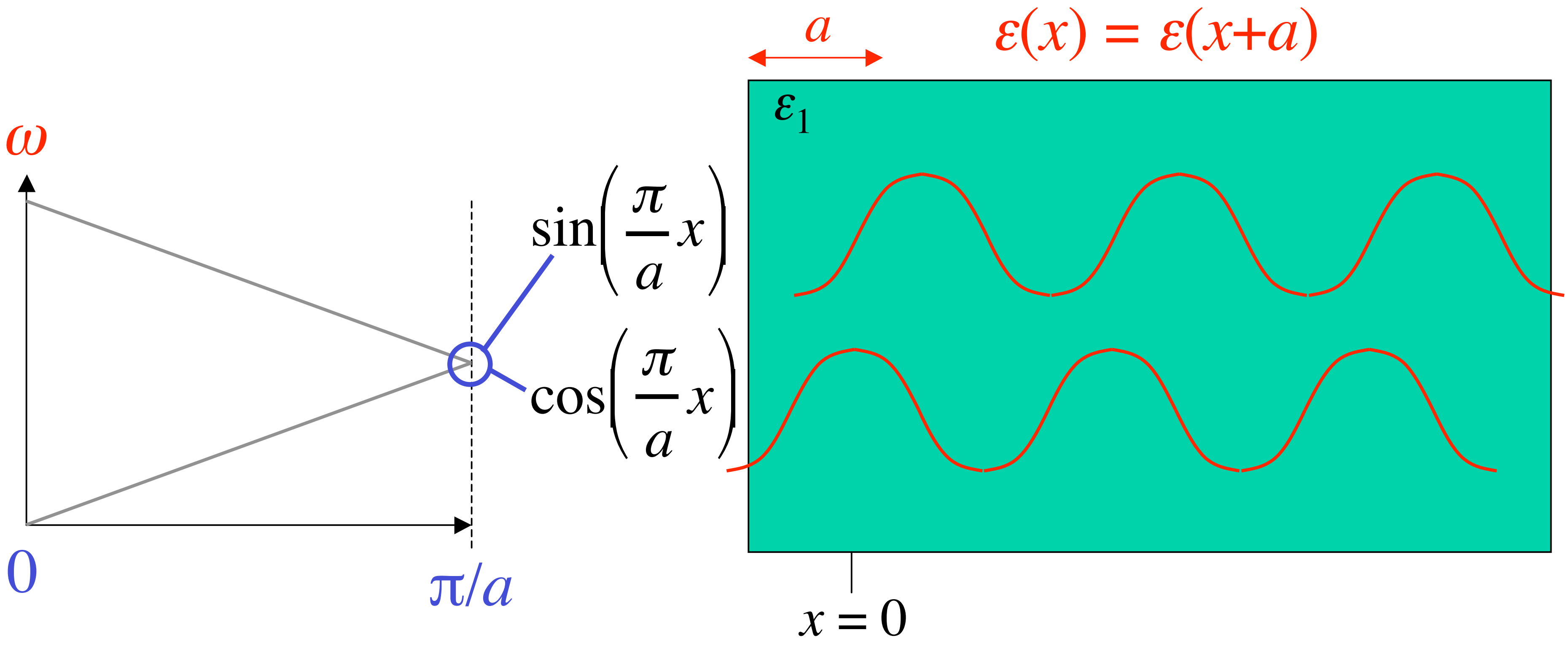
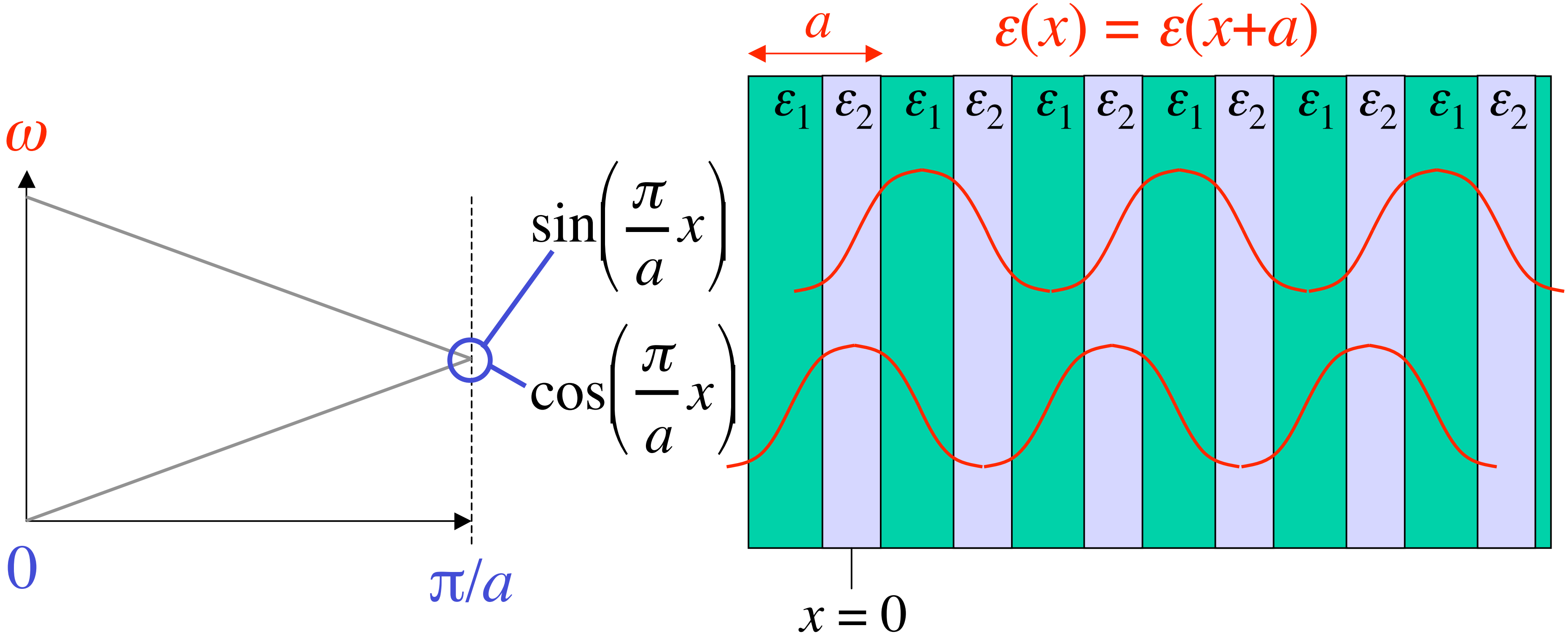


Any 1d Periodic System has a Gap

[Lord Rayleigh, “On the maintenance of vibrations by forces of double frequency, and on the propagation of waves through a medium endowed with a periodic structure,” *Philosophical Magazine* **24**, 145–159 (1887).]

Add a small
“real” periodicity
$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$



Any 1d Periodic System has a Gap

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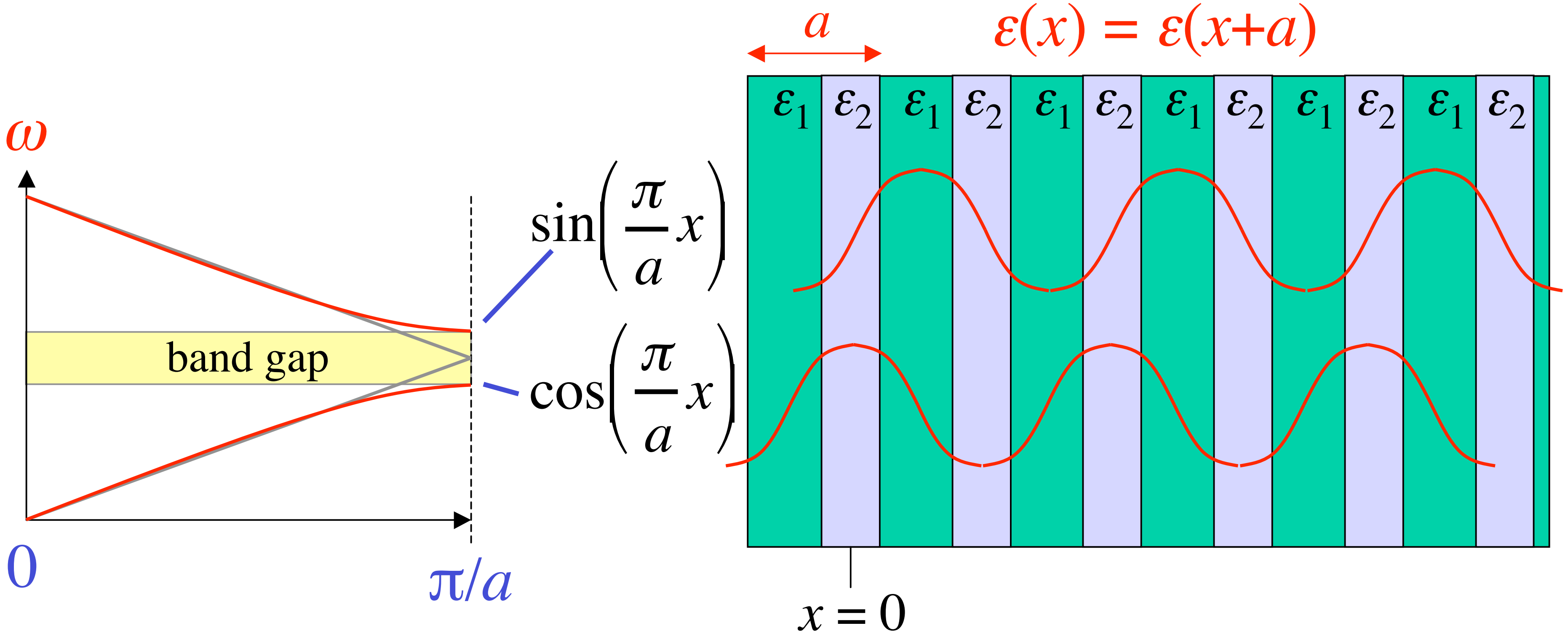
Add a small
“real” periodicity
$\varepsilon_2 = \varepsilon_1 + \Delta\varepsilon$

Splitting of degeneracy:
state concentrated in higher index (ε_2)
has lower frequency



Some 2d and 3d systems have gaps

- In general, eigen-frequencies satisfy Variational Theorem:

$$\omega_1(\vec{k})^2 = \min_{\substack{\vec{E}_1 \\ \nabla\cdot\varepsilon\vec{E}_1=0}} \frac{\int \left|\left(\nabla + i\vec{k}\right)\times\vec{E}_1\right|^2}{\int \varepsilon\left|\vec{E}_1\right|^2} c^2$$

"kinetic"

inverse
"potential"

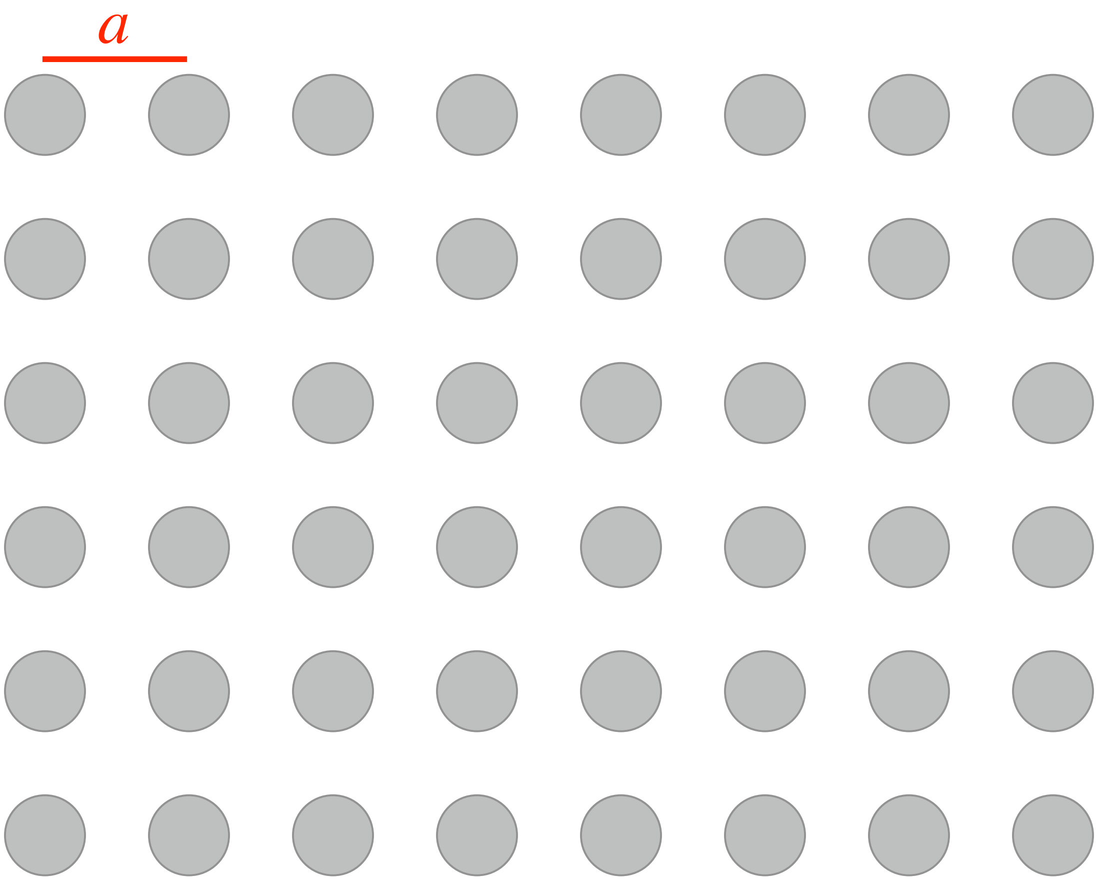
$$\omega_2(\vec{k})^2 = \min_{\substack{\vec{E}_2 \\ \nabla\cdot\varepsilon\vec{E}_2=0 \\ \int\varepsilon E_1^{*}\cdot E_2=0}} \text{"}\cdots\text{"}$$

bands "want" to be in high-ε

…but are forced out by orthogonality

–> band gap (maybe)

A 2d Model System

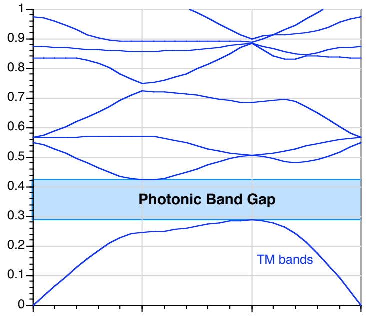


Square lattice of dielectric rods (ε = 12 ~ Si) in air (ε = 1)

Solving the Maxwell Eigenproblem

Finite cell ➔ *discrete* eigenvalues ω_n

Want to solve for $\omega_n(\mathbf{k})$,
& plot vs. “all” **k** for “all” *n*,



$$(\nabla + i\mathbf{k}) \times \frac{1}{\varepsilon}(\nabla + i\mathbf{k}) \times \mathbf{H}_n = \frac{\omega_n^2}{c^2}\mathbf{H}_n$$

constraint: $(\nabla + i\mathbf{k}) \cdot \mathbf{H} = 0$

where: $\mathbf{H}(x,y)\ e^{i(\mathbf{k}\cdot\mathbf{x} - \omega t)}$

① Limit range of **k**: irreducible Brillouin zone

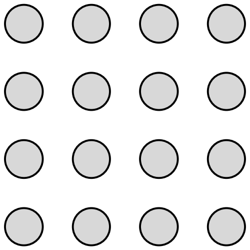
② Limit degrees of freedom: expand **H** in finite basis

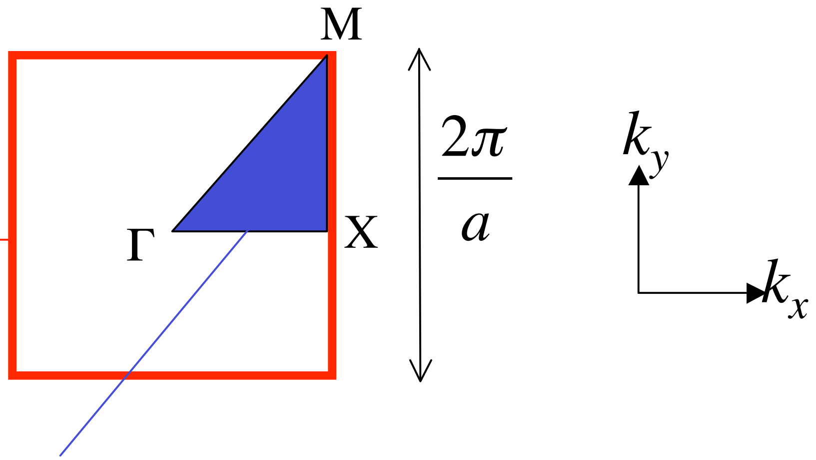
③ Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 1

① Limit range of **k**: irreducible Brillouin zone

—Bloch's theorem: solutions are periodic in **k**





first Brillouin zone

= minimum |**k**| "primitive cell"

irreducible Brillouin zone: reduced by symmetry

② Limit degrees of freedom: expand **H** in finite basis

③ Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 2a

① Limit range of **k**: irreducible Brillouin zone

② Limit degrees of freedom: expand **H** in finite basis (N)

$$|\mathbf{H}\rangle = \mathbf{H}(\mathbf{x}_t) = \sum_{m=1}^{N} h_m \mathbf{b}_m(\mathbf{x}_t)$$

solve: $\hat{A}|\mathbf{H}\rangle = \omega^2 |\mathbf{H}\rangle$

finite matrix problem: $Ah = \omega^2 Bh$

$$\langle \mathbf{f}|\mathbf{g}\rangle = \int \mathbf{f}^* \cdot \mathbf{g} \qquad A_{m\ell} = \langle \mathbf{b}_m|\hat{A}|\mathbf{b}_\ell\rangle \qquad B_{m\ell} = \langle \mathbf{b}_m|\mathbf{b}_\ell\rangle$$

③ Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 2b

① Limit range of **k**: irreducible Brillouin zone

② Limit degrees of freedom: expand **H** in finite basis

— must satisfy constraint: $(\nabla + i\mathbf{k}) \cdot \mathbf{H} = 0$

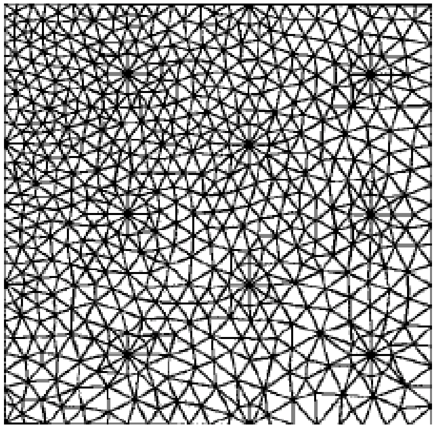
Planewave (FFT) basis

$$\mathbf{H}(\mathbf{x}_t) = \sum_{\mathbf{G}} \mathbf{H}_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{x}_t}$$

constraint: $\mathbf{H}_{\mathbf{G}} \cdot (\mathbf{G} + \mathbf{k}) = 0$

uniform "grid," periodic boundaries,
simple code, O(N log N)

Finite-element basis



[figure: Peyrilloux *et al.*,
J. Lightwave Tech.
21, 536 (2003)]

constraint, boundary conditions:

Nédélec elements

[Nédélec, *Numerische Math.*
35, 315 (1980)]

nonuniform mesh,
more arbitrary boundaries,
complex code & mesh, O(N)

③ Efficiently solve eigenproblem: iterative methods

Solving the Maxwell Eigenproblem: 3a

① Limit range of **k**: irreducible Brillouin zone

② Limit degrees of freedom: expand **H** in finite basis

③ Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Slow way: compute A & B, ask LAPACK for eigenvalues

— requires O(N^2) storage, O(N^3) time

Faster way:

— start with *initial guess* eigenvector h_0

— *iteratively* improve

— O(Np) storage, ~ O(Np^2) time for p eigenvectors

(p smallest eigenvalues)

Solving the Maxwell Eigenproblem: 3b

① Limit range of **k**: irreducible Brillouin zone

② Limit degrees of freedom: expand **H** in finite basis

③ Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Many iterative methods:

— Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …, Rayleigh-quotient minimization

Solving the Maxwell Eigenproblem: 3c

① Limit range of **k**: irreducible Brillouin zone

② Limit degrees of freedom: expand **H** in finite basis

③ Efficiently solve eigenproblem: iterative methods

$$Ah = \omega^2 Bh$$

Many iterative methods:

— Arnoldi, Lanczos, Davidson, Jacobi-Davidson, …, Rayleigh-quotient minimization

for Hermitian matrices, smallest eigenvalue ω_0 minimizes:

"variational theorem"

$$\omega_0^2 = \min_h \frac{h' Ah}{h' Bh}$$

minimize by preconditioned conjugate-gradient (or…)

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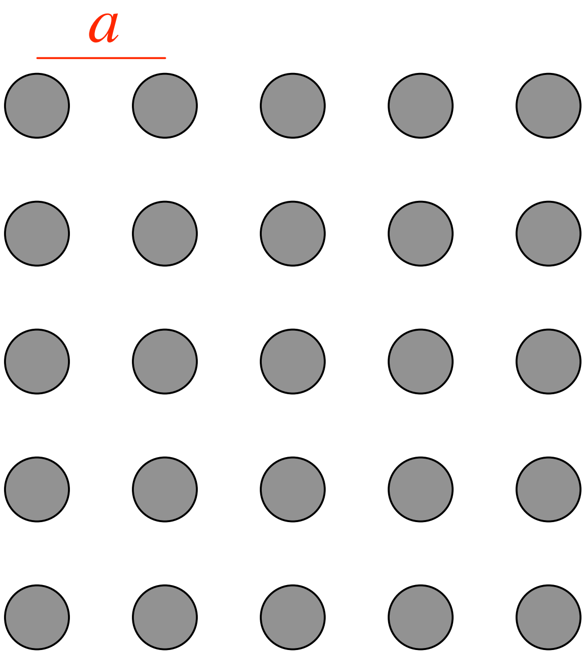
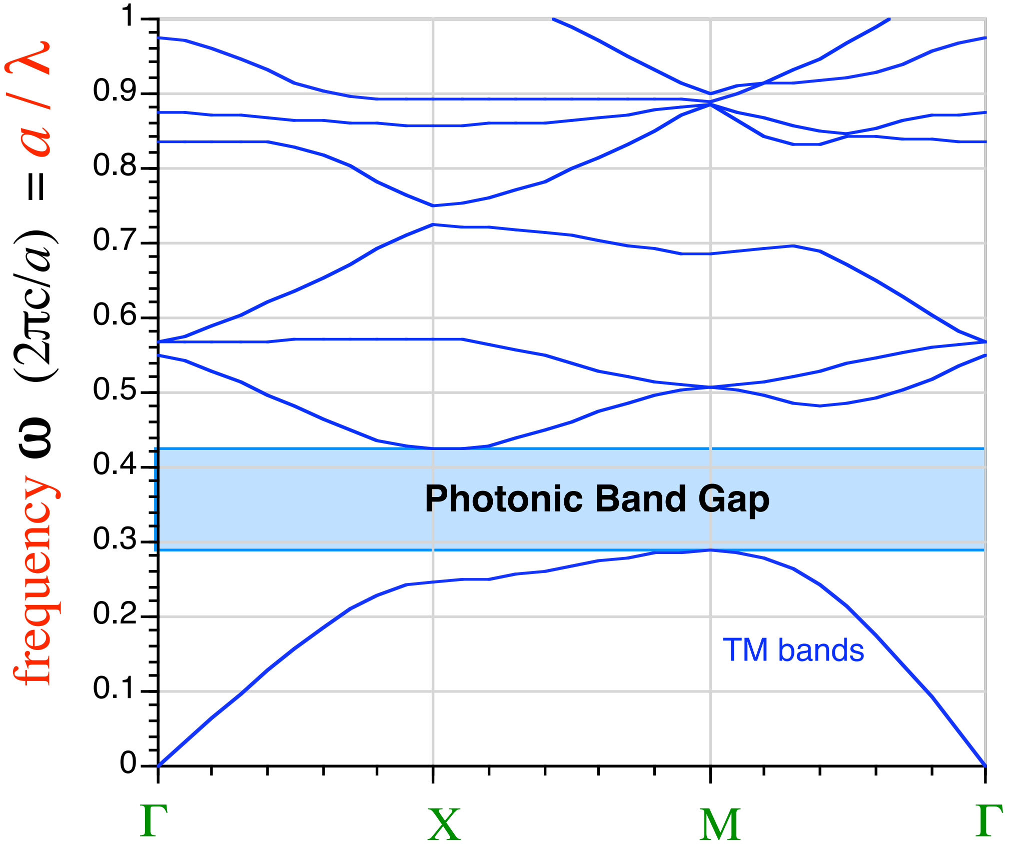
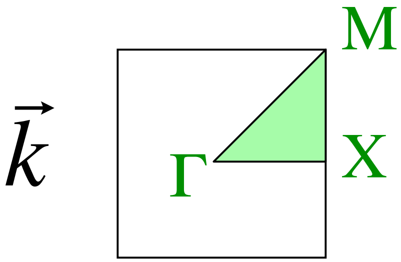
2d periodicity, ε=12:1

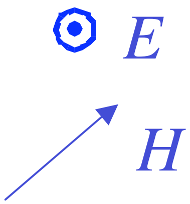




irreducible Brillouin zone

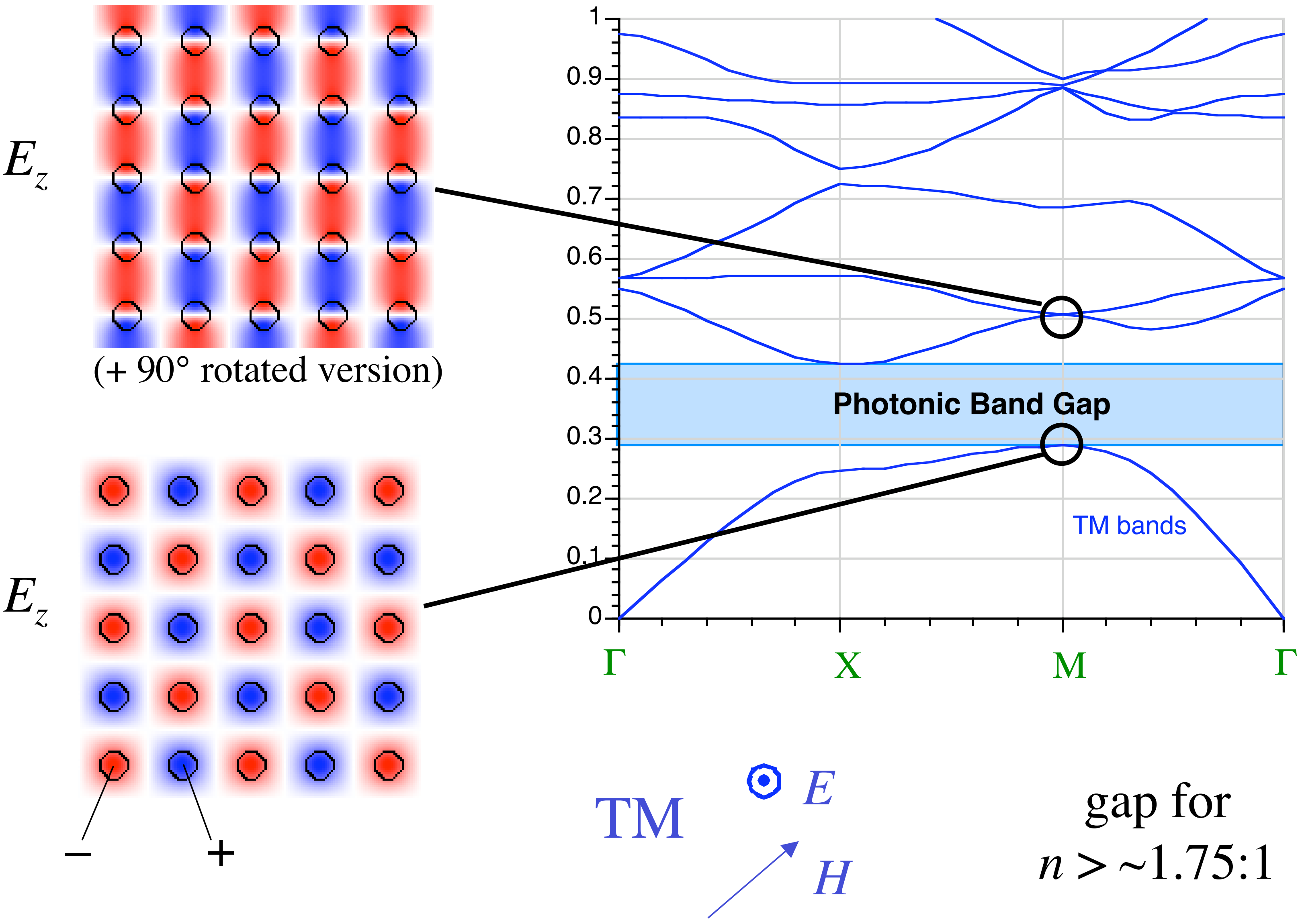


TM



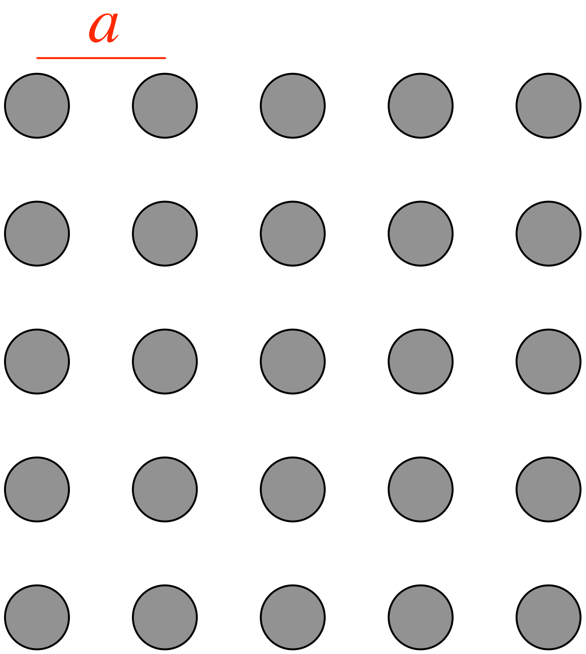
gap for
$n > \sim 1.75:1$

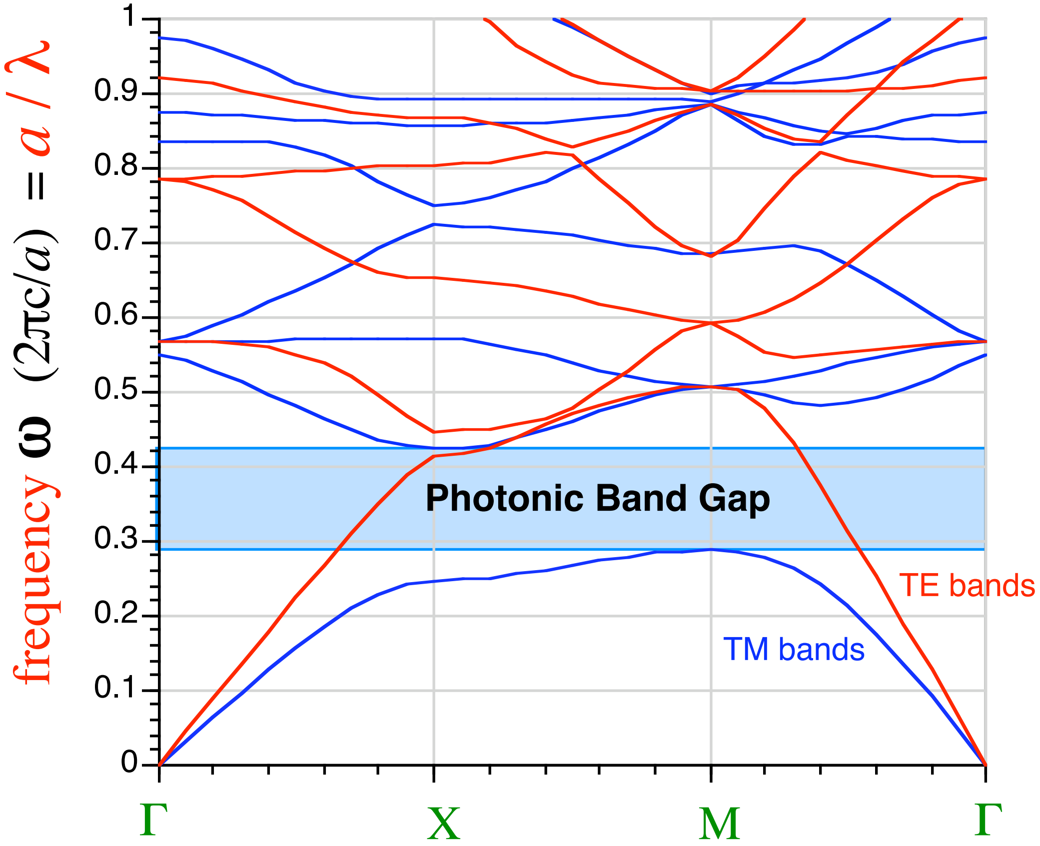
2d periodicity, ε=12:1



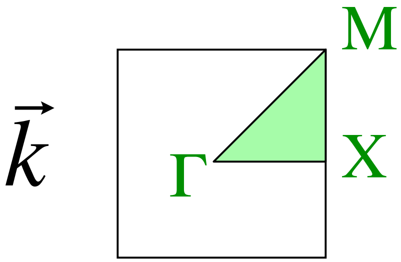
gap for
n > ~1.75:1

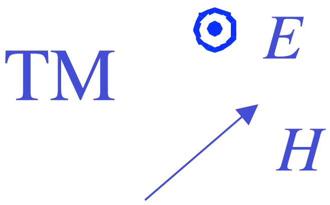
2d periodicity, ε=12:1

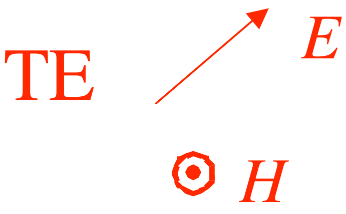




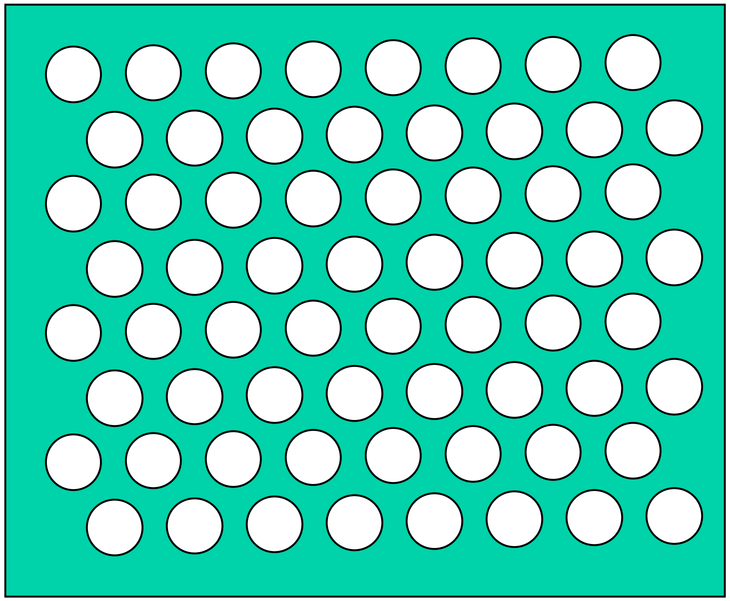
irreducible Brillouin zone

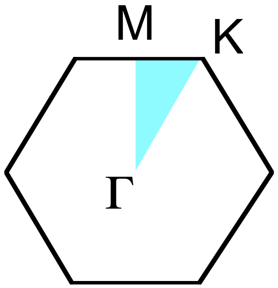


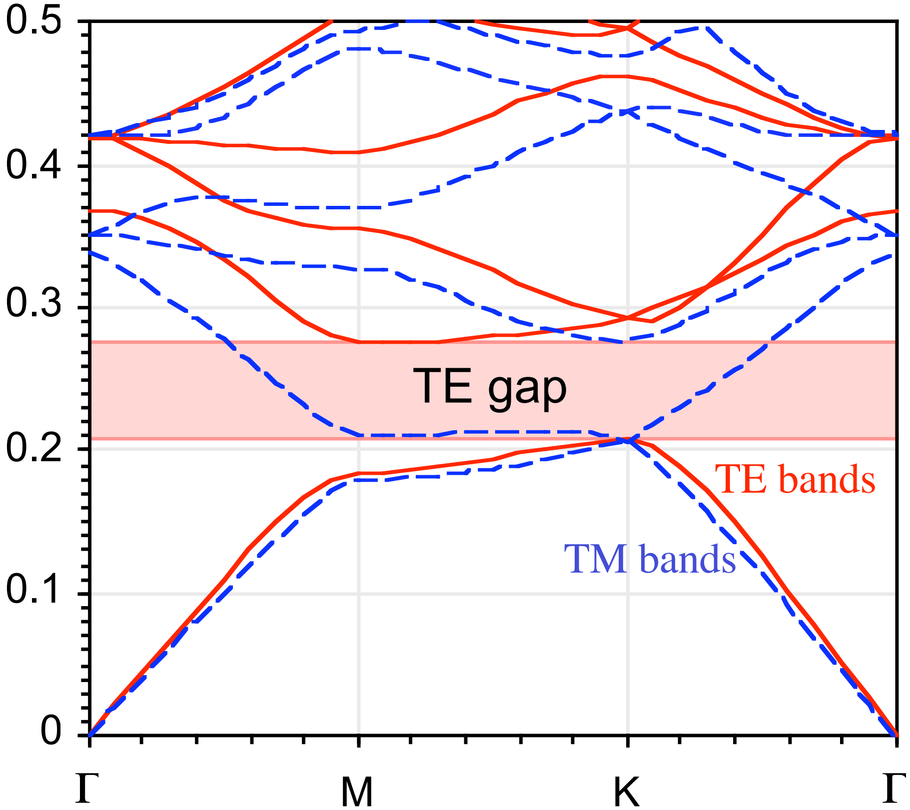


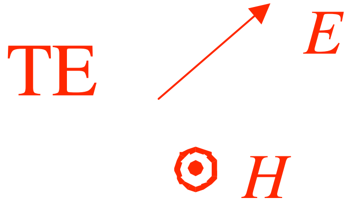


2d photonic crystal: TE gap, ε=12:1



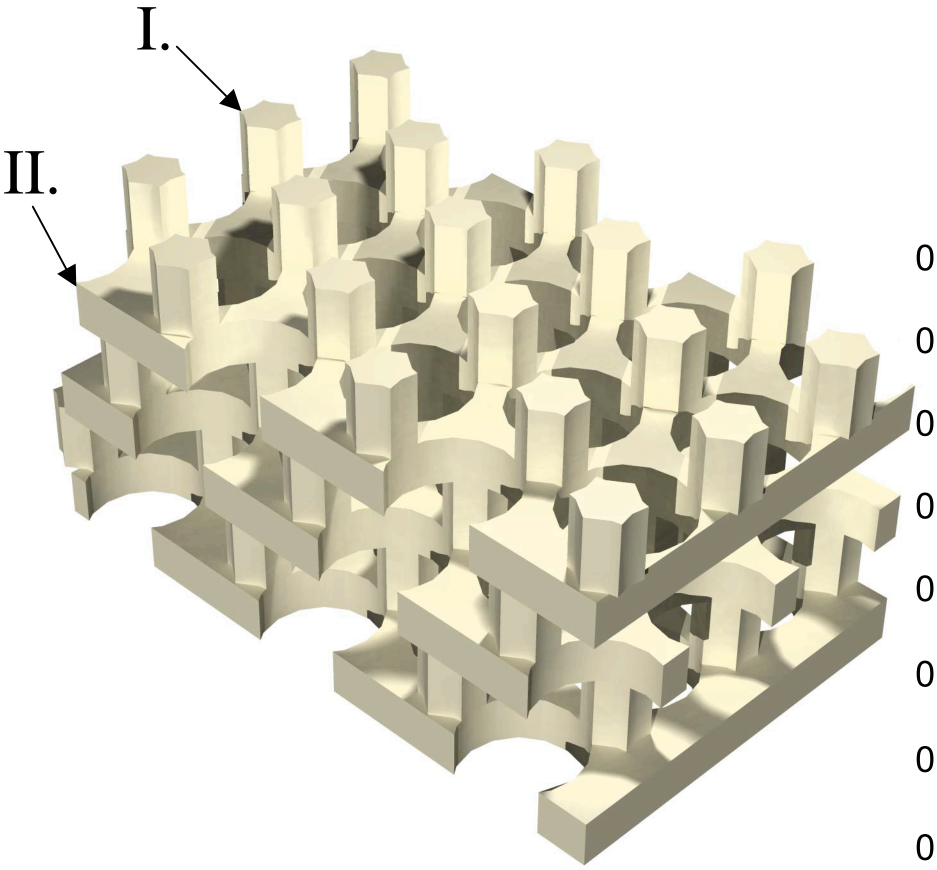




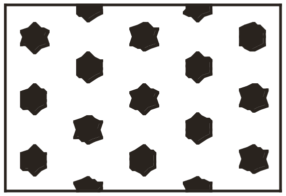


gap for n > ~1.4:1

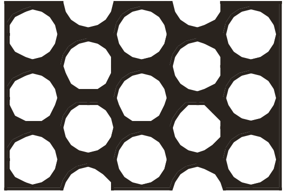
3d photonic crystal: complete gap , ε=12:1

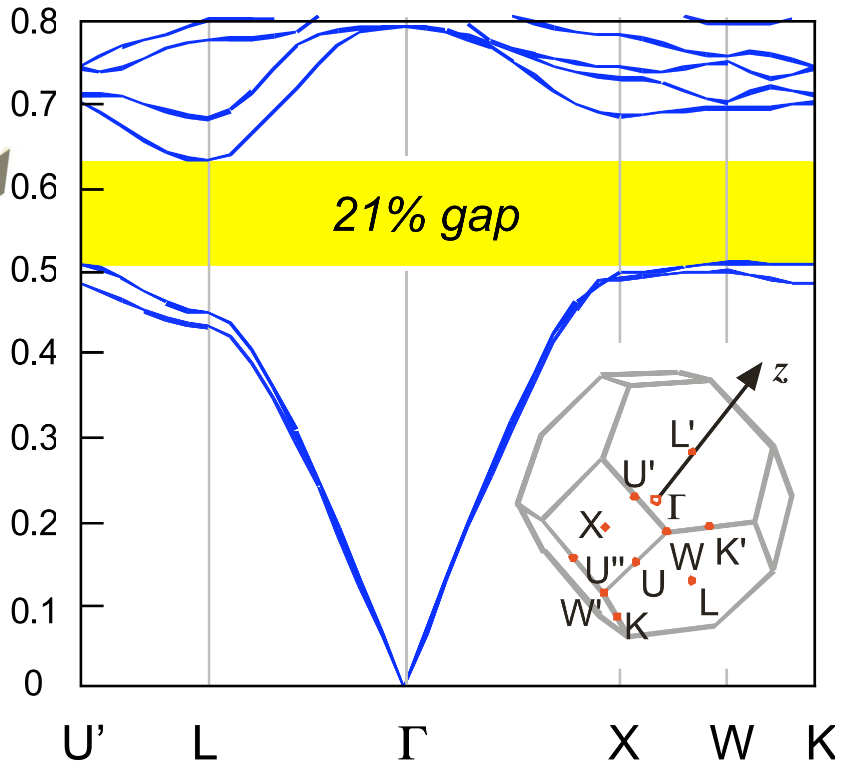


I: rod layer



II: hole layer





gap for $n > \sim 4:1$

[S. G. Johnson *et al.*, *Appl. Phys. Lett.* **77**, 3490 (2000)]

You, too, can compute photonic eigenmodes!

MIT Photonic-Bands (MPB) package:

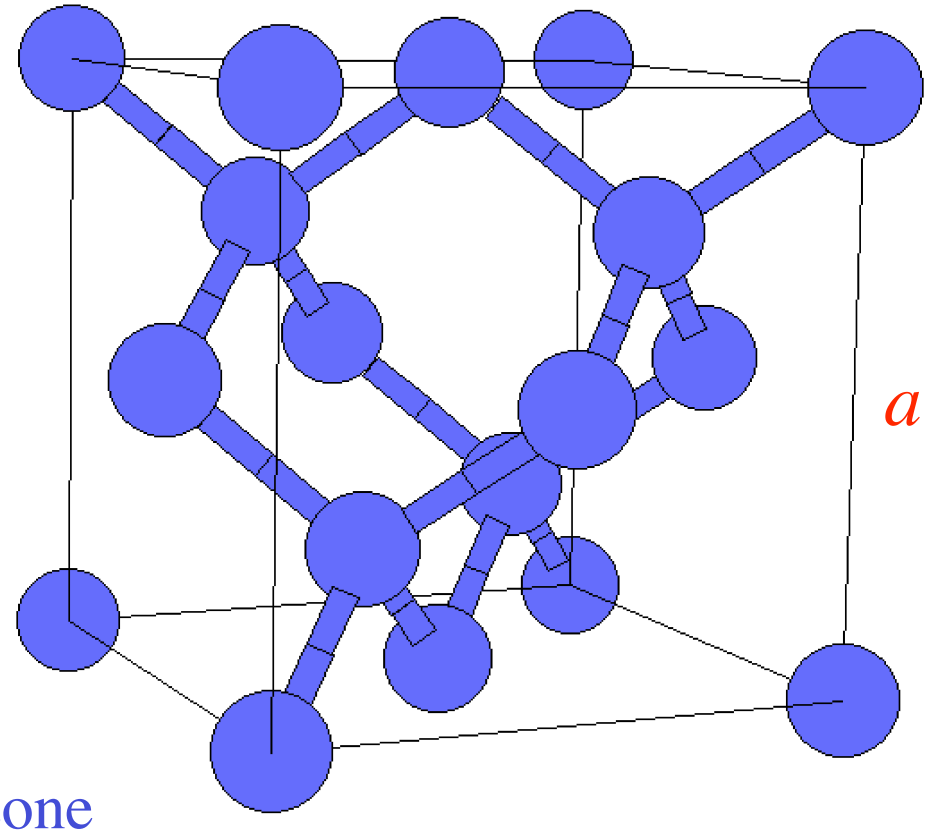
`http://ab-initio.mit.edu/mpb`

The Mother of (almost) All Bandgaps

The diamond lattice:

fcc (face-centered-cubic)
with two "atoms" per unit cell

(primitive)



Recipe for a complete gap:

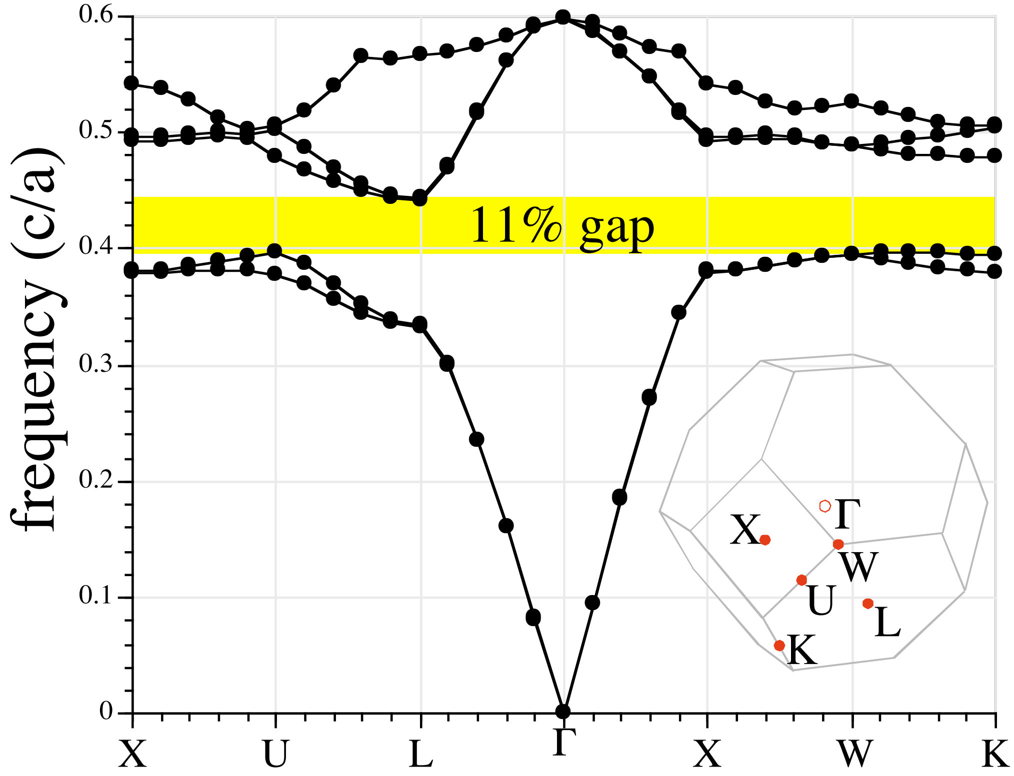
fcc = most-spherical Brillouin zone

+ diamond "bonds" = lowest (two) bands can concentrate in lines

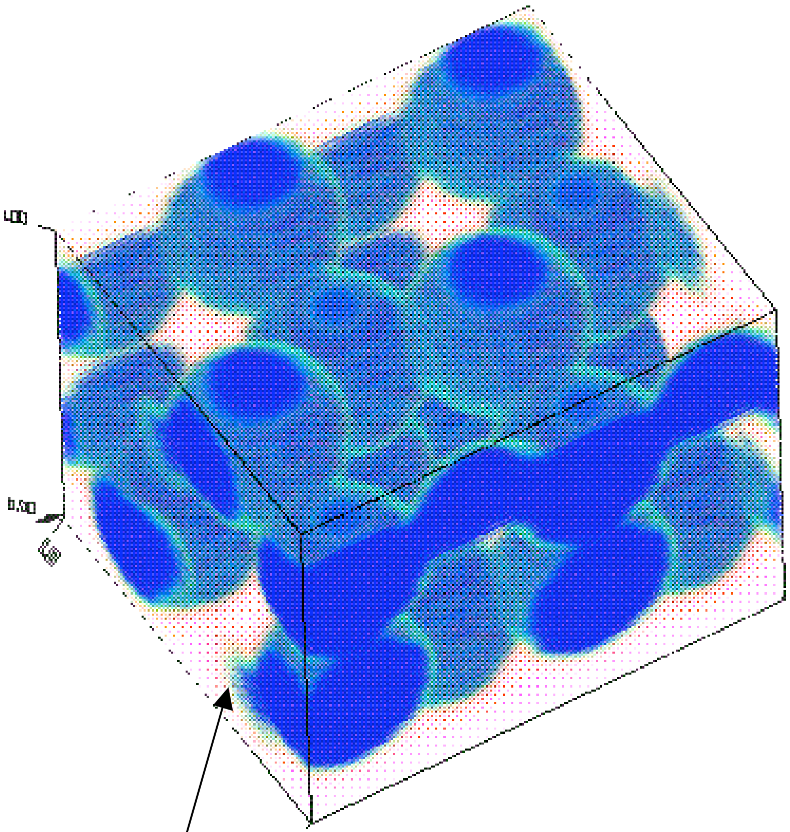
Image: **`http://cst-www.nrl.navy.mil/lattice/struk/a4.html`**

The First 3d Bandgap Structure

K. M. Ho, C. T. Chan, and C. M. Soukoulis, *Phys. Rev. Lett.* **65**, 3152 (1990).



for gap at $\lambda = 1.55\mu m$,
sphere diameter ~ 330nm



overlapping Si spheres

MPB tutorial, `http://ab-initio.mit.edu/mpb`

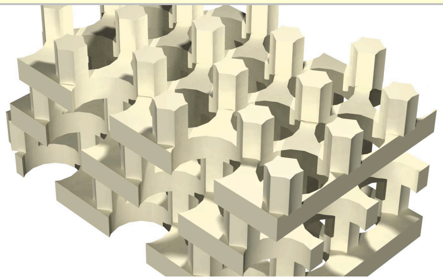
Layer-by-Layer Lithography

- Fabrication of 2d patterns in Si or GaAs is very advanced
(think: Pentium IV, 50 million transistors)

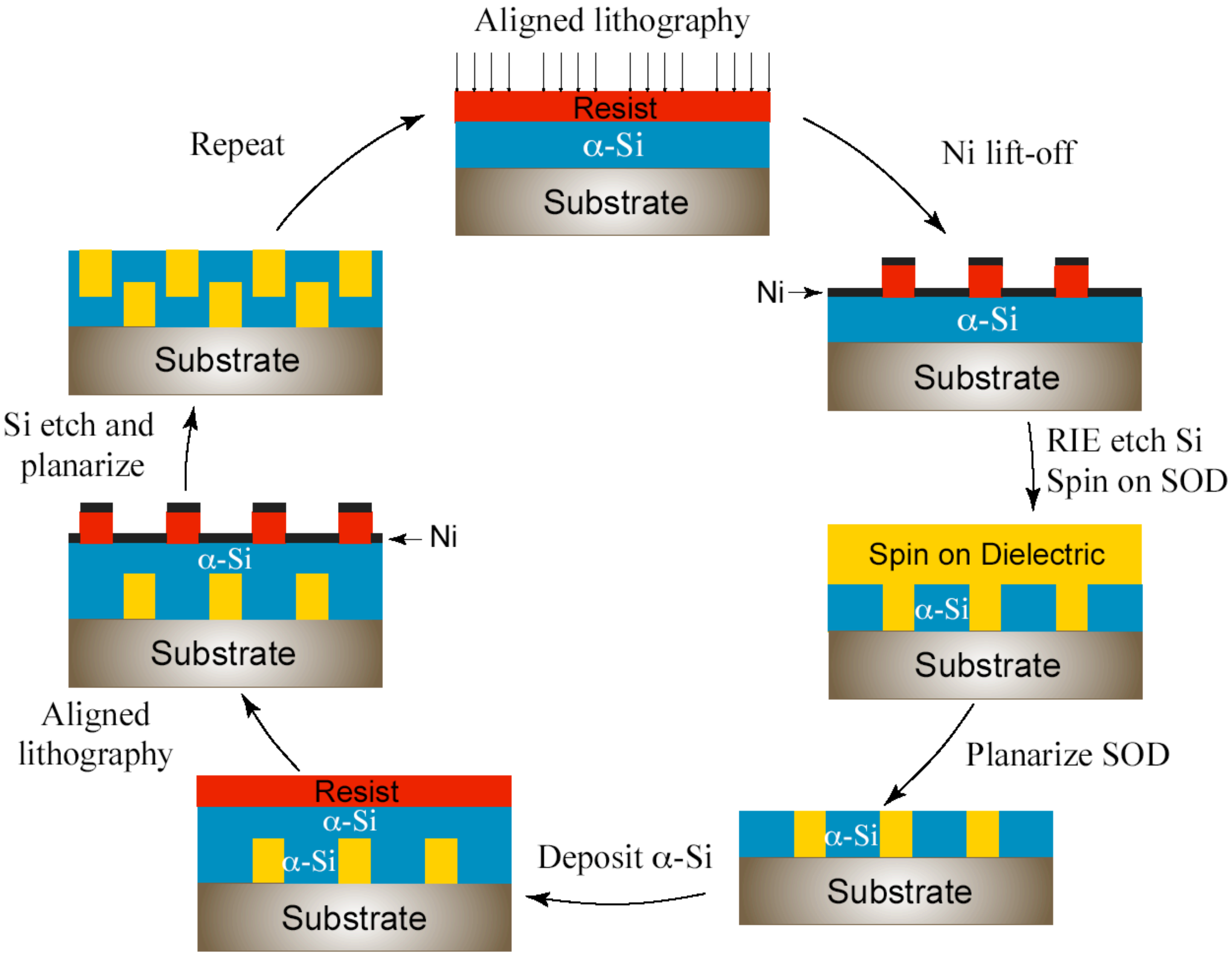
…inter-layer alignment techniques are only slightly more exotic

So, make 3d structure one layer at a time

Need a 3d crystal with constant cross-section layers

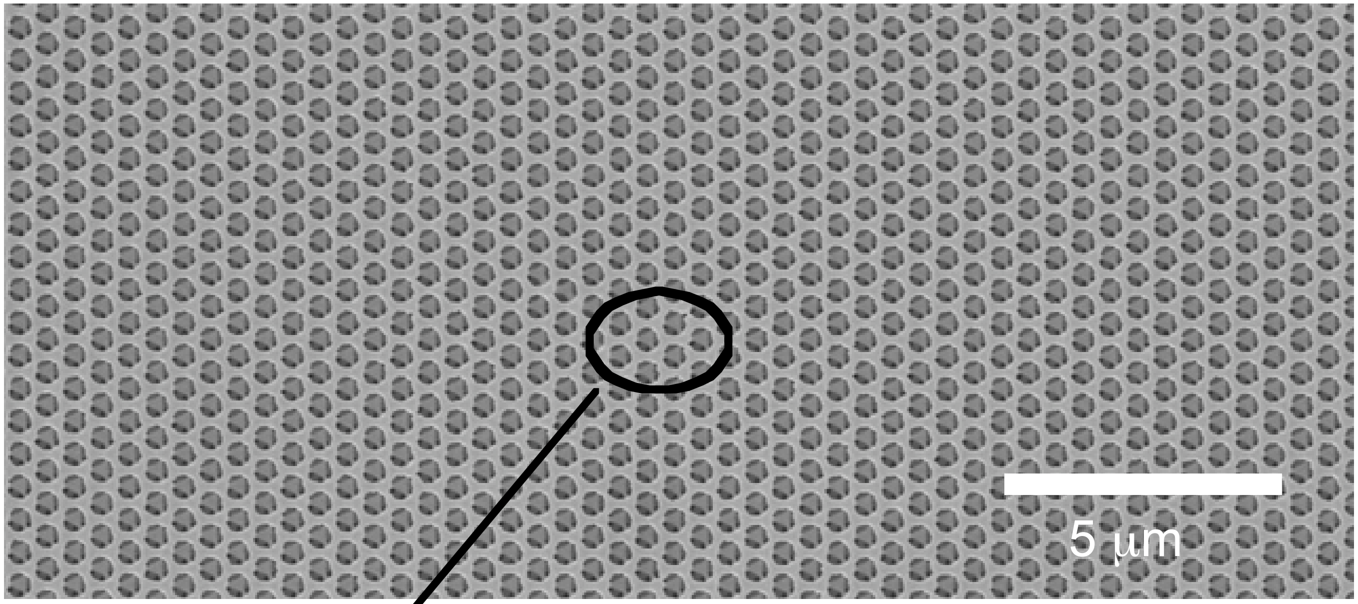


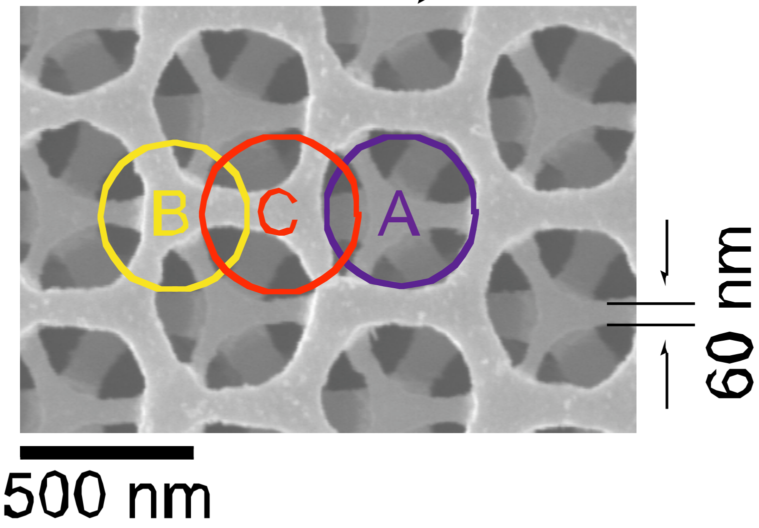
A Schematic

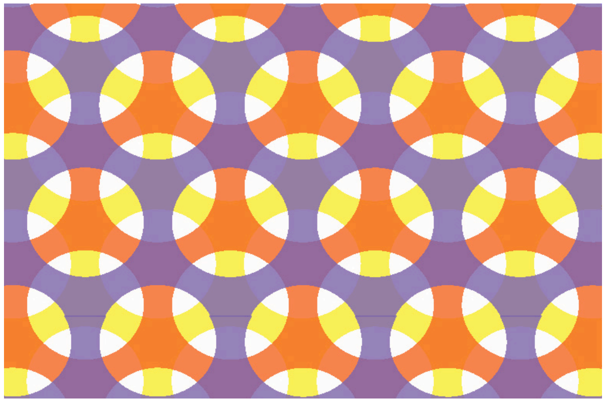


[M. Qi, H. Smith, MIT]

7-layer E-Beam Fabrication







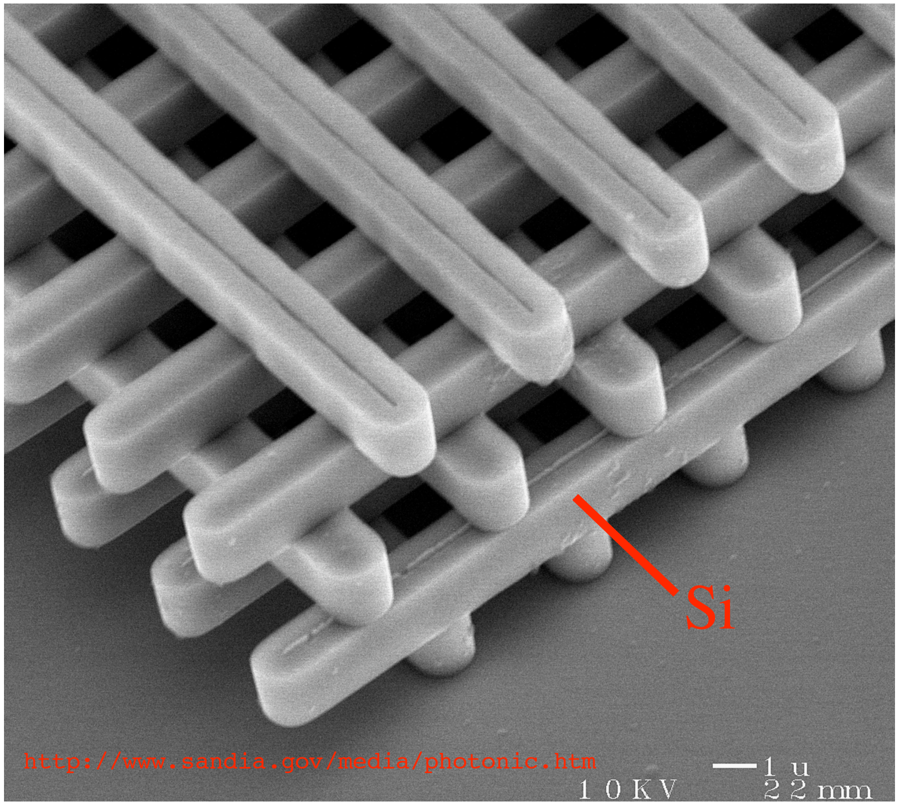
[M. Qi, *et al.*, *Nature* **429**, 538 (2004)]

an earlier design:
(& currently more popular)

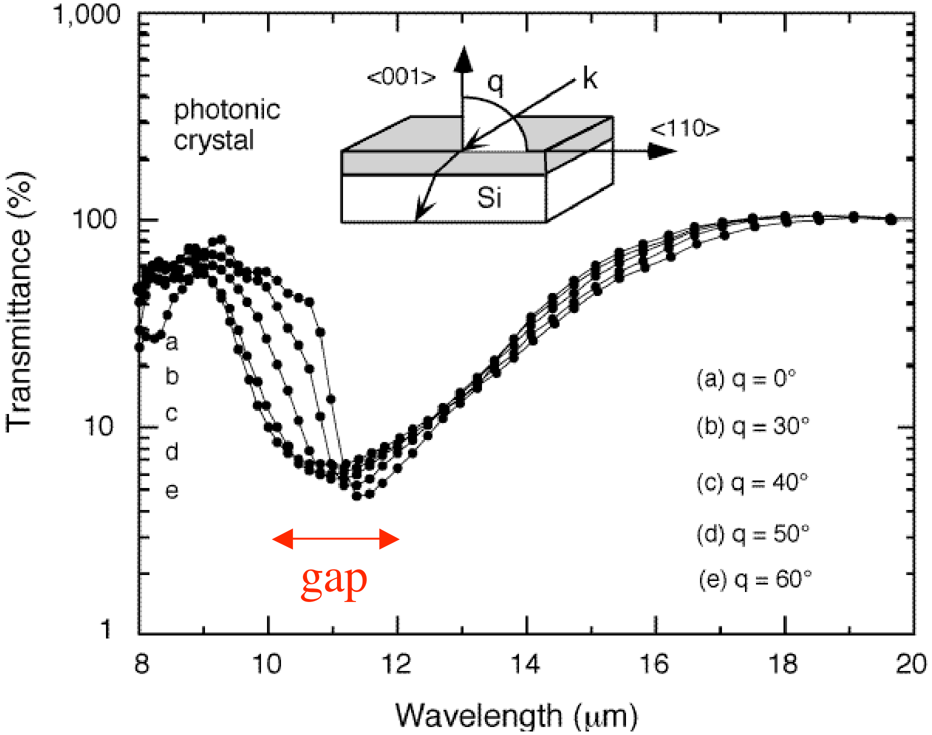
The Woodpile Crystal

[K. Ho *et al.*, *Solid State Comm*. **89**, 413 (1994)] [H. S. Sözüer *et al.*, *J. Mod. Opt*. **41**, 231 (1994)]

(4 “log” layers = 1 period)



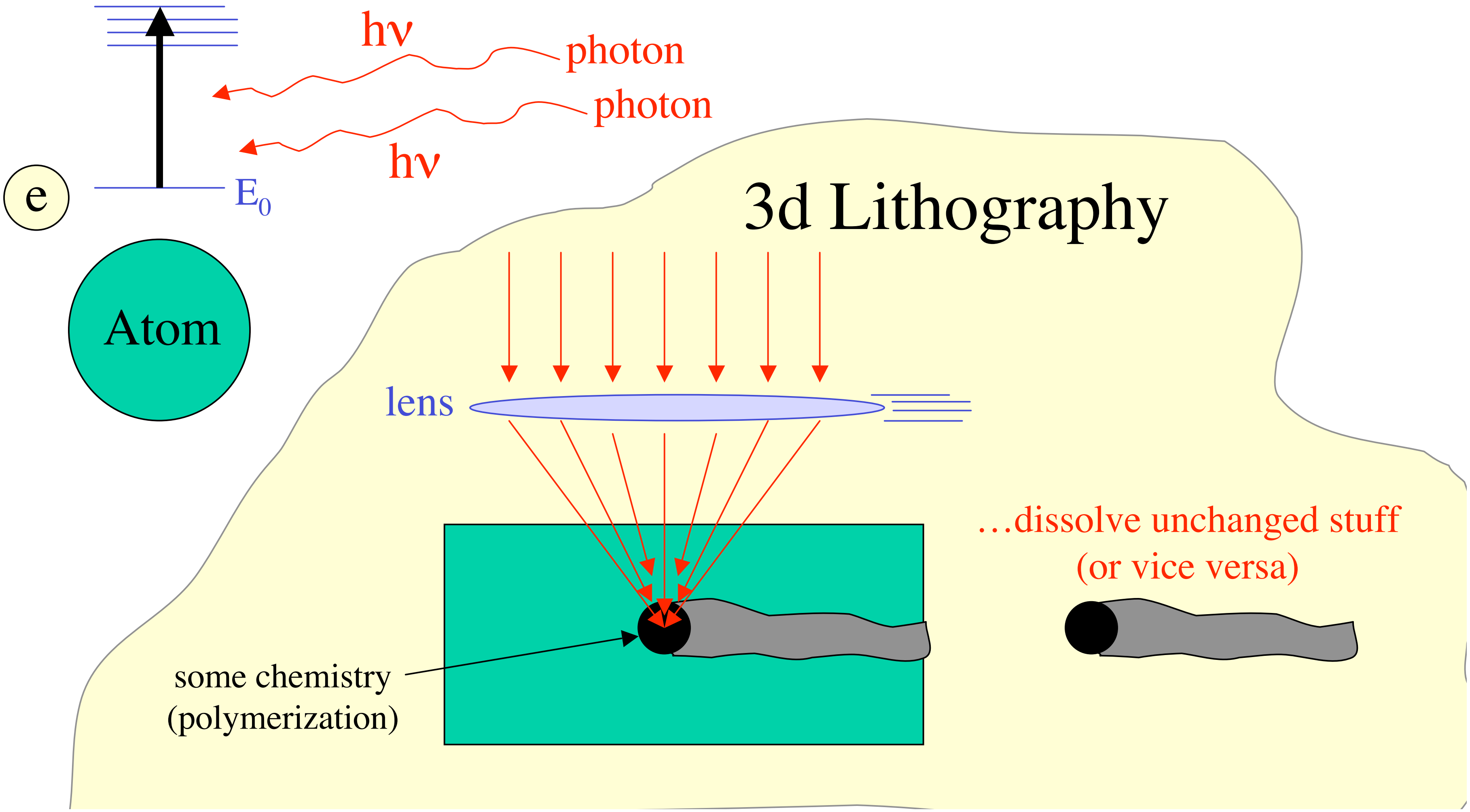
[S. Y. Lin *et al.*, *Nature* **394**, 251 (1998)]



Two-Photon Lithography

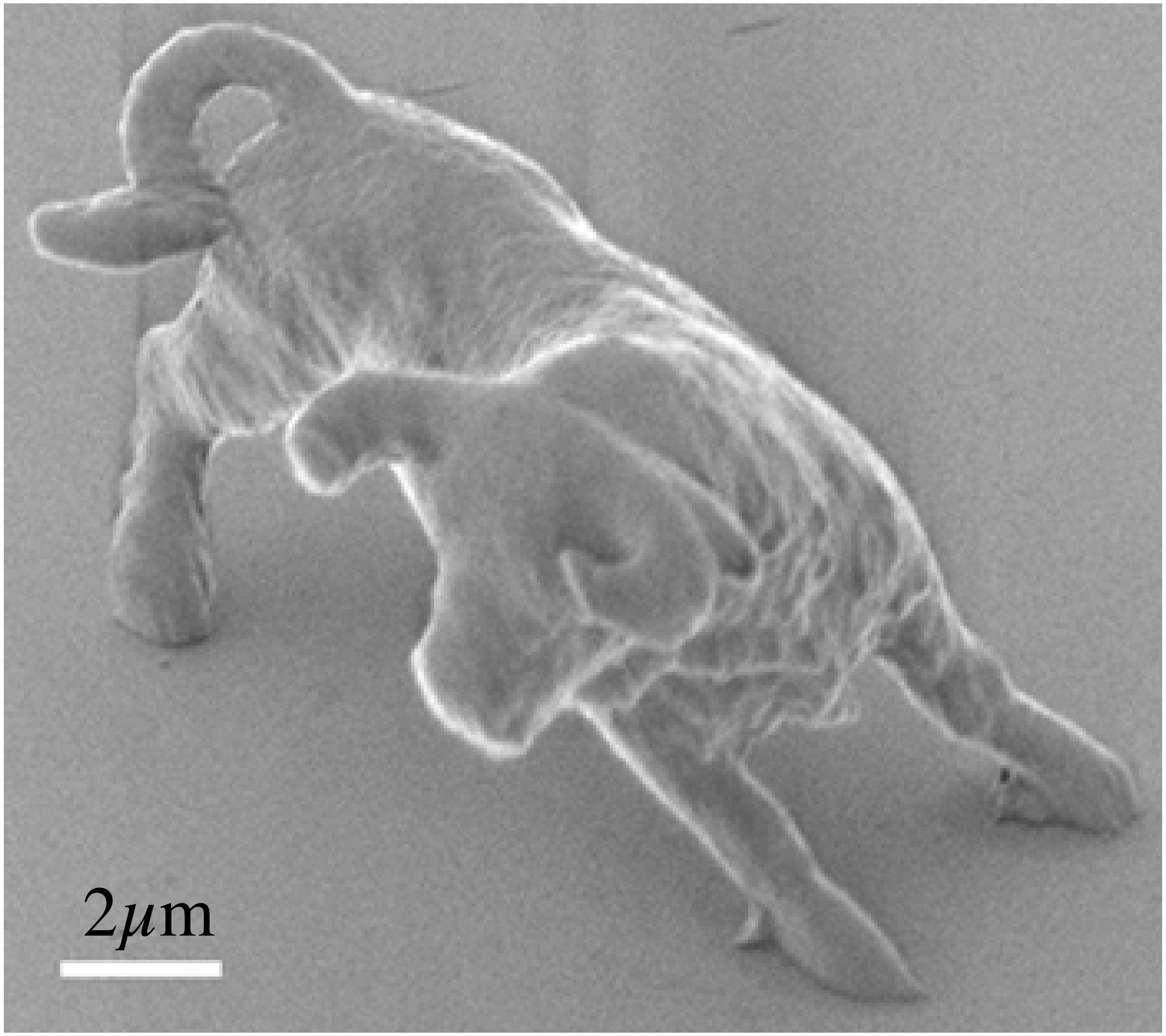
$2\ h\nu = \Delta E$

2-photon probability ~ (light intensity)^2



Lithography is a Beast

[S. Kawata *et al.*, *Nature* **412**, 697 (2001)]



λ = 780nm

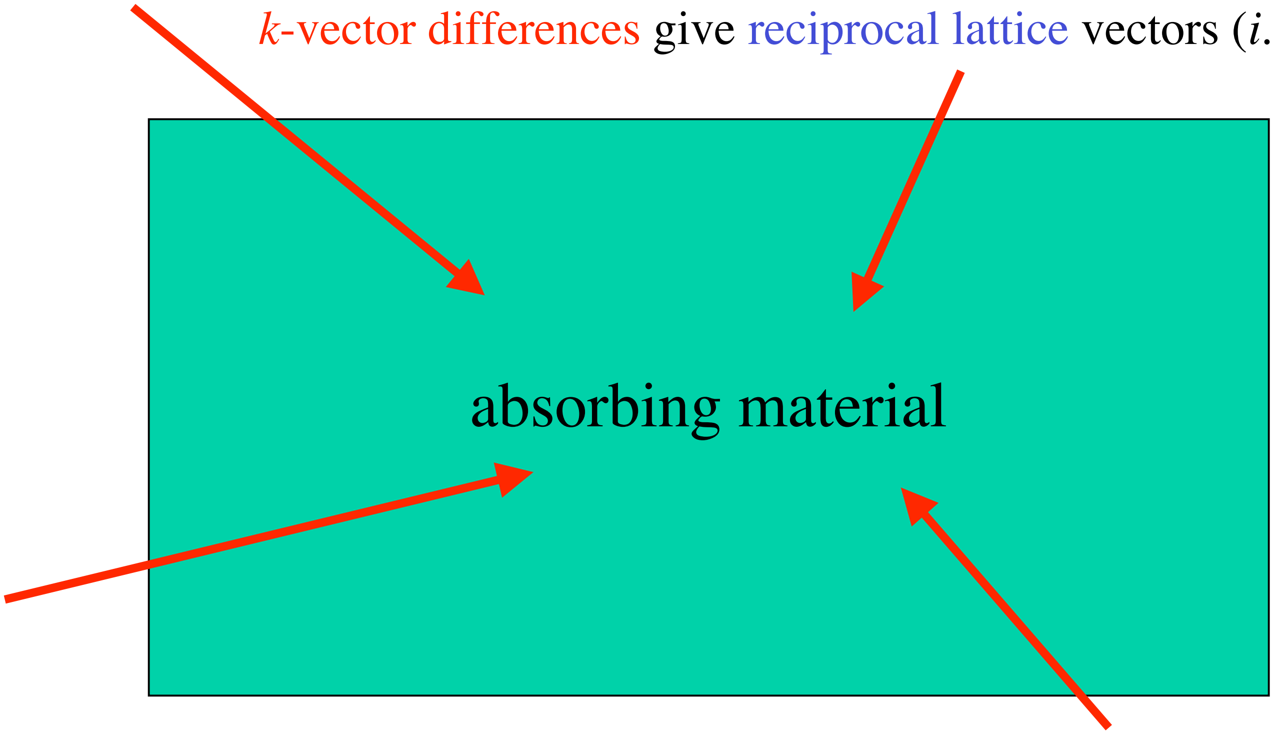
resolution = 150nm

7μm

(3 hours to make)

Holographic Lithography

[D. N. Sharp *et al.*, *Opt. Quant. Elec.* **34**, 3 (2002)]



Four beams make 3d-periodic interference pattern

k-vector differences give reciprocal lattice vectors (*i.e.* periodicity)

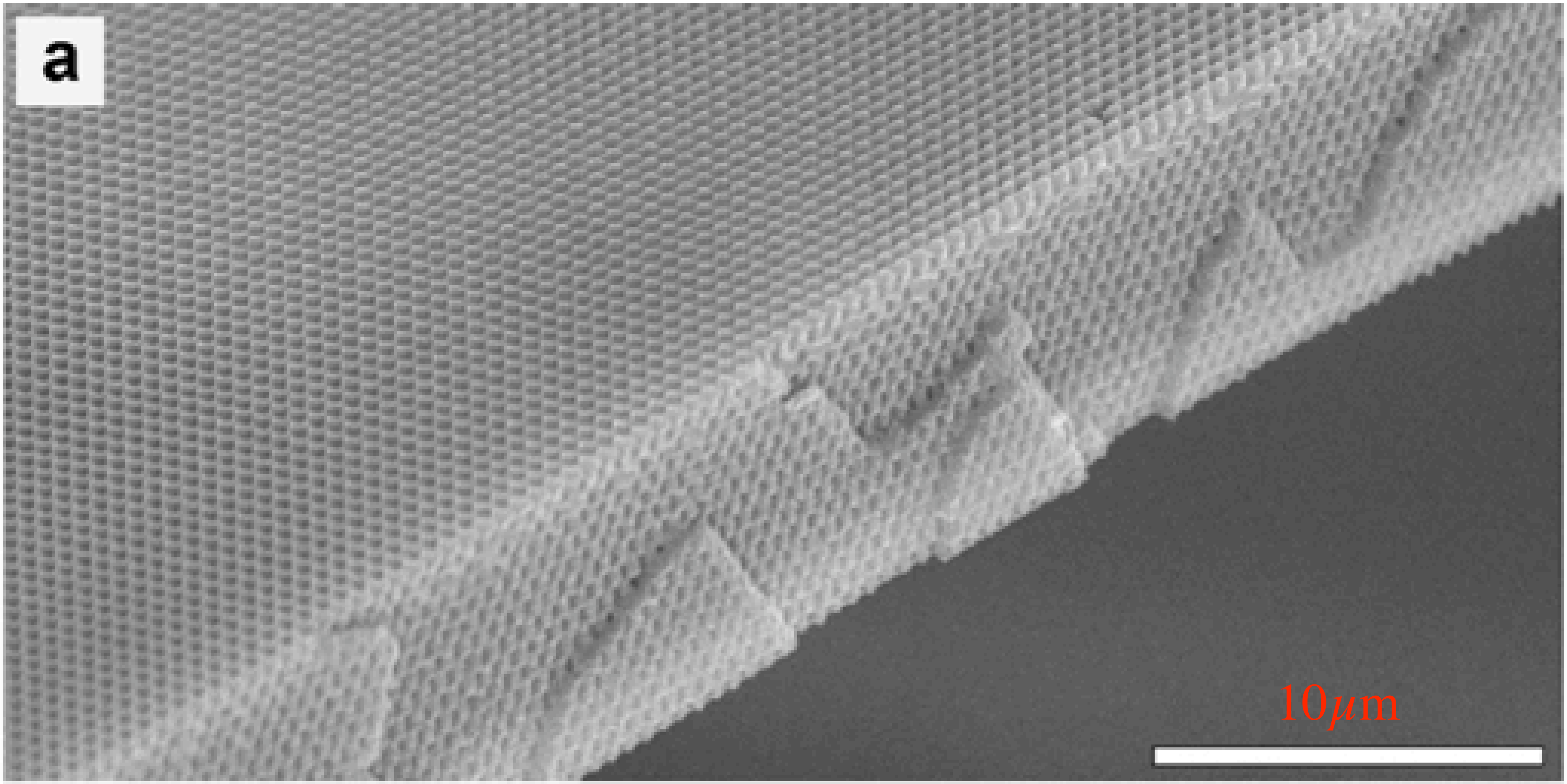
absorbing material

(1.4μm)

beam polarizations + amplitudes (8 parameters) give unit cell

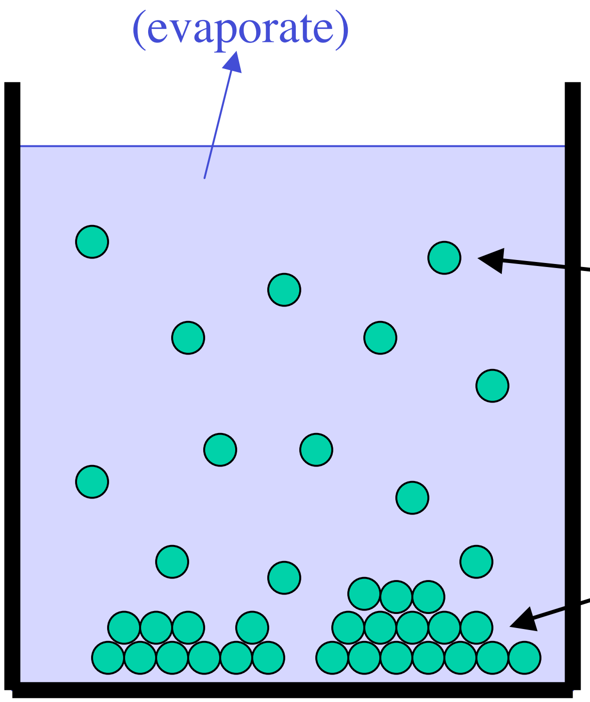
One-Photon
Holographic Lithography

[D. N. Sharp *et al.*, *Opt. Quant. Elec.* **34**, 3 (2002)]



huge volumes, long-range periodic, fcc lattice…backfill for high contrast

Mass-production II: Colloids



silica (SiO_2)
microspheres (diameter < 1μm)

sediment by gravity into
close-packed fcc lattice!

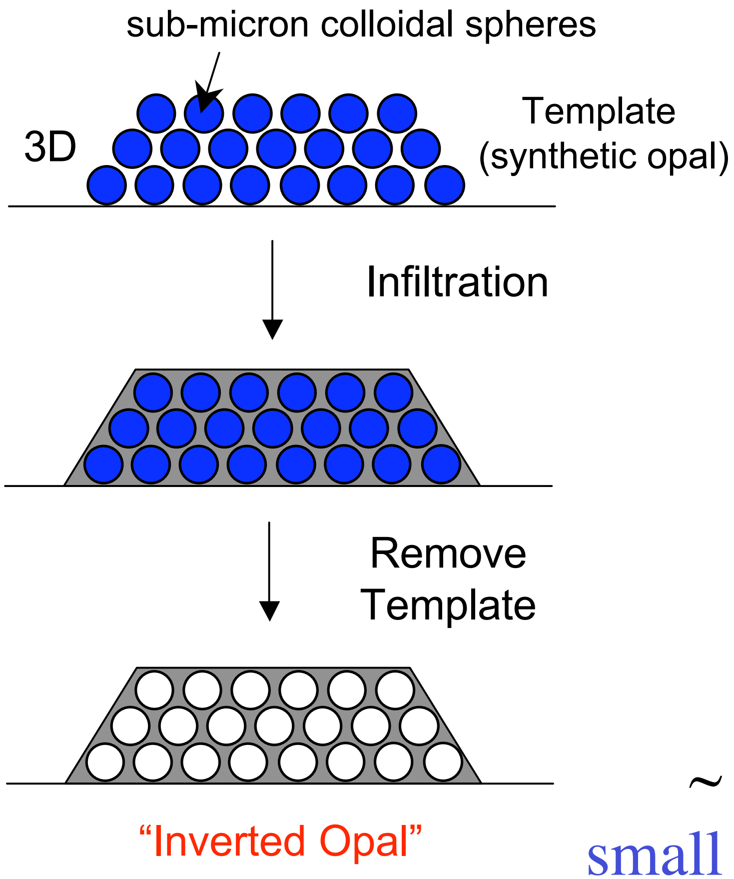
Inverse Opals

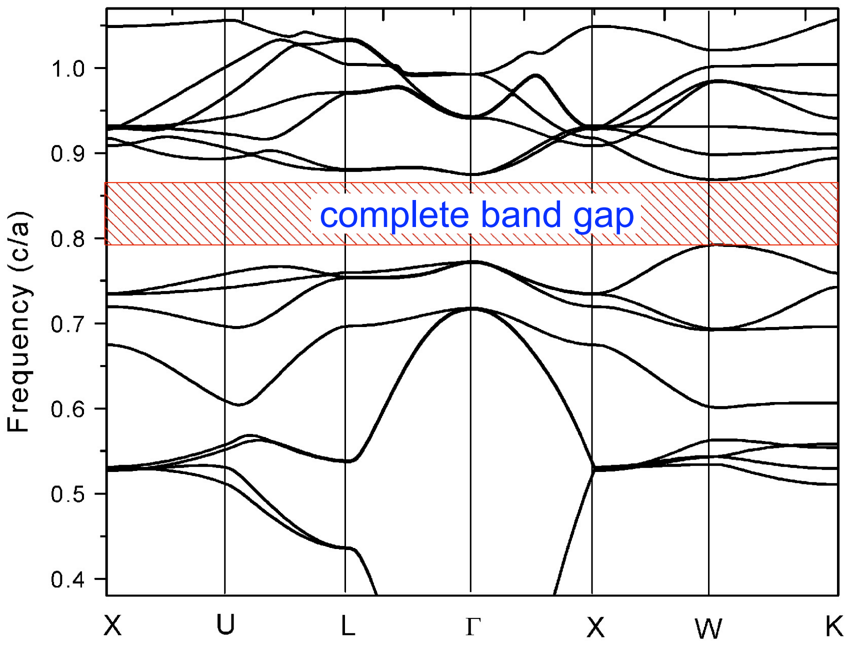
[figs courtesy
D. Norris, UMN]

[H. S. Sözüer, *PRB* **45**, 13962 (1992)]

fcc solid spheres do not have a gap…

…but fcc spherical holes in Si *do* have a gap



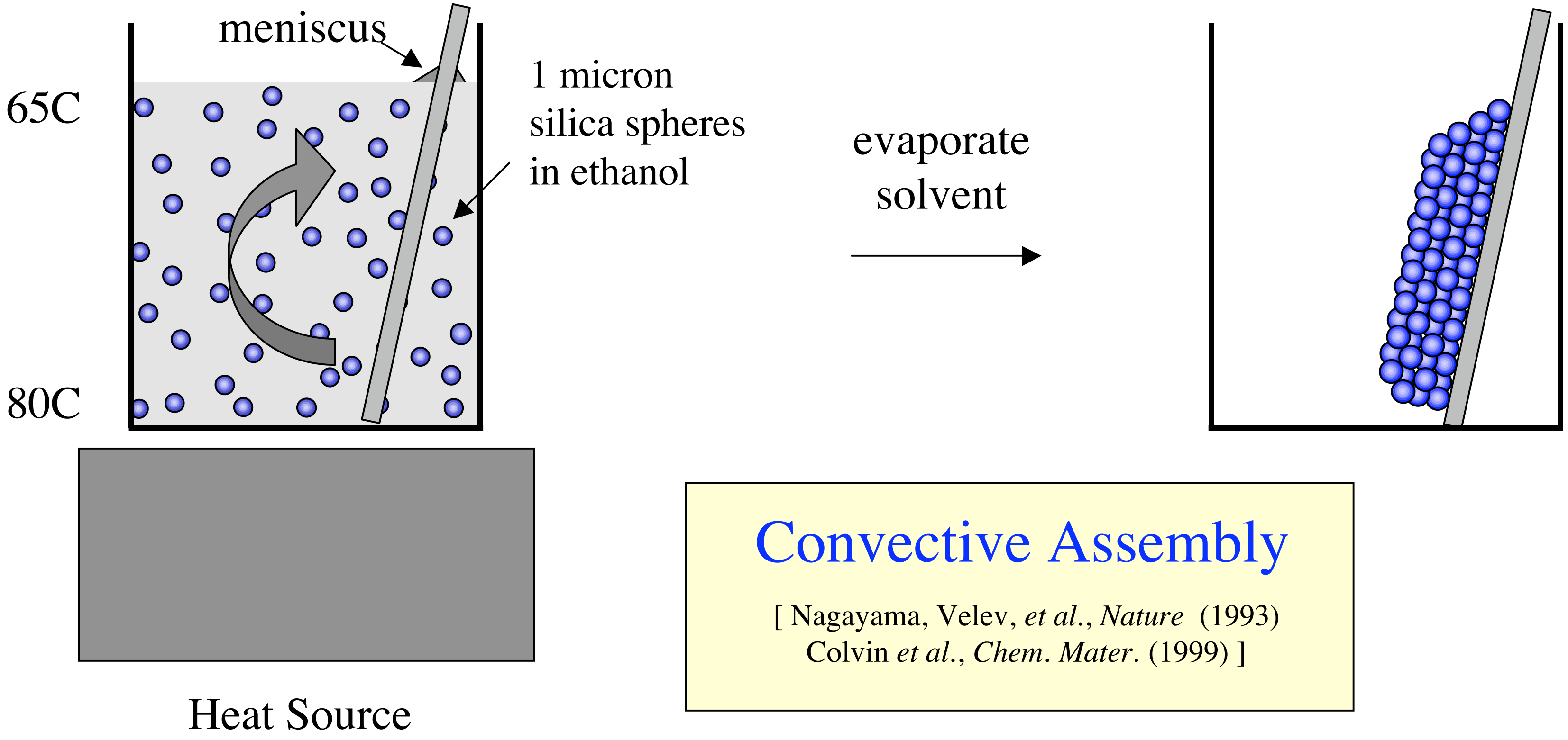


~ 10% gap between 8th & 9th bands

small gap, upper bands: sensitive to disorder

In Order To Form a More Perfect Crystal…

[figs courtesy D. Norris, UMN]



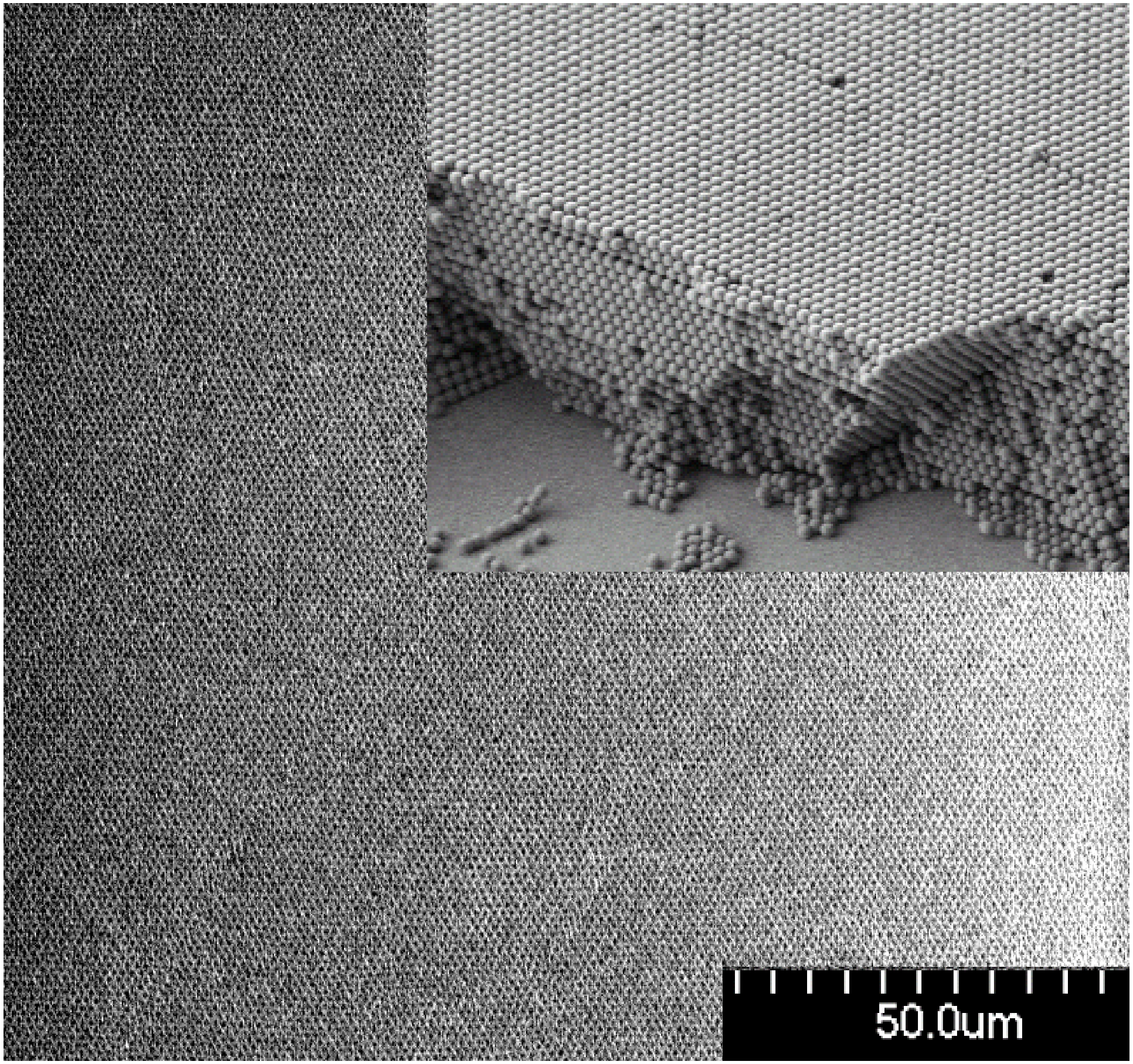
Convective Assembly

[Nagayama, Velev, *et al.*, *Nature* (1993)
Colvin *et al.*, *Chem. Mater.* (1999)]

- Capillary forces during drying cause assembly in the meniscus
- Extremely flat, large-area opals of controllable thickness

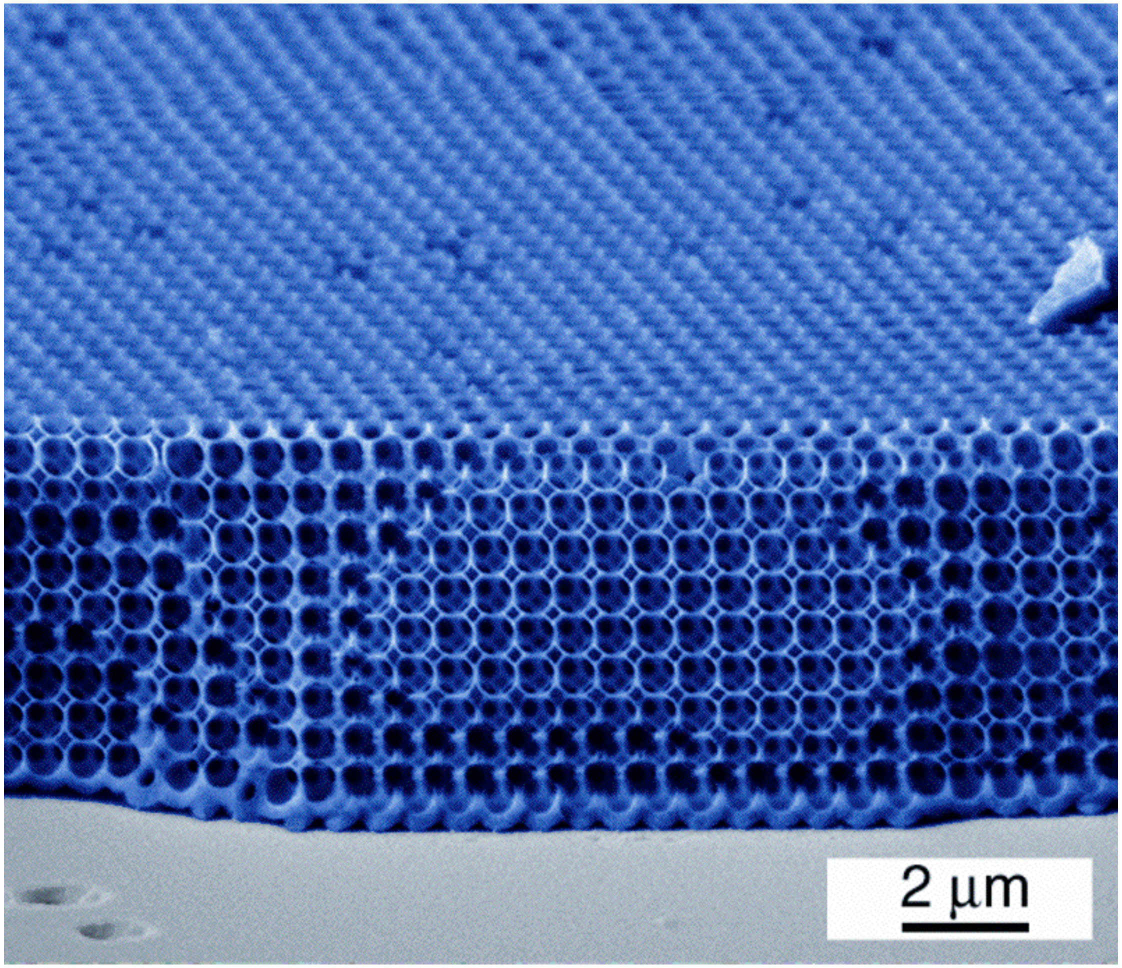
A Better Opal

[fig courtesy
D. Norris, UMN]



Inverse-Opal Photonic Crystal

[fig courtesy
D. Norris, UMN]



[Y. A. Vlasov *et al*., *Nature* **414**, 289 (2001).]

Outline

- Preliminaries: waves in periodic media
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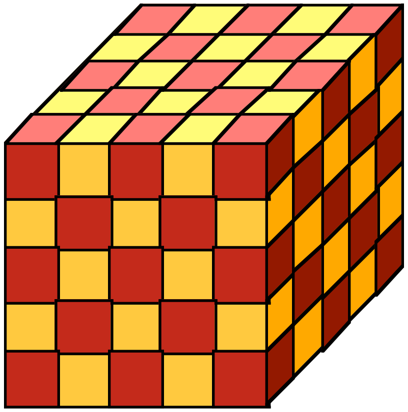
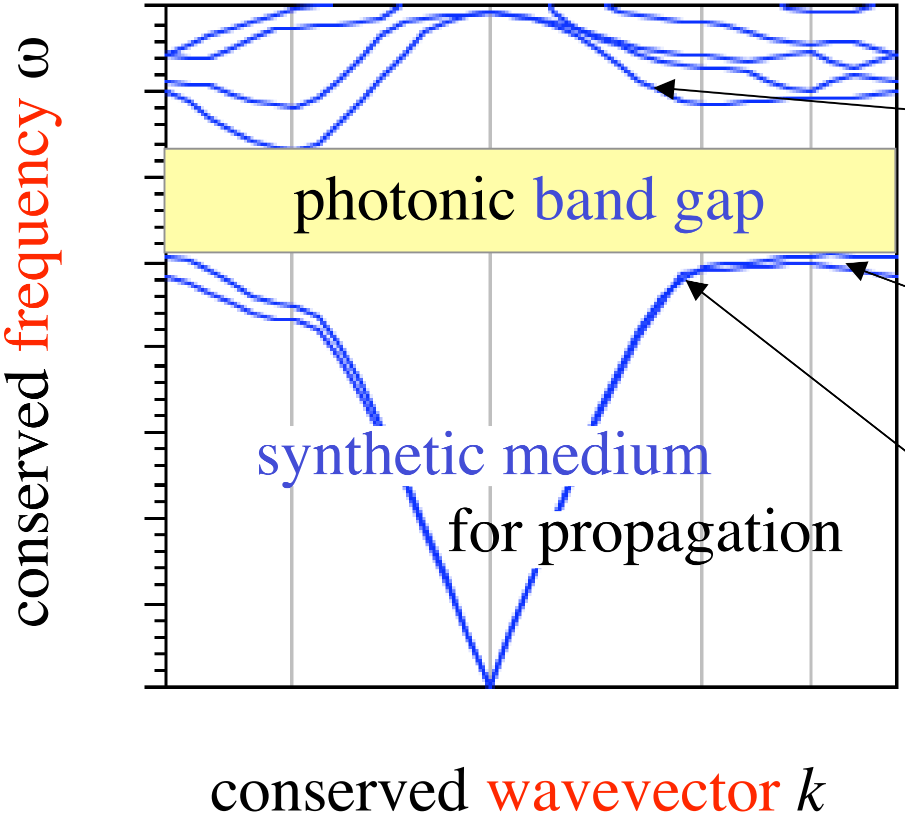
Properties of Bulk Crystals

by Bloch's theorem



(cartoon)

band diagram (dispersion relation)



backwards slope:

negative refraction

dω/d*k* → 0: slow light

(e.g. DFB lasers)

strong curvature:

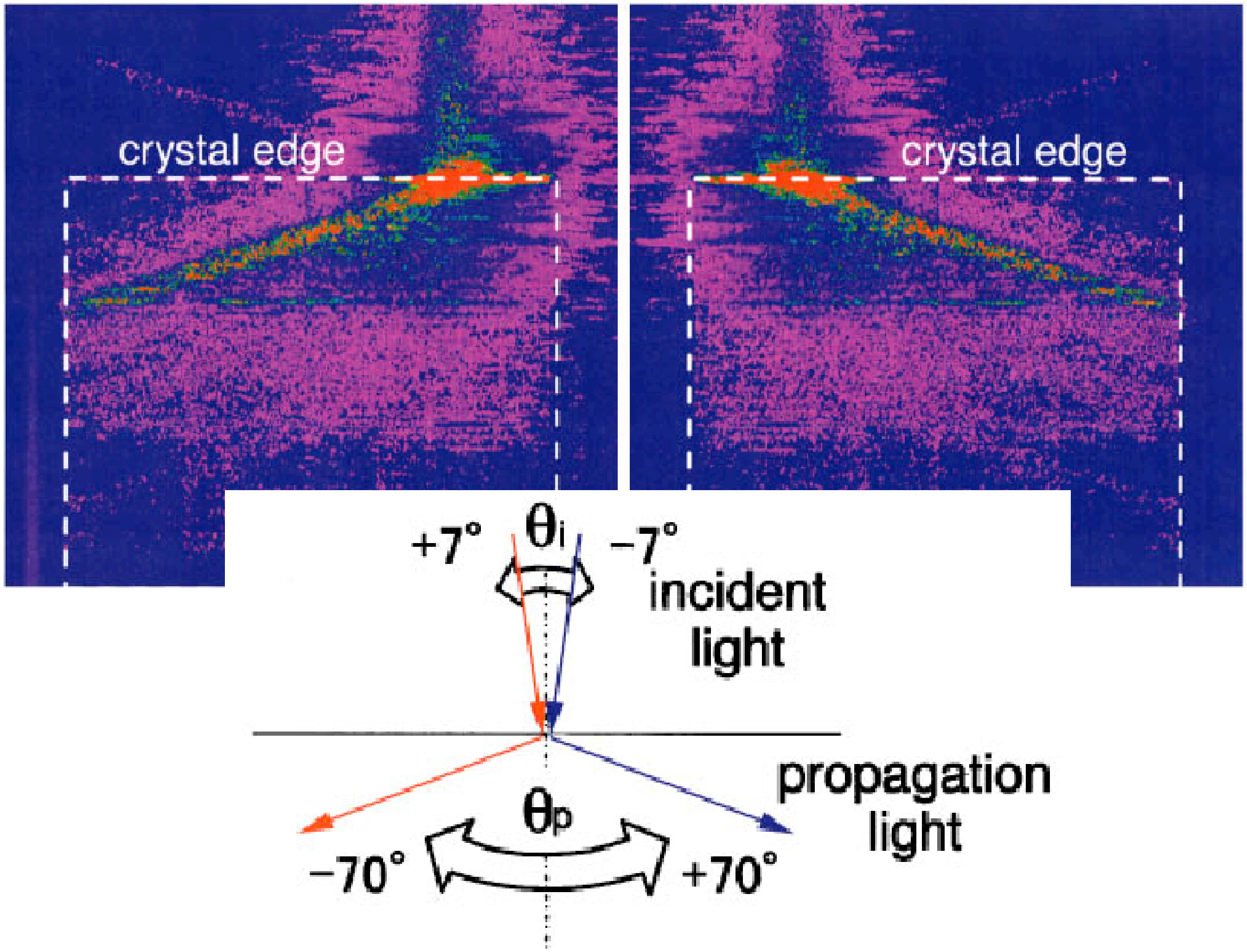
super-prisms, …

(+ negative refraction)

Superprisms

from divergent dispersion (band curvature)

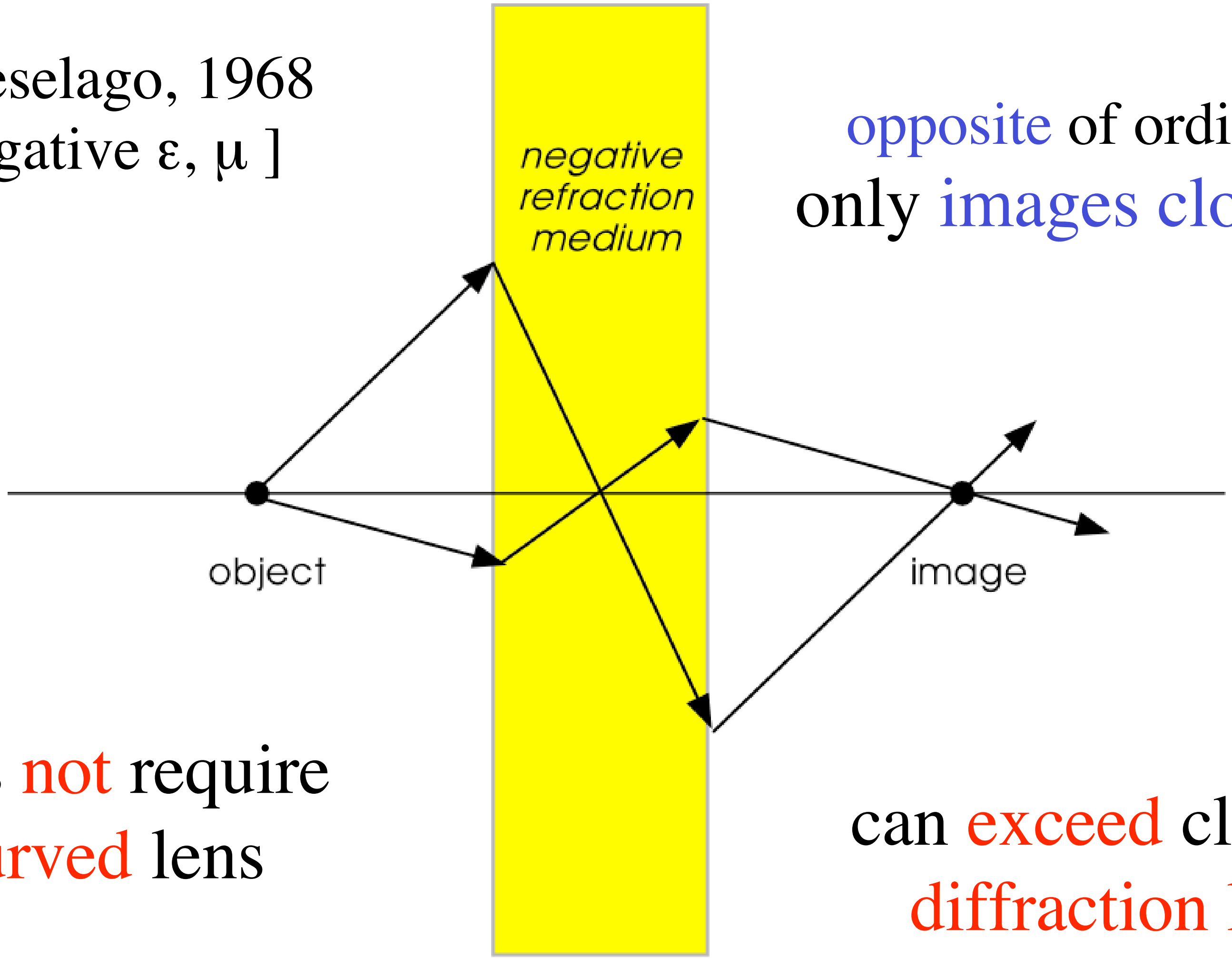
[Kosaka, *PRB* **58**, R10096 (1998).]



Negative Refraction

[Veselago, 1968
negative ε, μ]

opposite of ordinary lens:
only images close objects

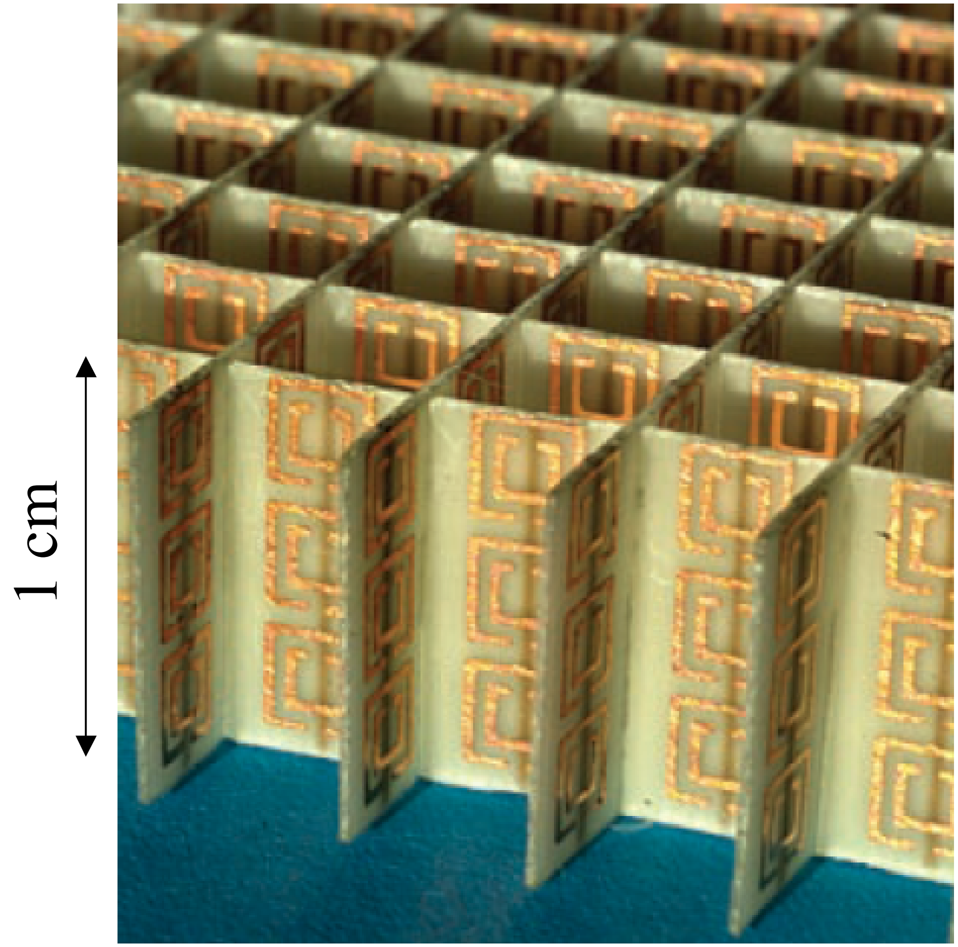


does not require
curved lens

can exceed classical
diffraction limit

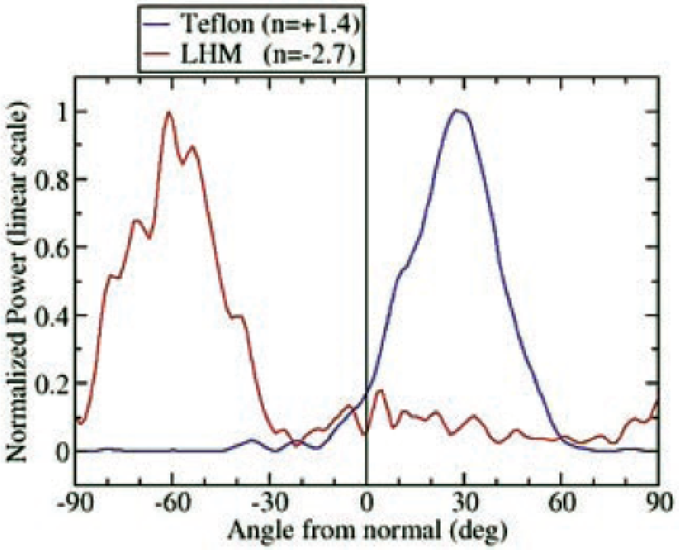
Microwave negative refraction

[D. R. Smith, J. B. Pendry, M. C. K. Wiltshire, *Science* **305**, 788 (2004)]

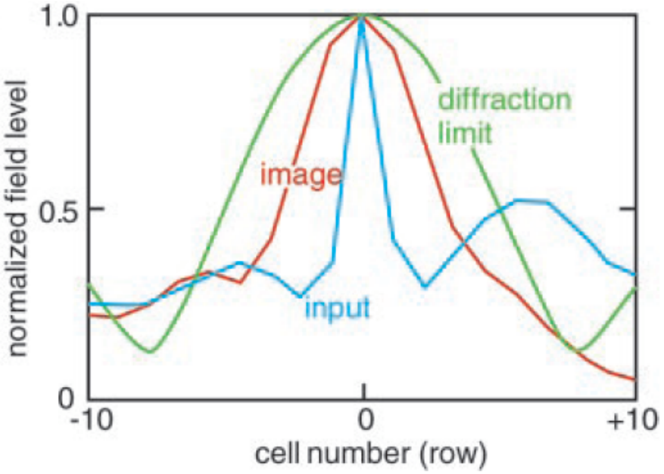


Magnetic (ring) + Electric (strip) resonances

negative refraction

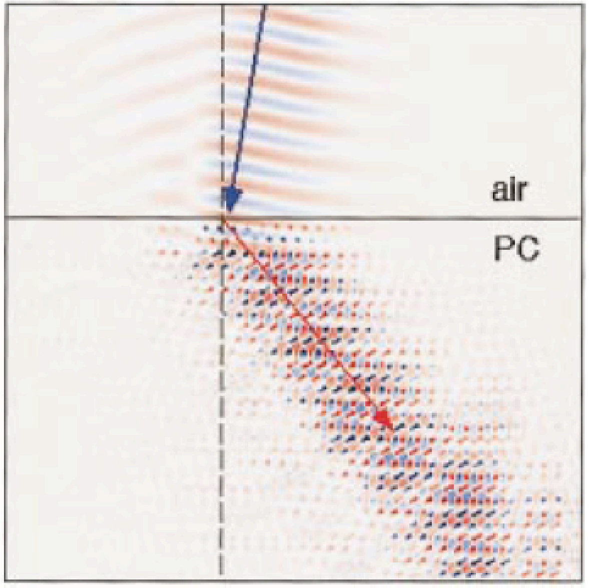


superlensing

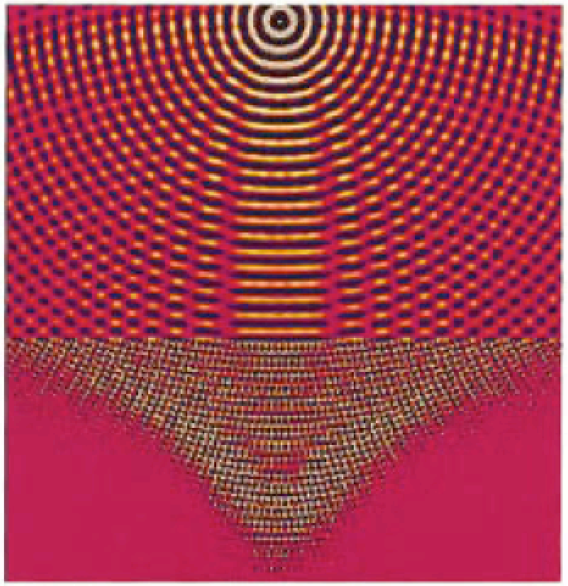


Negative-refractive all-dielectric photonic crystals

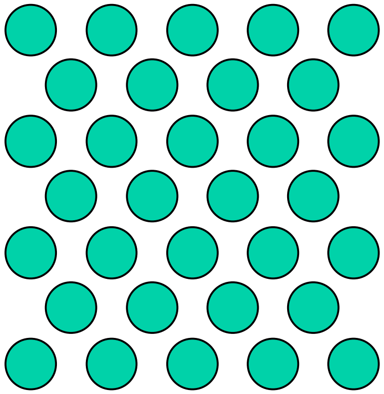
negative refraction



focussing



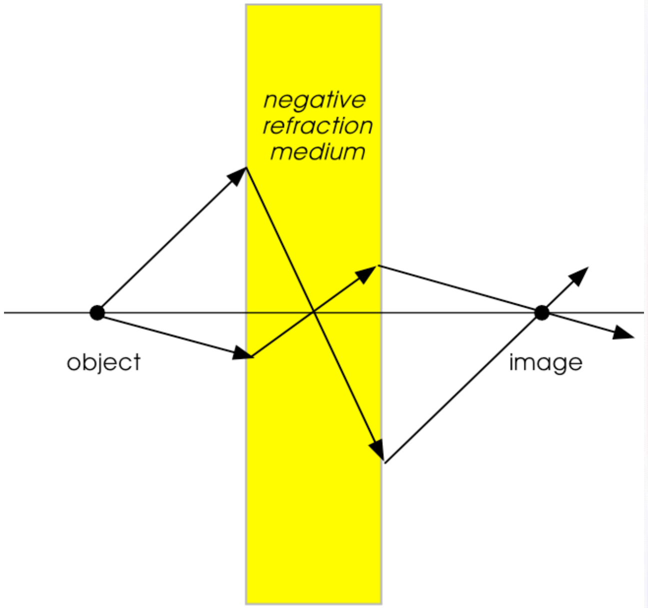
(2d rods in air, TE)

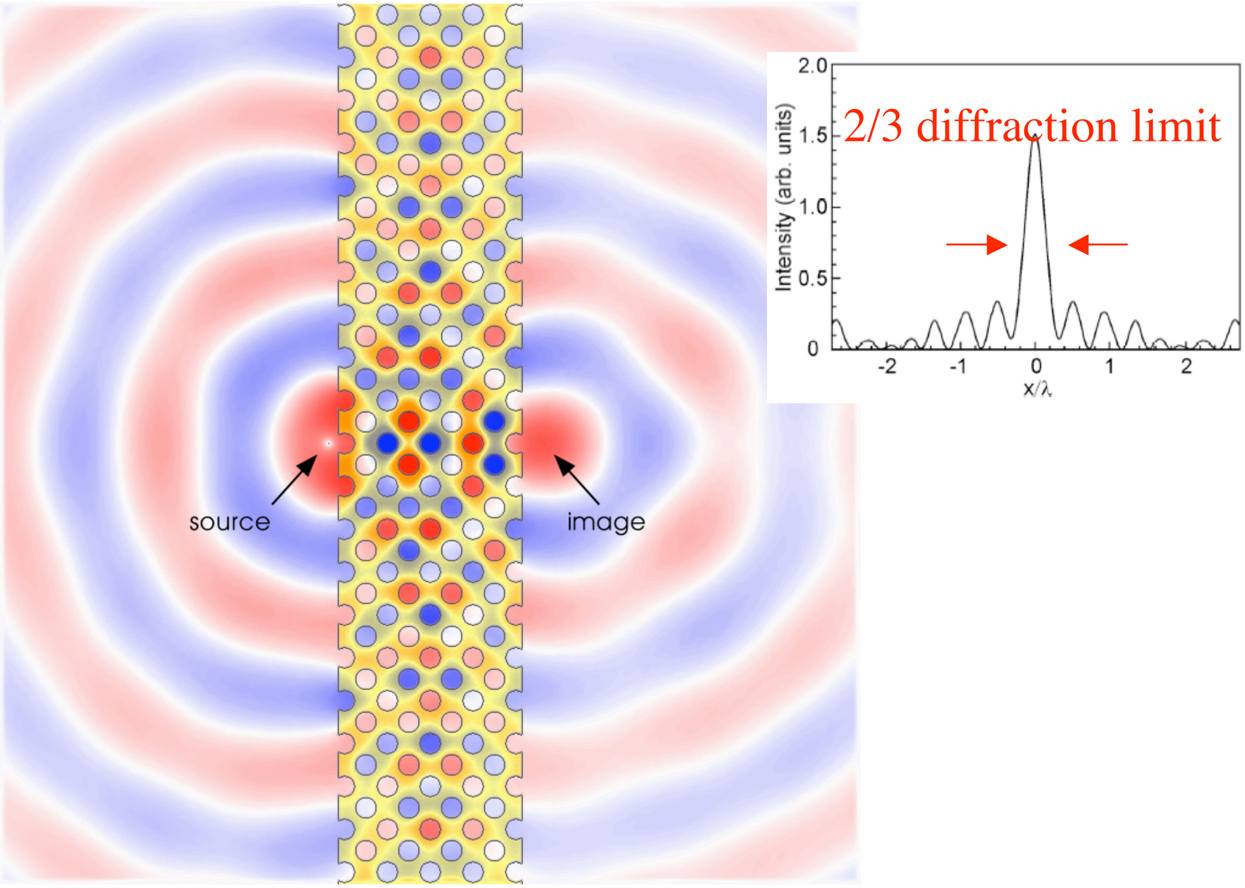


[M. Notomi, *PRB* **62**, 10696 (2000).]

Superlensing with Photonic Crystals

[Luo *et al*, *PRB* **68**, 045115 (2003).]

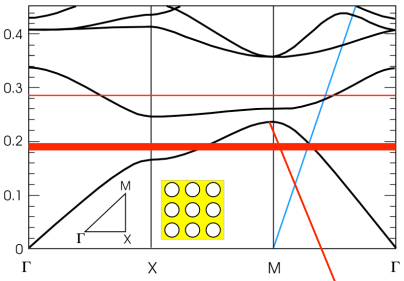


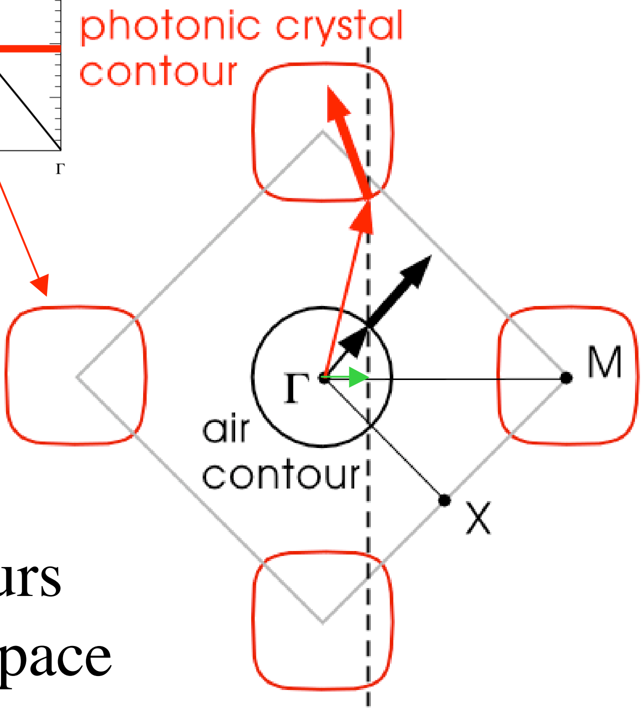


Here, using *positive* effective index but negative "effective mass"…

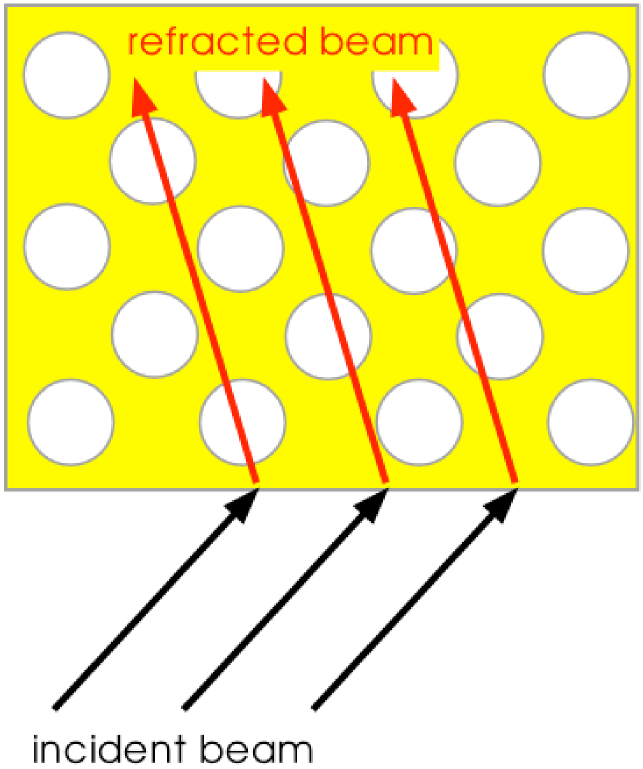
Negative Refraction
with negative *or* positive “index”

[Luo *et al*, *PRB* **65**, 2001104 (2002).]





ω contours
in (k_x,k_y) space



→ $k_{||}$ is conserved

Here, using *positive* effective index but negative “effective mass”

the magic of periodicity:
unusual dispersion without scattering

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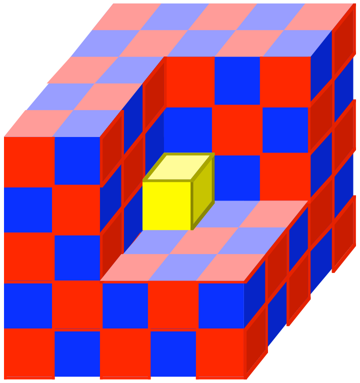
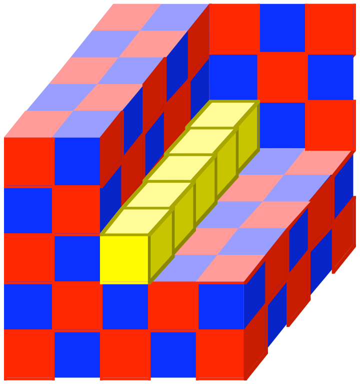
cavity

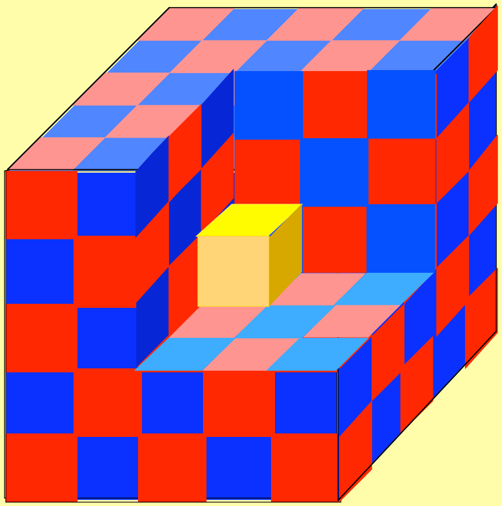


waveguide

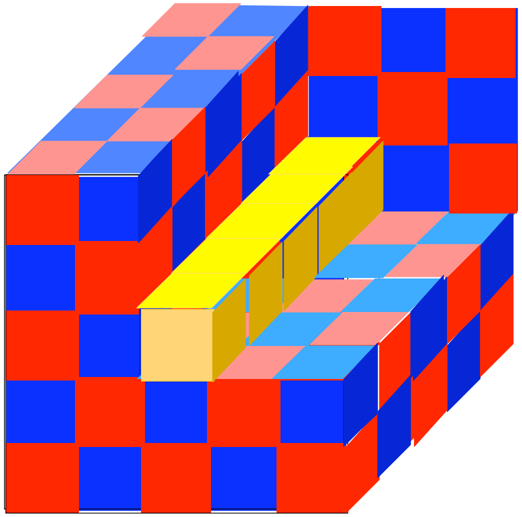


Intentional “defects” are good

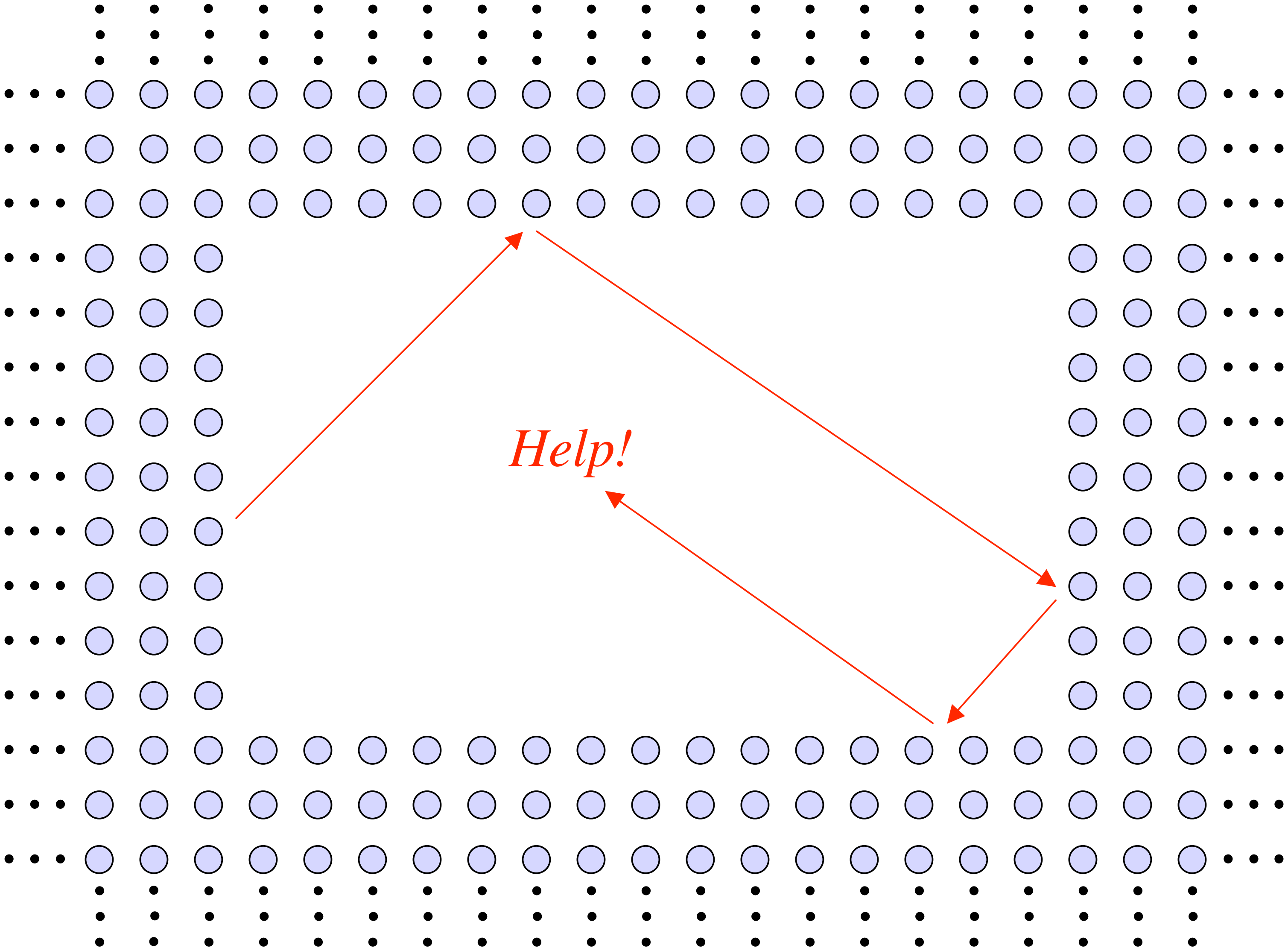
microcavities



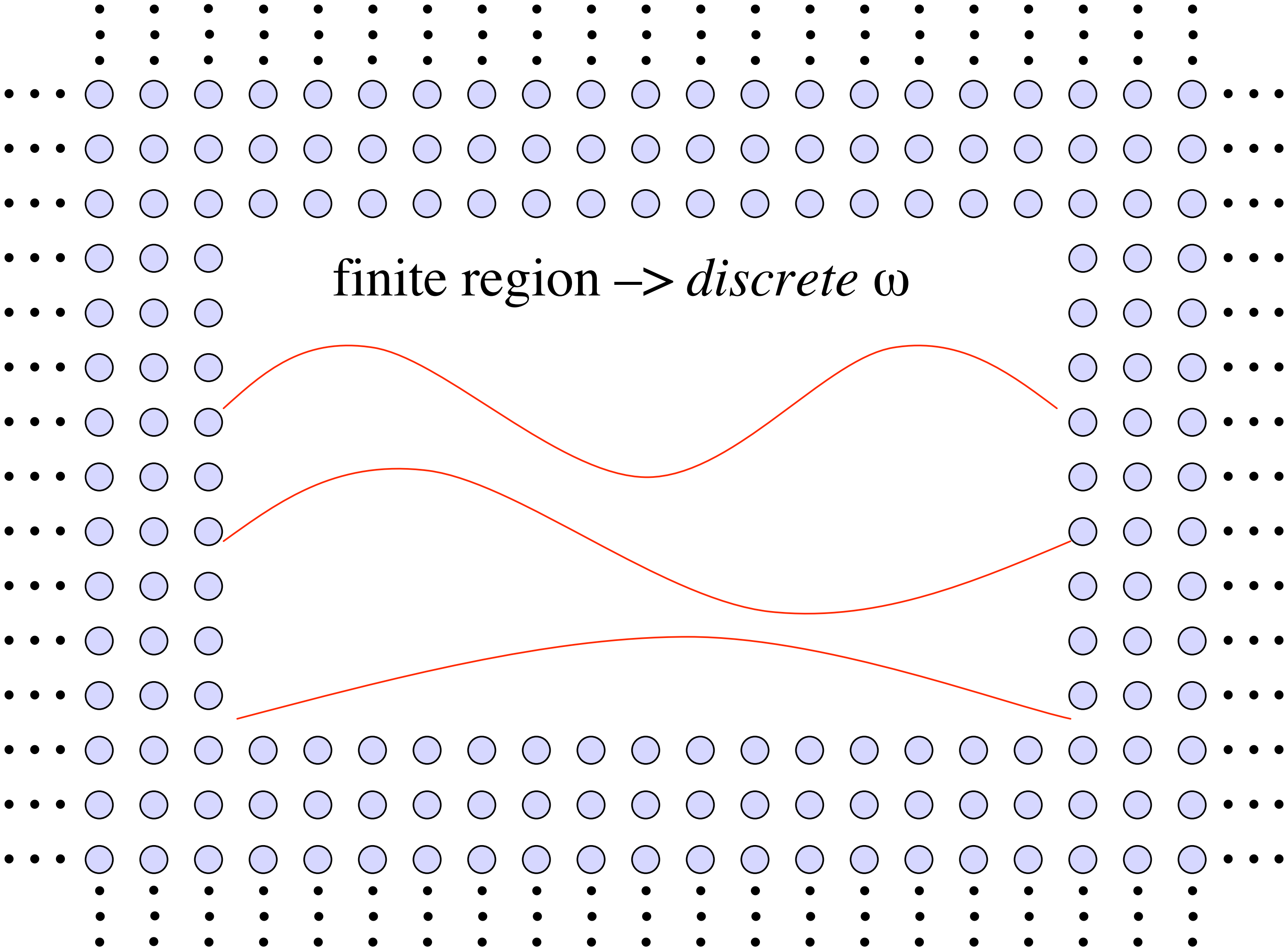
waveguides (“wires”)



Cavity Modes

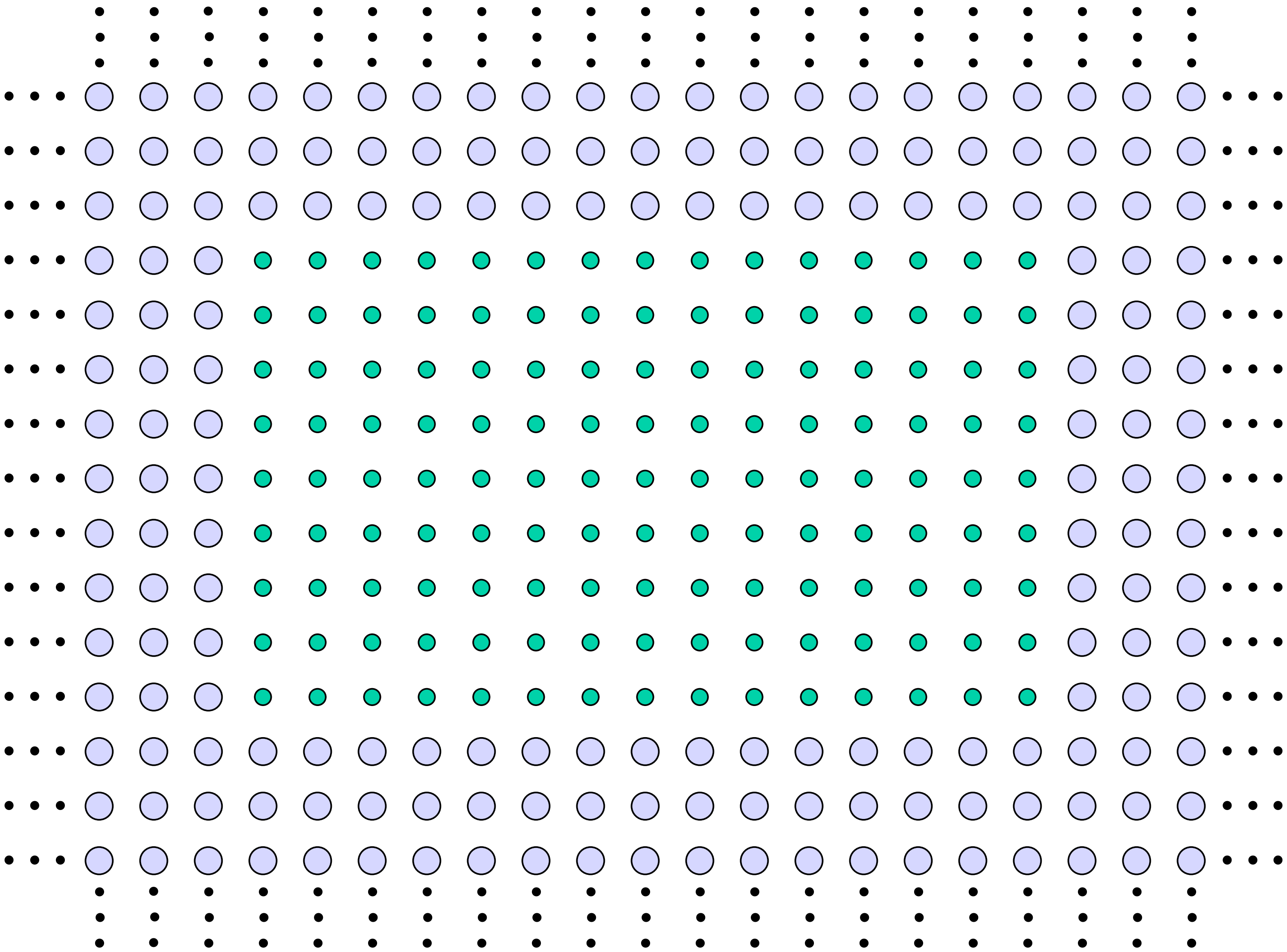


Cavity Modes

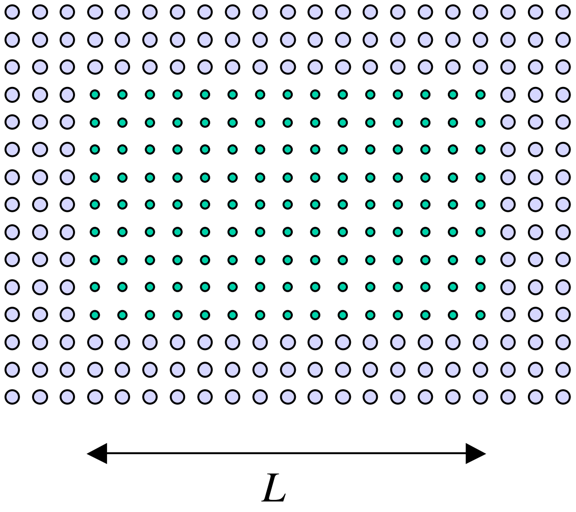


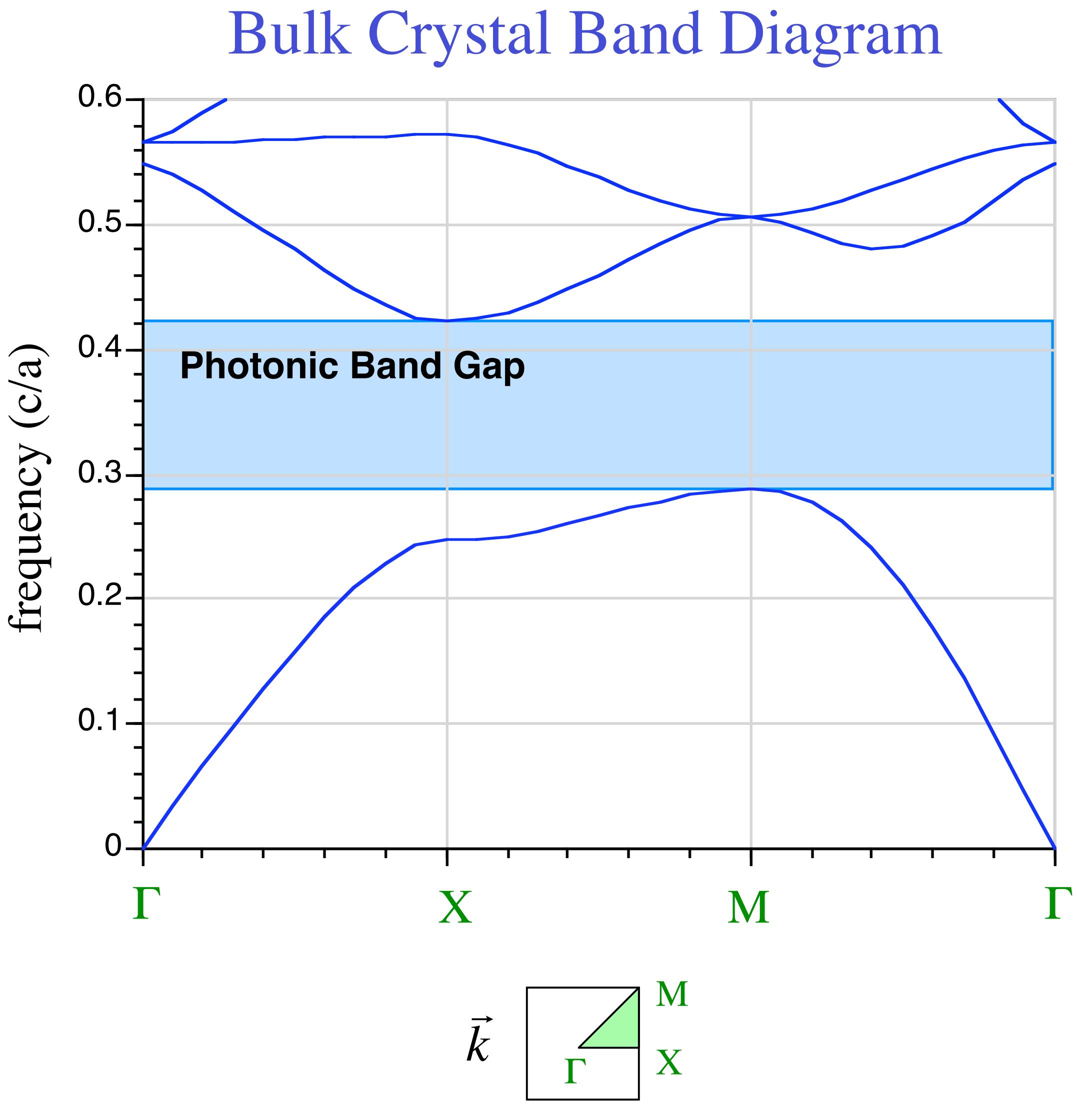
finite region –> *discrete* ω

Cavity Modes: Smaller Change

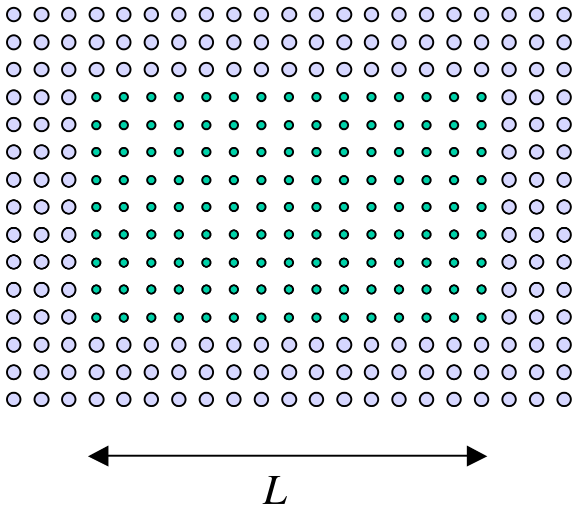


Cavity Modes: Smaller Change

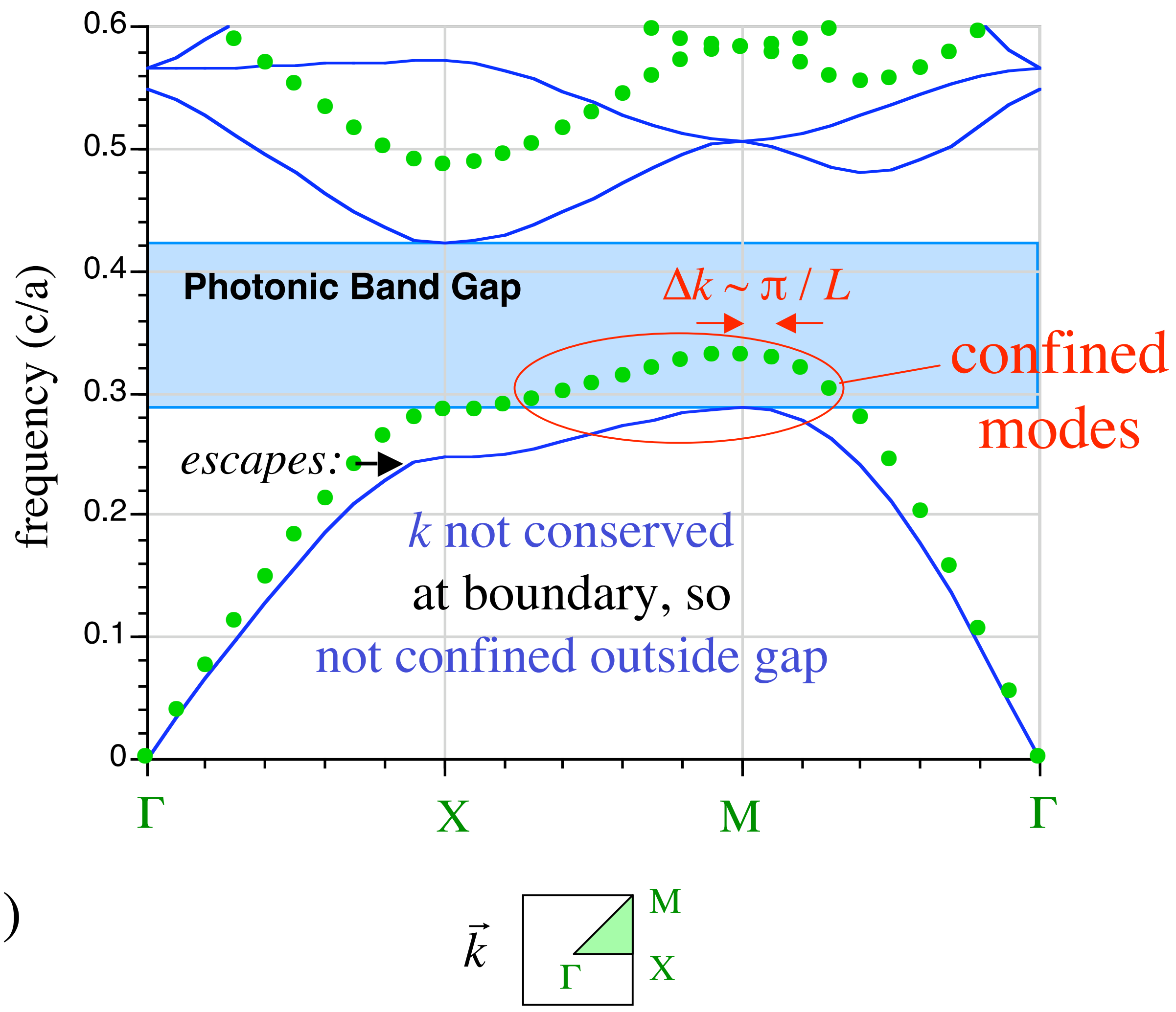




Cavity Modes: Smaller Change



Defect Crystal Band Diagram



$\Delta k \sim \pi / L$

confined modes

escapes:

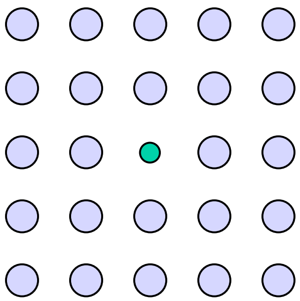
k not conserved at boundary, so not confined outside gap

Defect bands are shifted *up* (less ε)

with *discrete k*

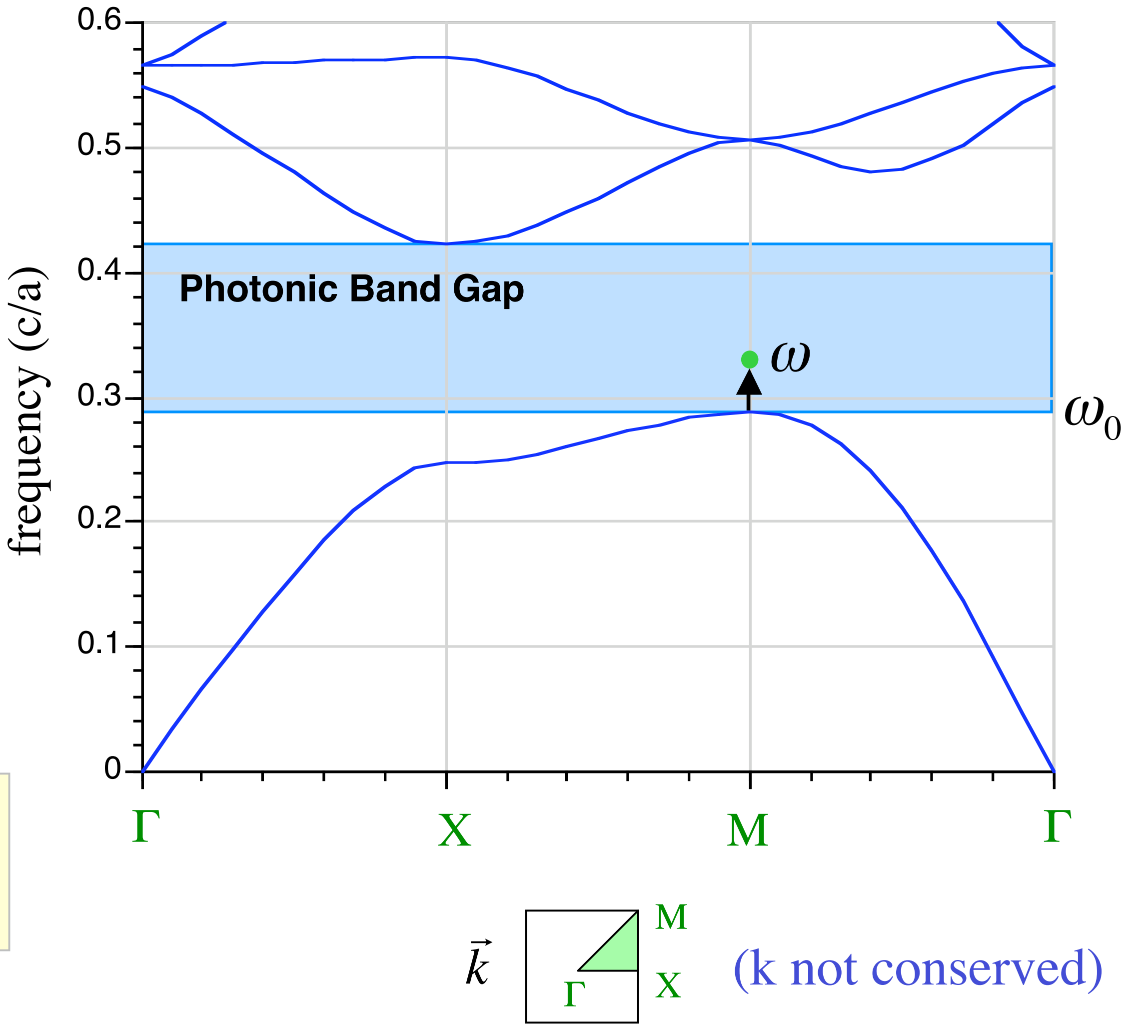
$$\# \cdot \frac{\lambda}{2} \sim L \qquad (k \sim 2\pi / \lambda)$$

Single-Mode Cavity



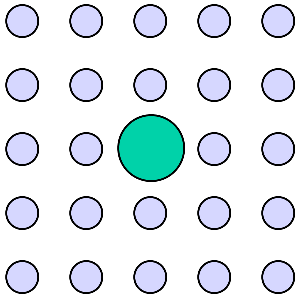
A *point defect*
can push up
a single mode
from the band edge

$$\text{field decay} \sim \sqrt{\frac{\omega - \omega_0}{\text{curvature}}}$$


Bulk Crystal Band Diagram

(k not conserved)

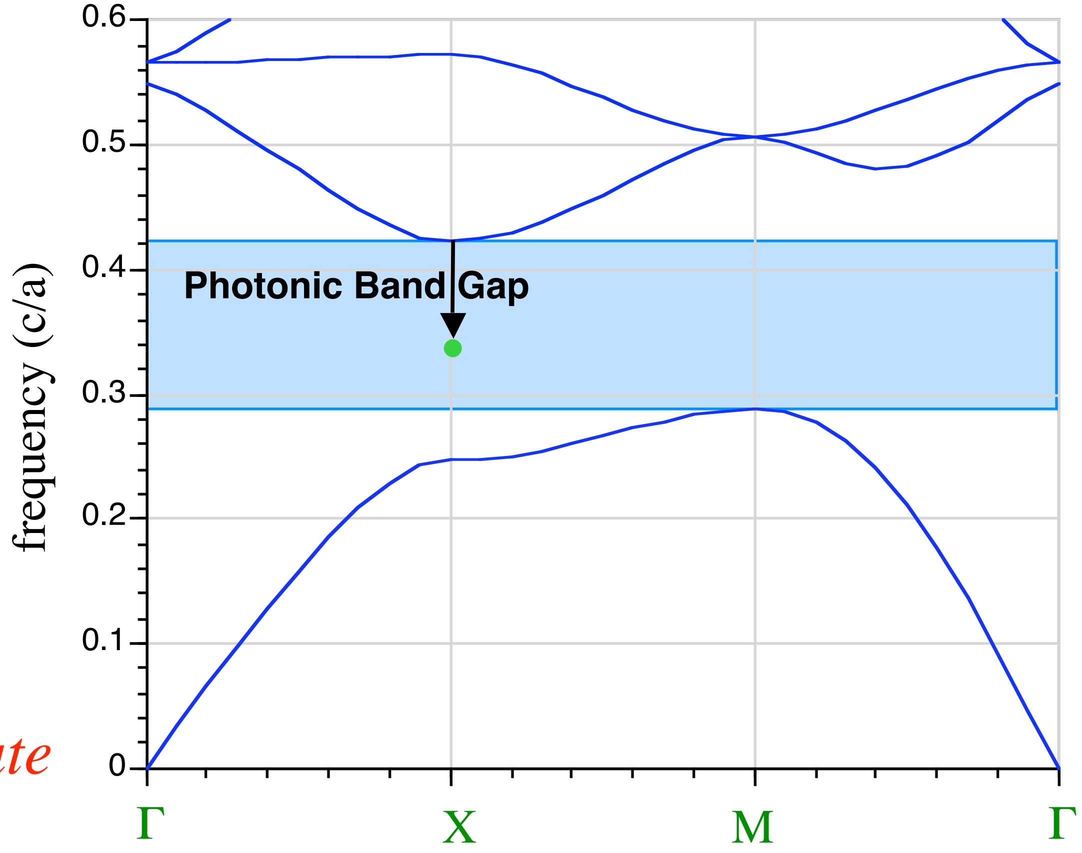
“Single”-Mode Cavity

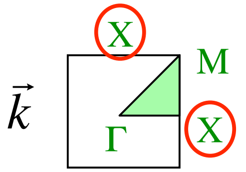


A *point defect*
can pull down
a “single” mode

…here, doubly-*degenerate*
(*two* states at *same* ω)

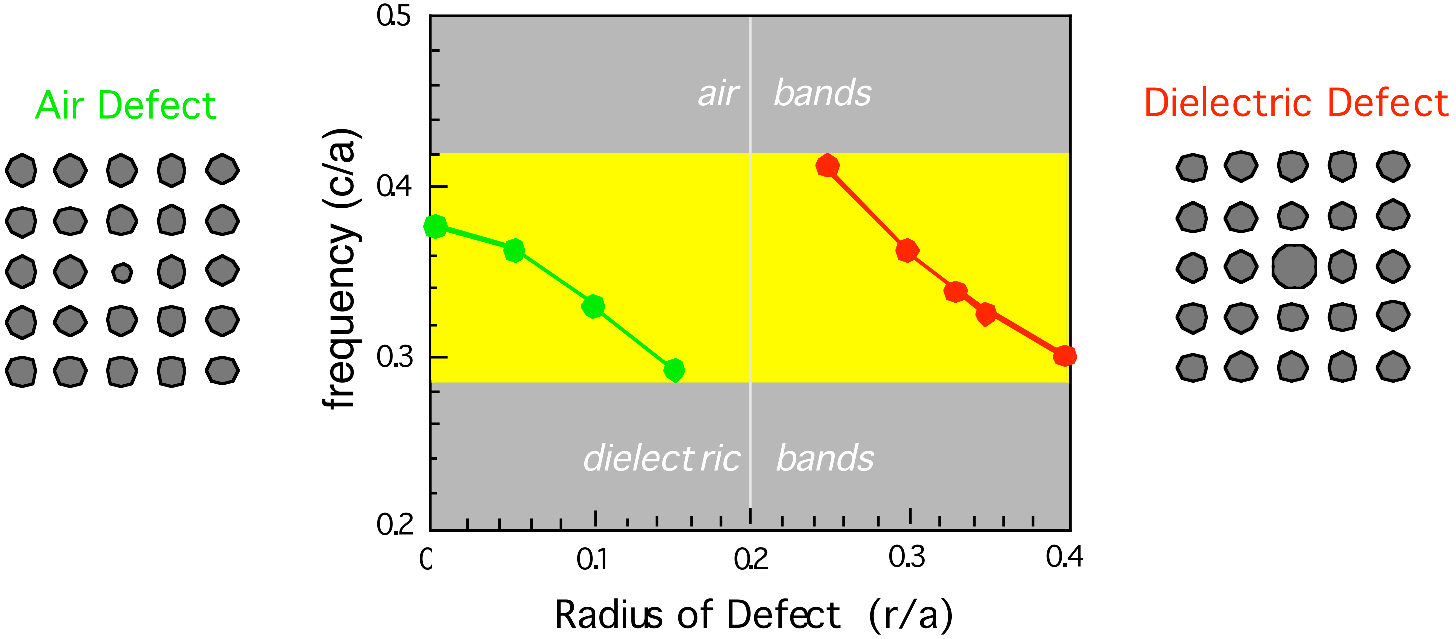
Bulk Crystal Band Diagram

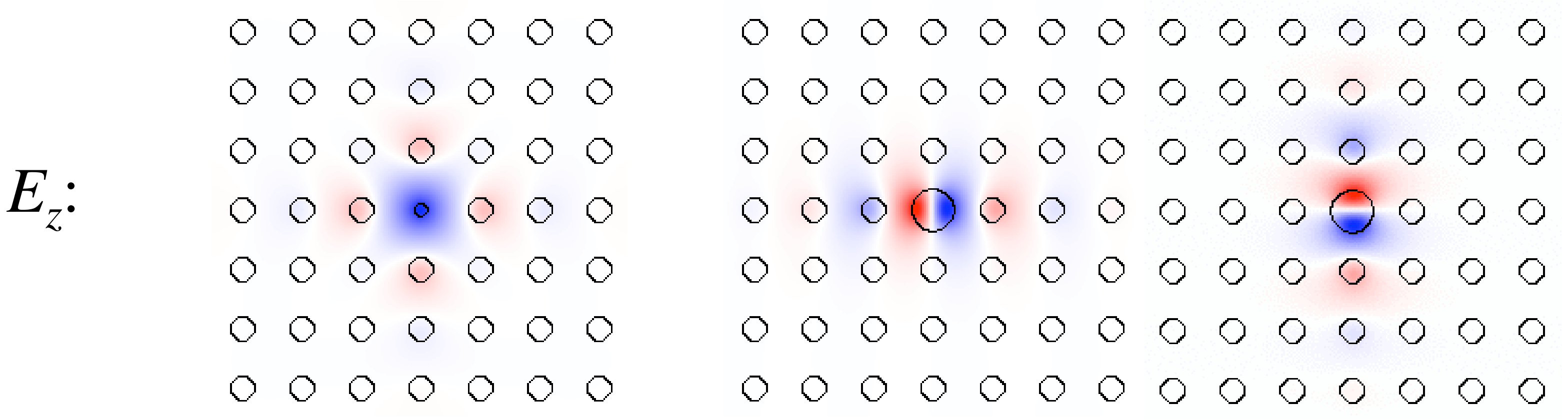




(k not conserved)

Tunable Cavity Modes

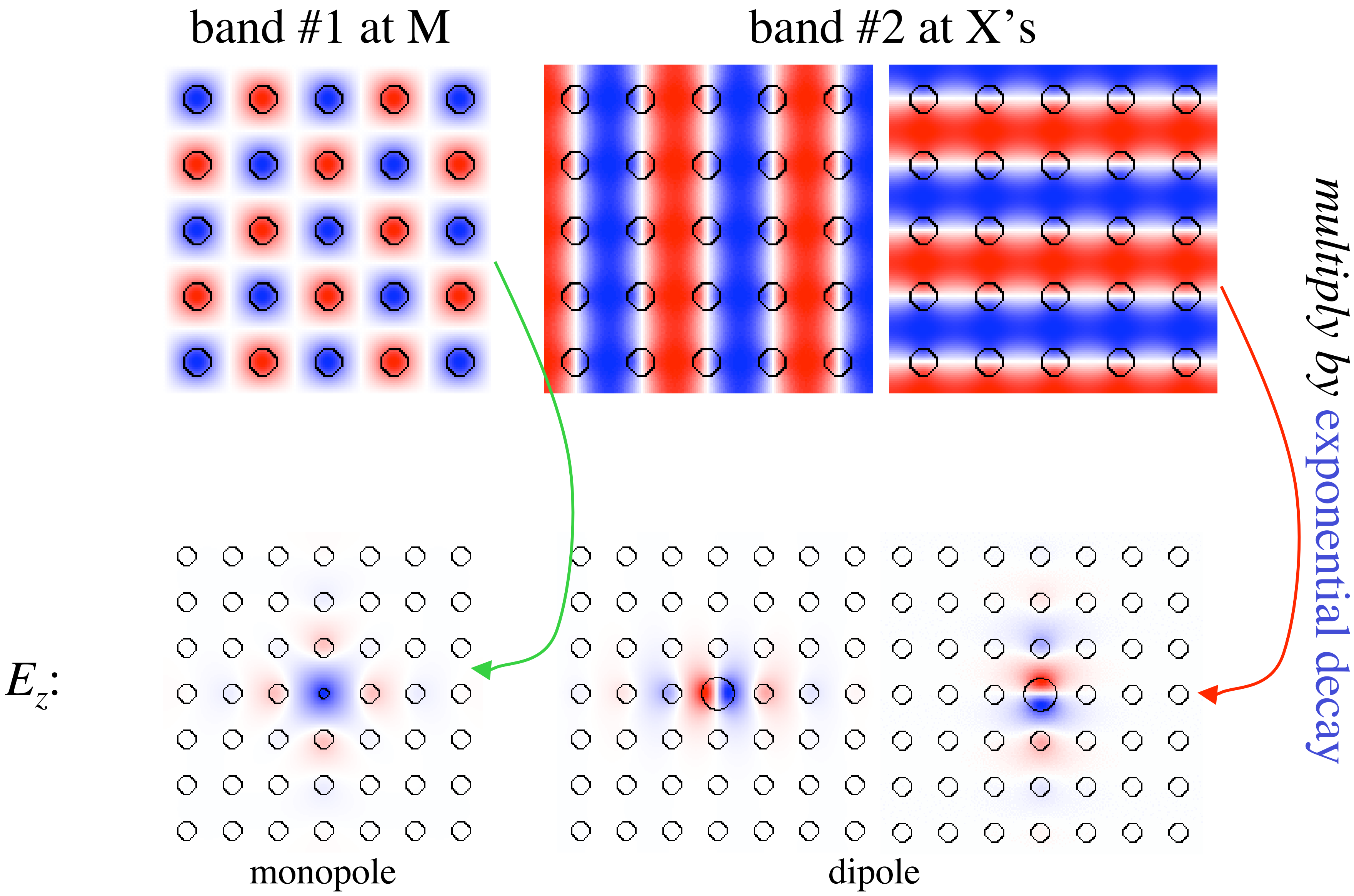




monopole

dipole

Tunable Cavity Modes



band #1 at M

band #2 at X's

multiply by exponential decay

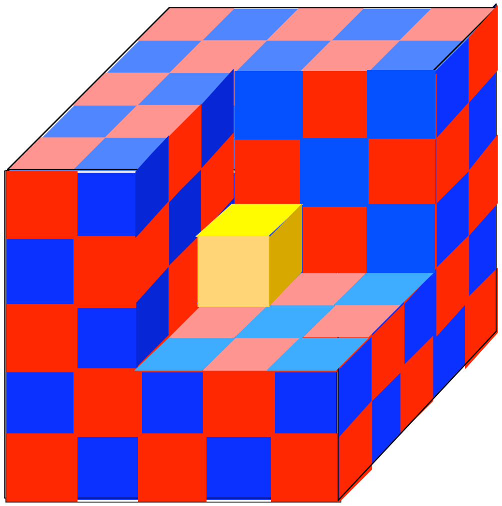
E_z:

monopole

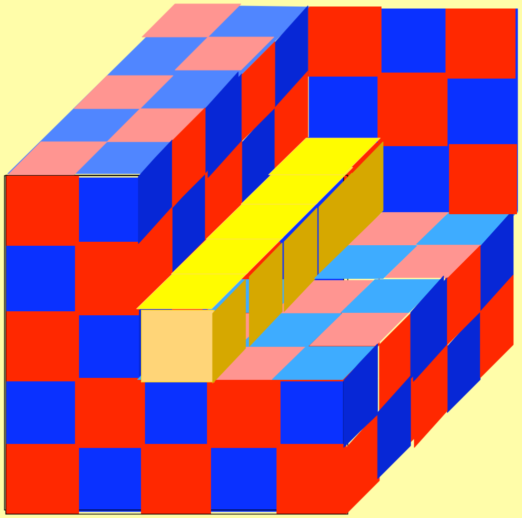
dipole

Intentional “defects” are good

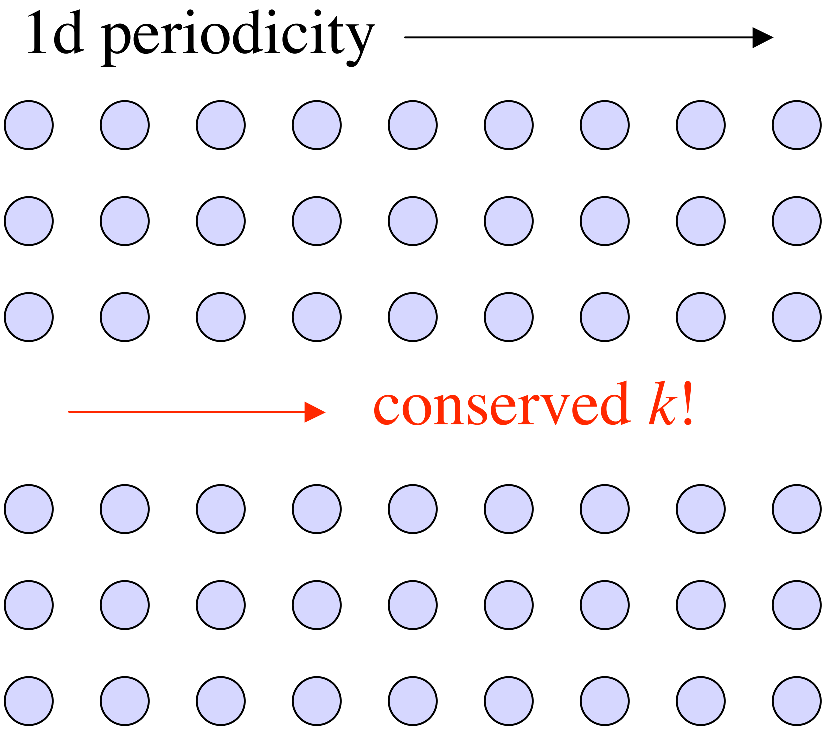
microcavities



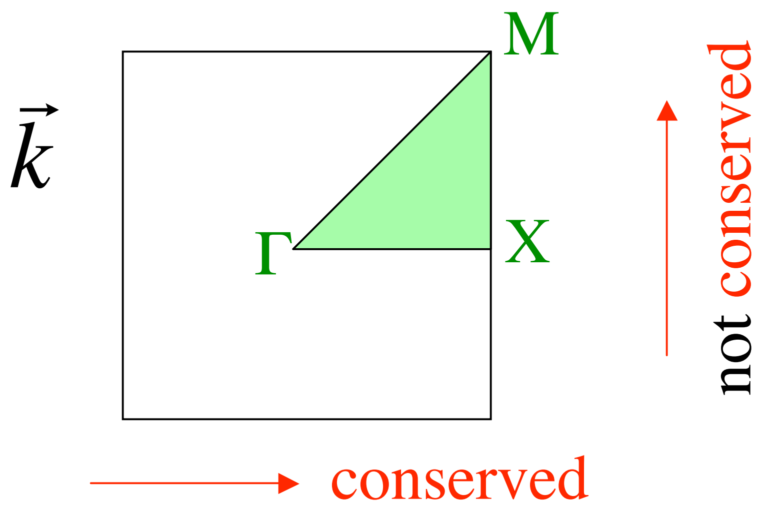
waveguides (“wires”)



Projected Band Diagrams


1d periodicity

conserved k!


$\vec{k}$

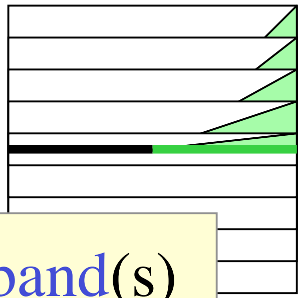
M

Γ

X

not conserved

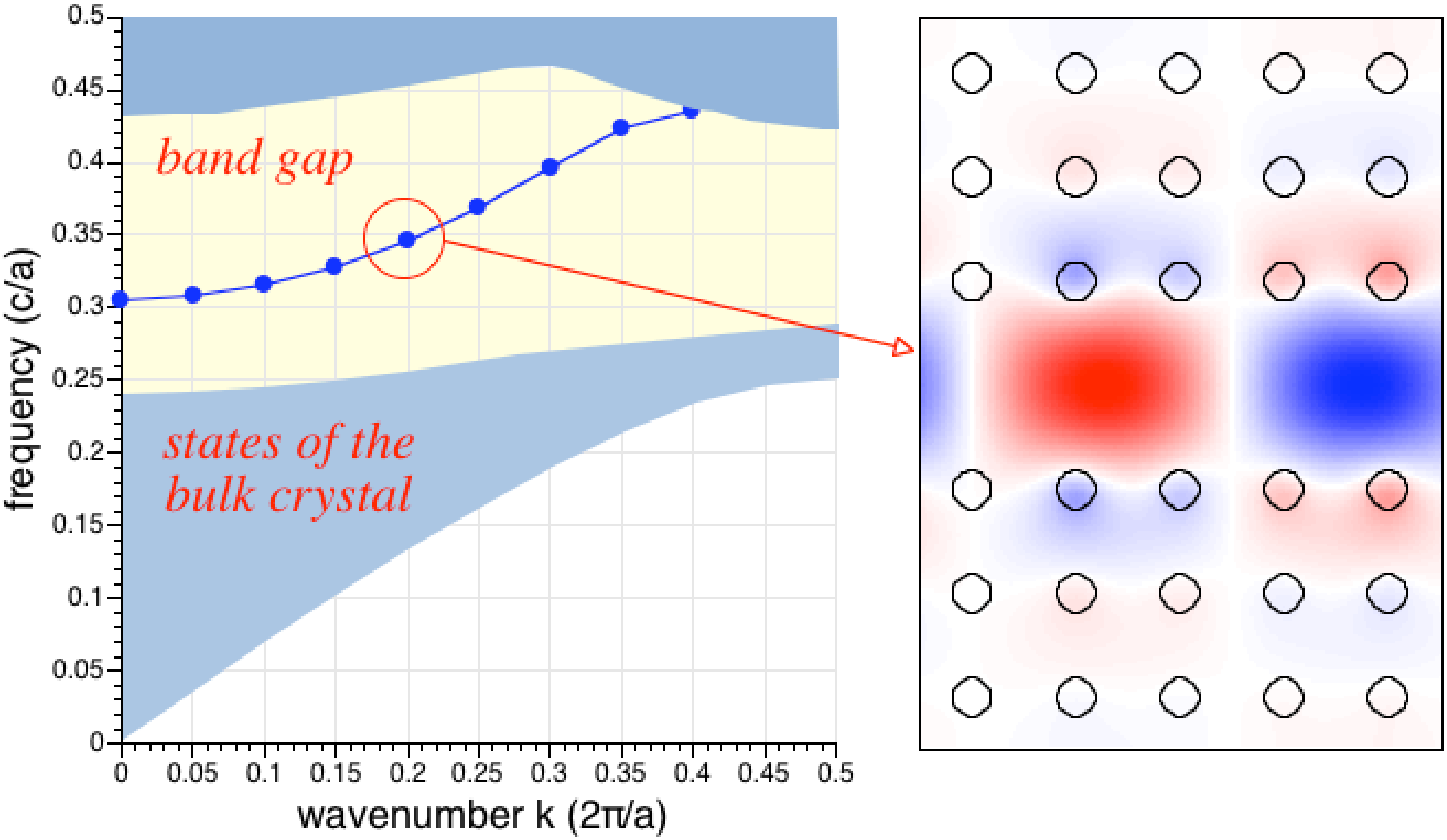
conserved



So, plot ω vs. k_x only…project Brillouin zone onto Γ–X:

gives continuum of bulk states + discrete guided band(s)

Air-waveguide Band Diagram



any state in the gap cannot couple to bulk crystal –> localized

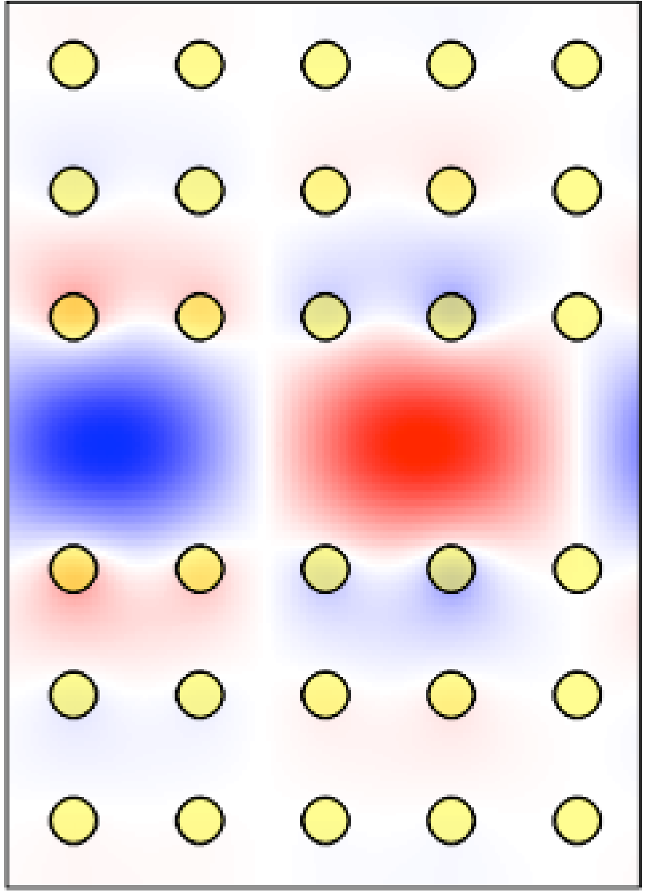
(Waveguides don’t really need a *complete* gap)

Fabry-Perot waveguide:

This is exploited *e.g*. for photonic-crystal fibers…

Guiding Light in Air!

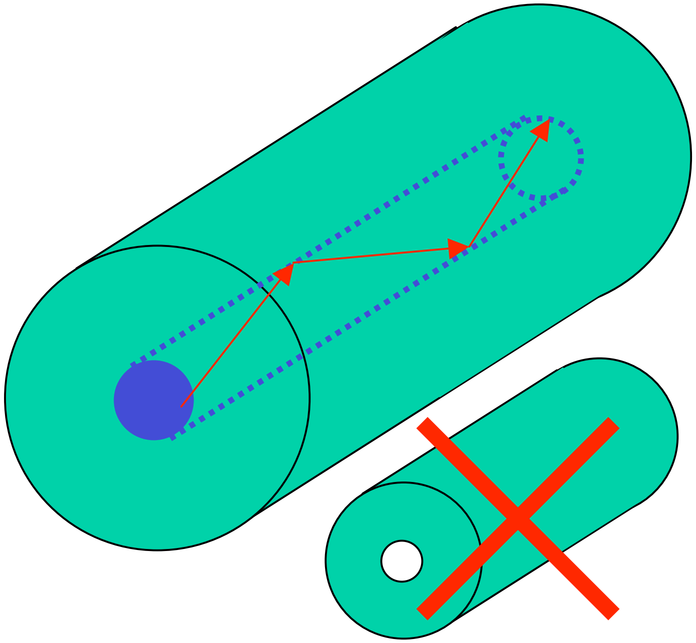
mechanism is gap only



vs. standard optical fiber:

"total internal reflection"

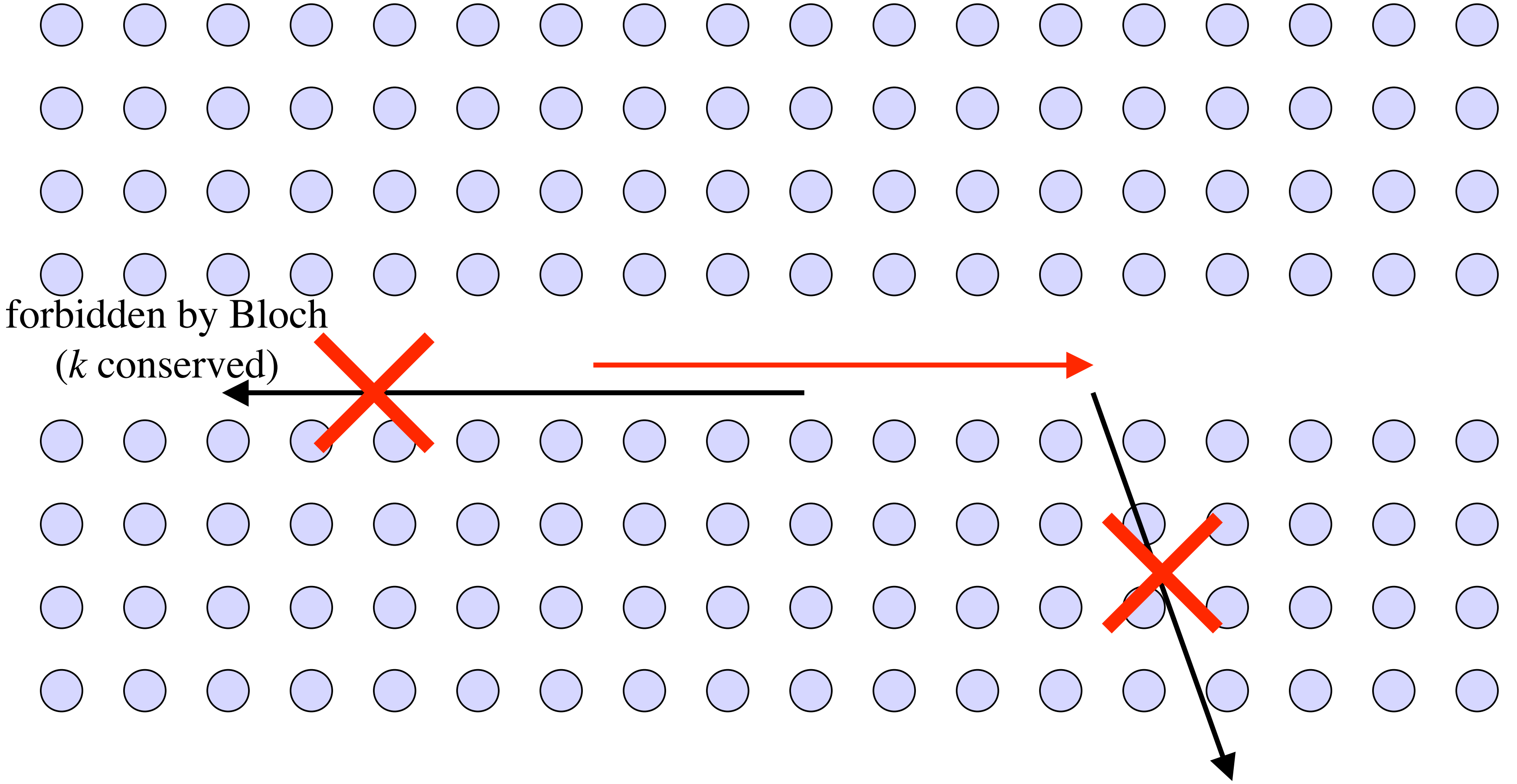
— requires *higher*-index core



no hollow core!

hollow = lower absorption, lower nonlinearities, higher power

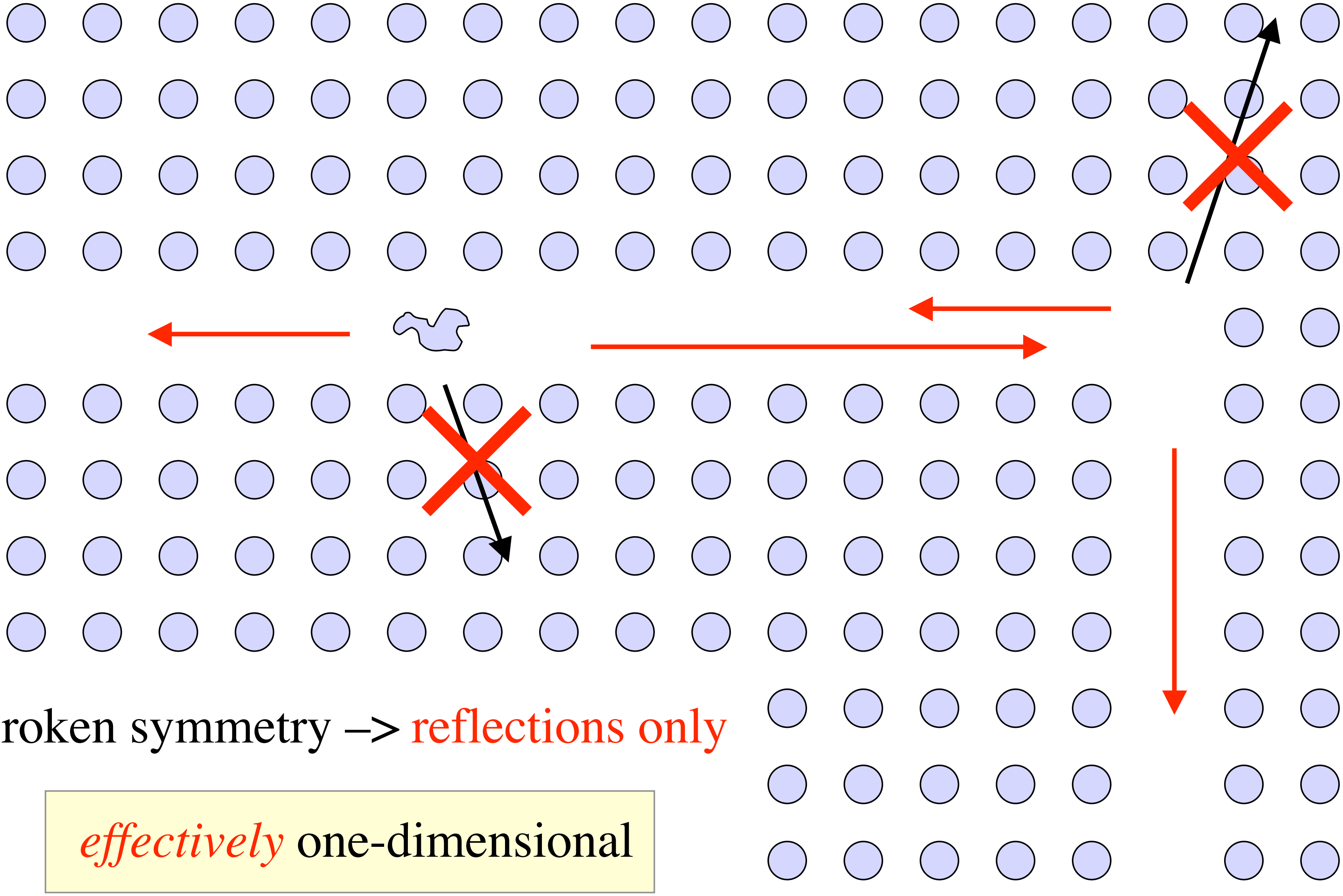
Review: Why no scattering?



forbidden by Bloch
(*k* conserved)

forbidden by gap
(except for finite-crystal tunneling)

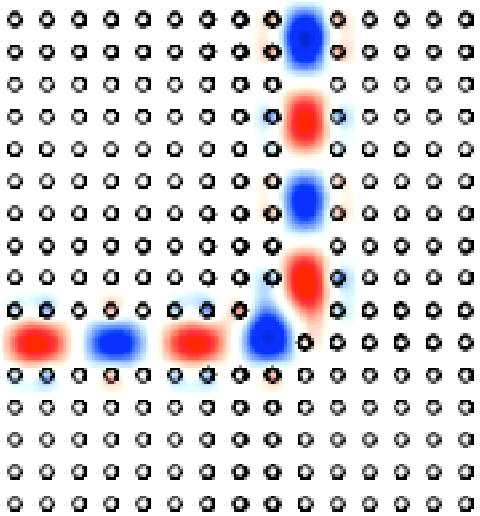
Benefits of a complete gap…

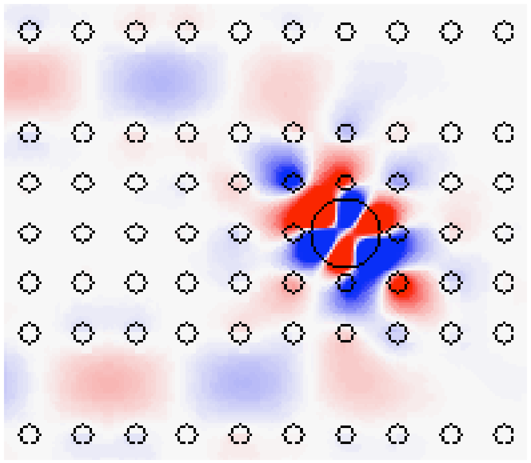


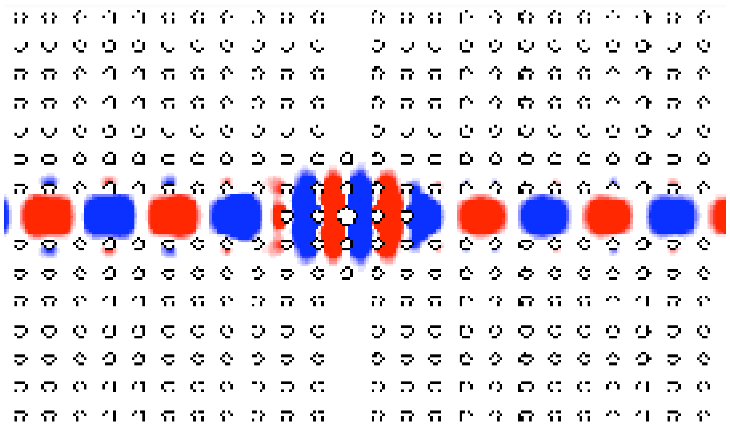
broken symmetry –> reflections only

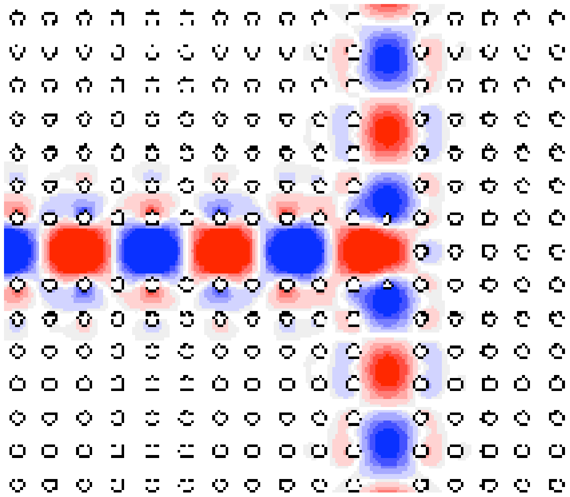
effectively one-dimensional

“1d” Waveguides + Cavities = Devices

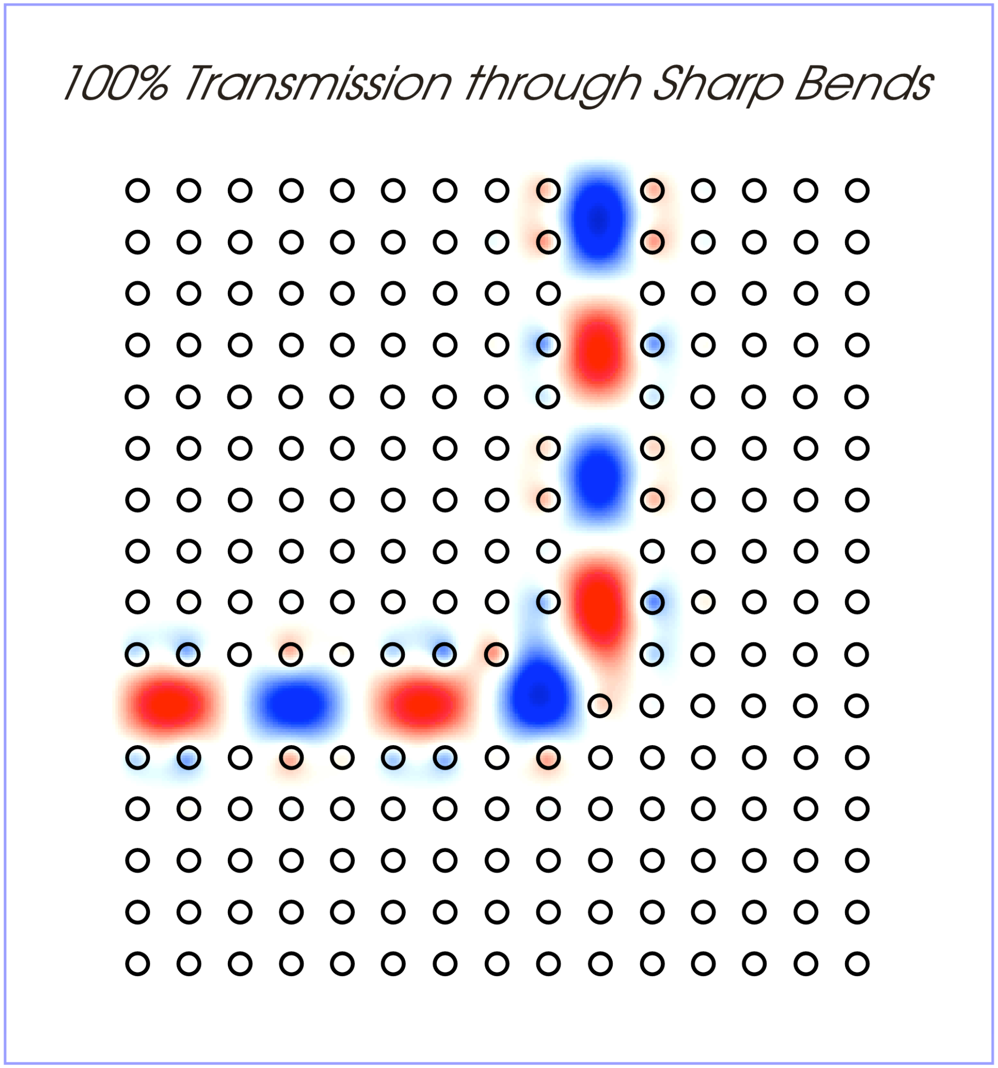

high transmission through sharp bends


channel-drop filter


elimination of waveguide crosstalk

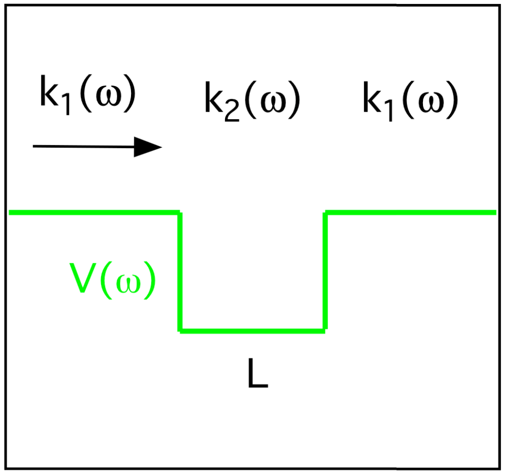

high transmission in wide-angle splitters

Lossless Bends



100% Transmission through Sharp Bends

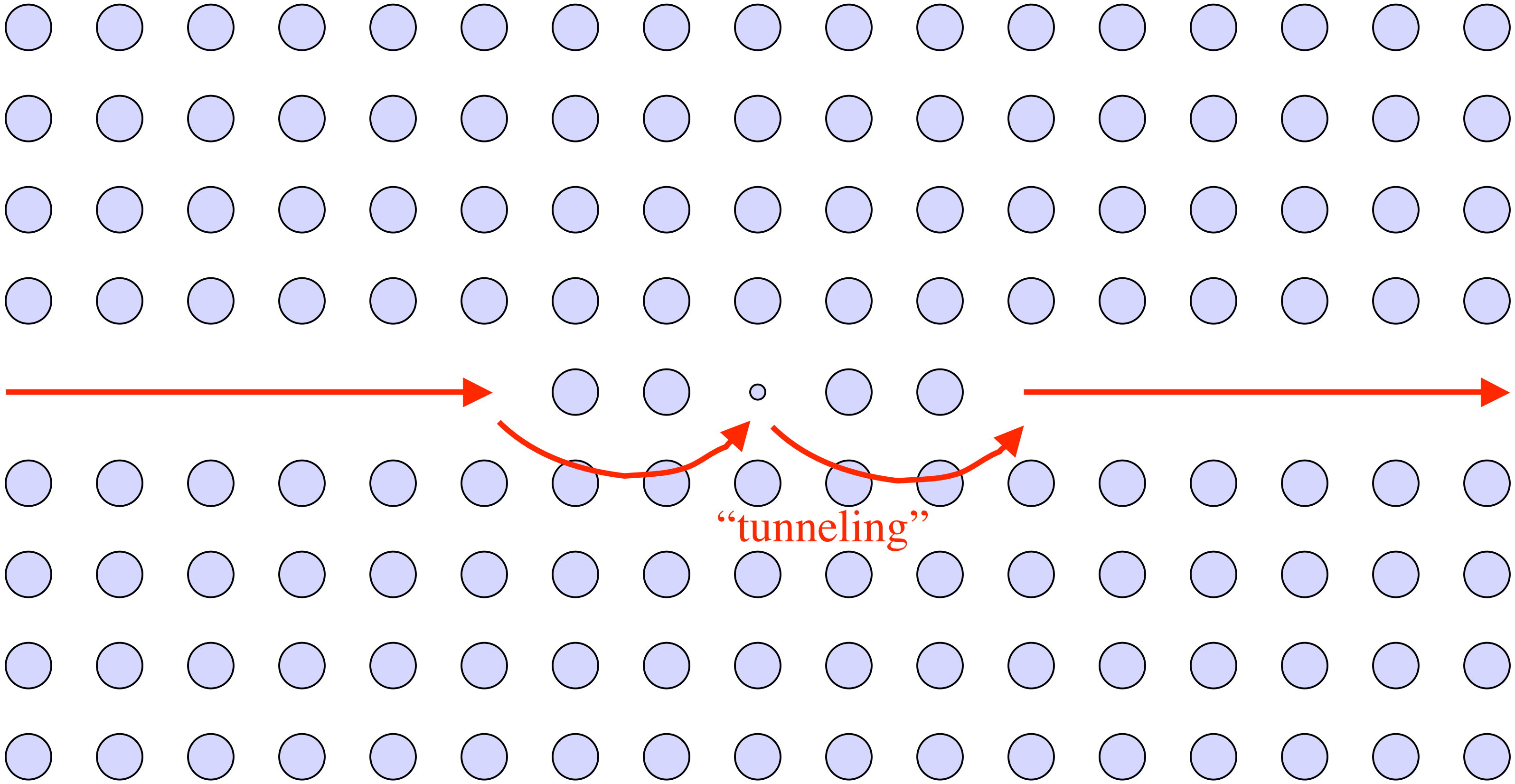
Maps onto problem of
Electron Resonant
Scattering in 1D



[A. Mekis *et al.*,
Phys. Rev. Lett. **77**, 3787 (1996)]

symmetry + single-mode + “1d” = resonances of 100% transmission

Waveguides + Cavities = Devices

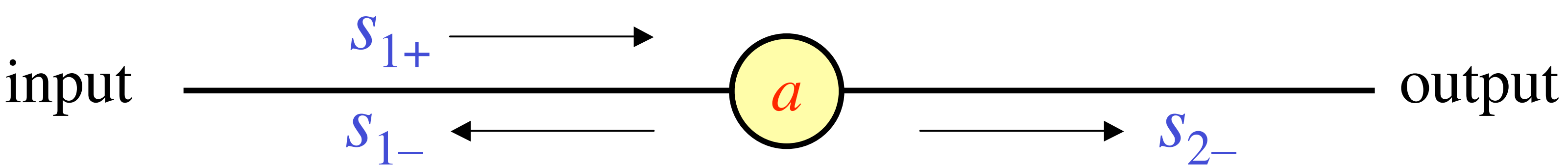


Ugh, must we simulate this to get the basic behavior?

“Coupling-of-Modes-in-Time”

(a form of coupled-mode theory)

[H. Haus, *Waves and Fields in Optoelectronics*]



input

s_{1+} $\rightarrow$

s_{1-} $\leftarrow$

a

$\rightarrow$ s_{2-}

output

resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = flux

$|a|^2$ = energy

$$\frac{da}{dt} = -i\omega_0 a - \frac{2}{\tau} a + \sqrt{\frac{2}{\tau}} s_{1+}$$

$$s_{1-} = -s_{1+} + \sqrt{\frac{2}{\tau}} a, \quad s_{2-} = \sqrt{\frac{2}{\tau}} a$$

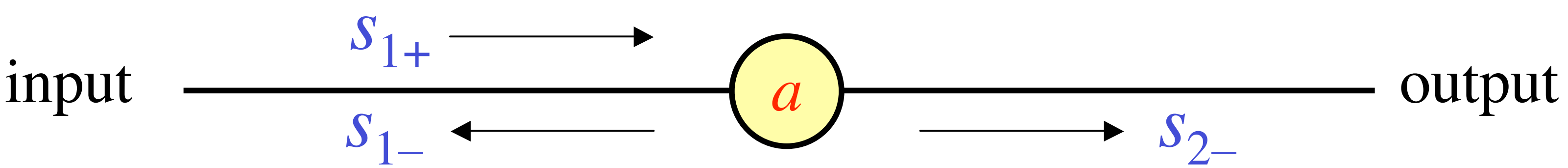
assumes only:

- exponential decay
  (strong confinement)
- conservation of energy
- time-reversal symmetry

“Coupling-of-Modes-in-Time”

(a form of coupled-mode theory)

[H. Haus, *Waves and Fields in Optoelectronics*]



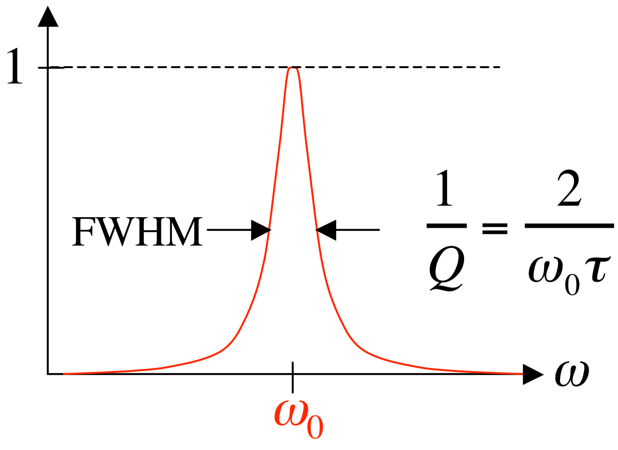
resonant cavity
frequency ω_0, lifetime τ

$|s|^2$ = flux

$|a|^2$ = energy

transmission T

$= | s_{2-} |^2 / | s_{1+} |^2$



$$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$$

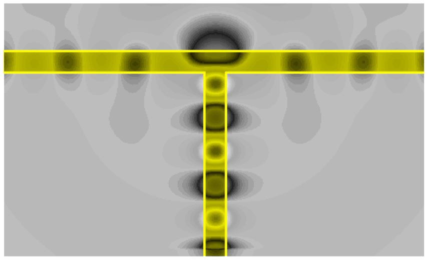
…quality factor Q

T = Lorentzian filter

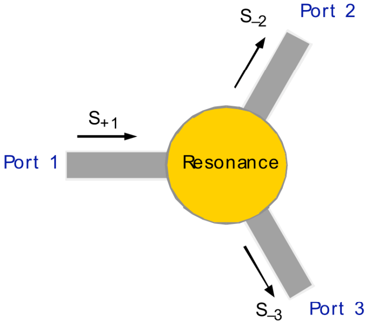
$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

Wide-angle Splitters

only 30%

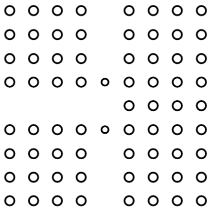


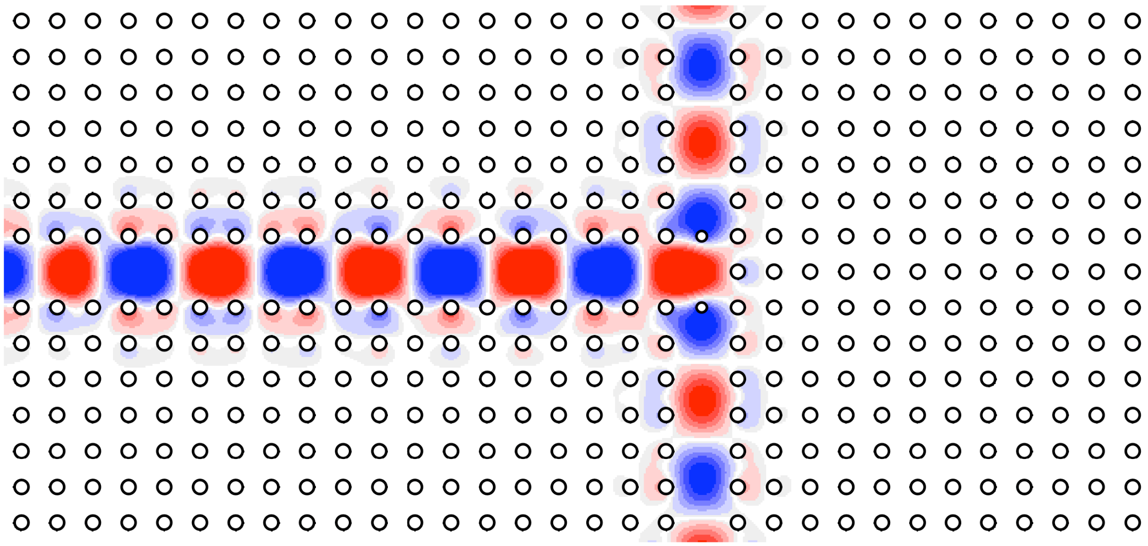
C. Manolatou



100% Transmission

$$1/\tau_1 = 1/\tau_2 + 1/\tau_3$$

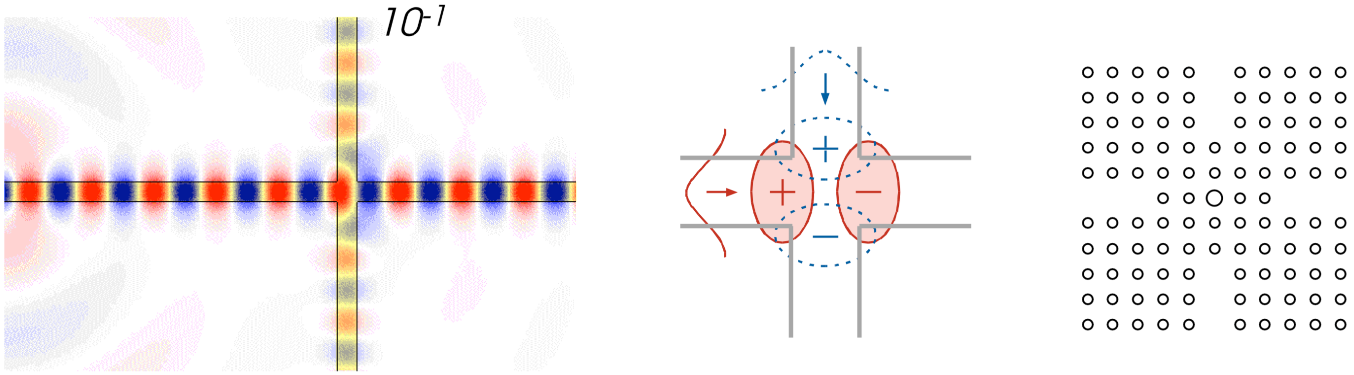


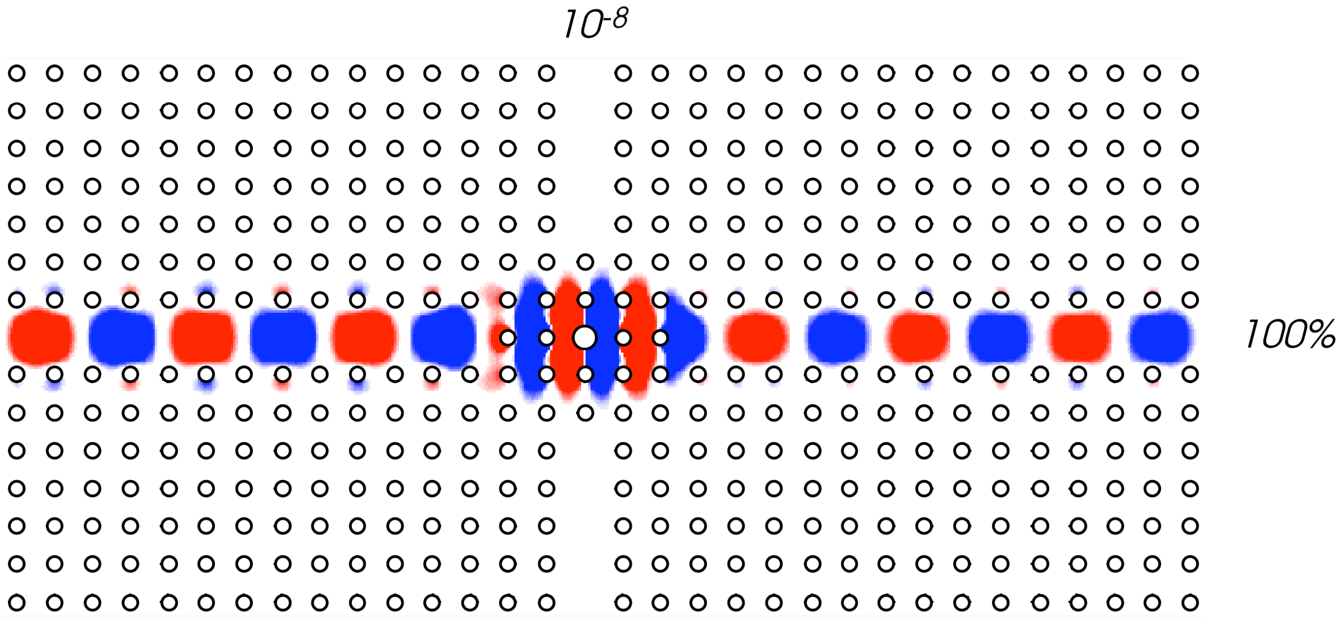


100%

[S. Fan *et al.*, *J. Opt. Soc. Am. B* **18**, 162 (2001)]

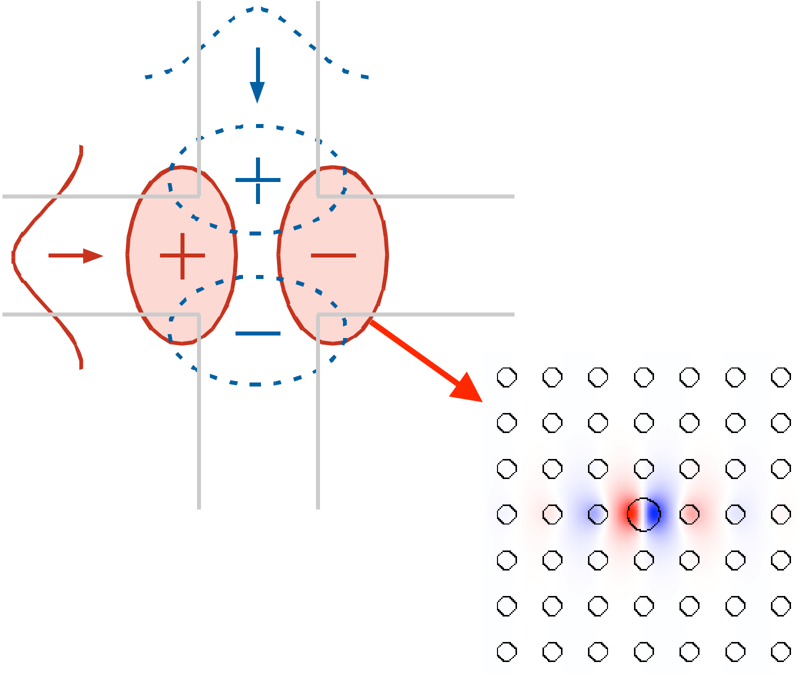
Waveguide Crossings

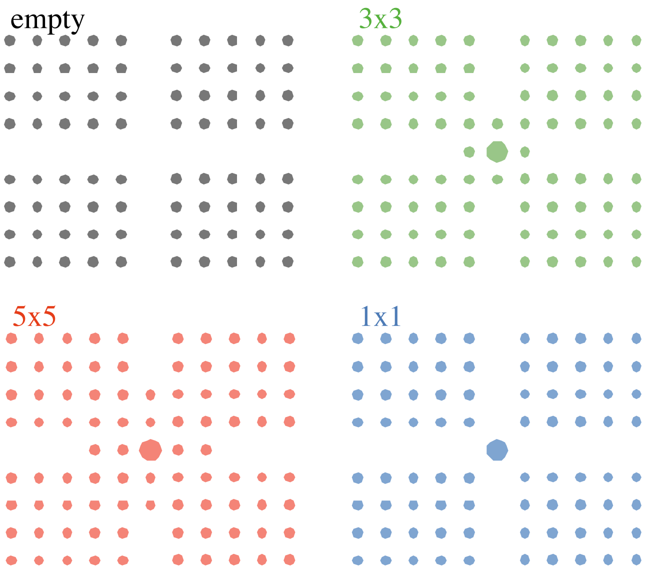


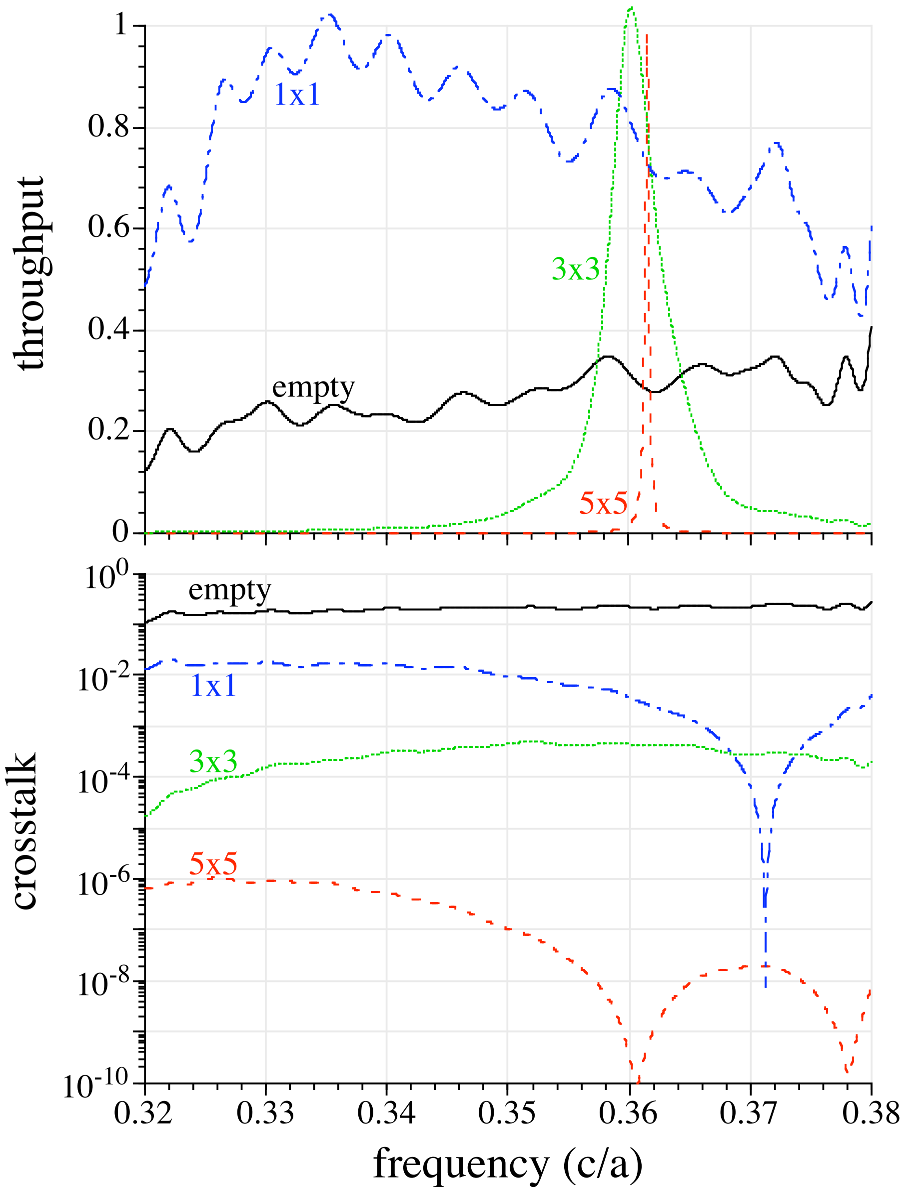


[S. G. Johnson *et al.*, *Opt. Lett.* **23**, 1855 (1998)]

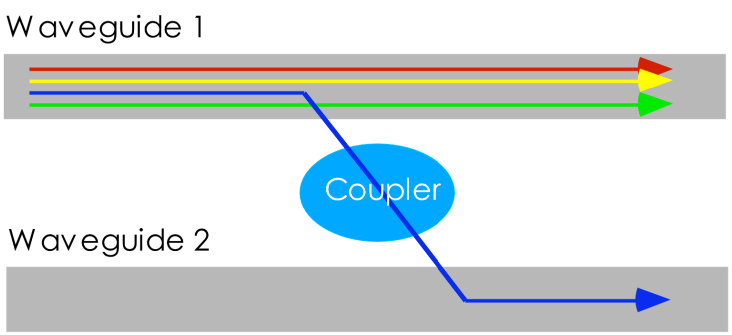
Waveguide Crossings







Channel-Drop Filters

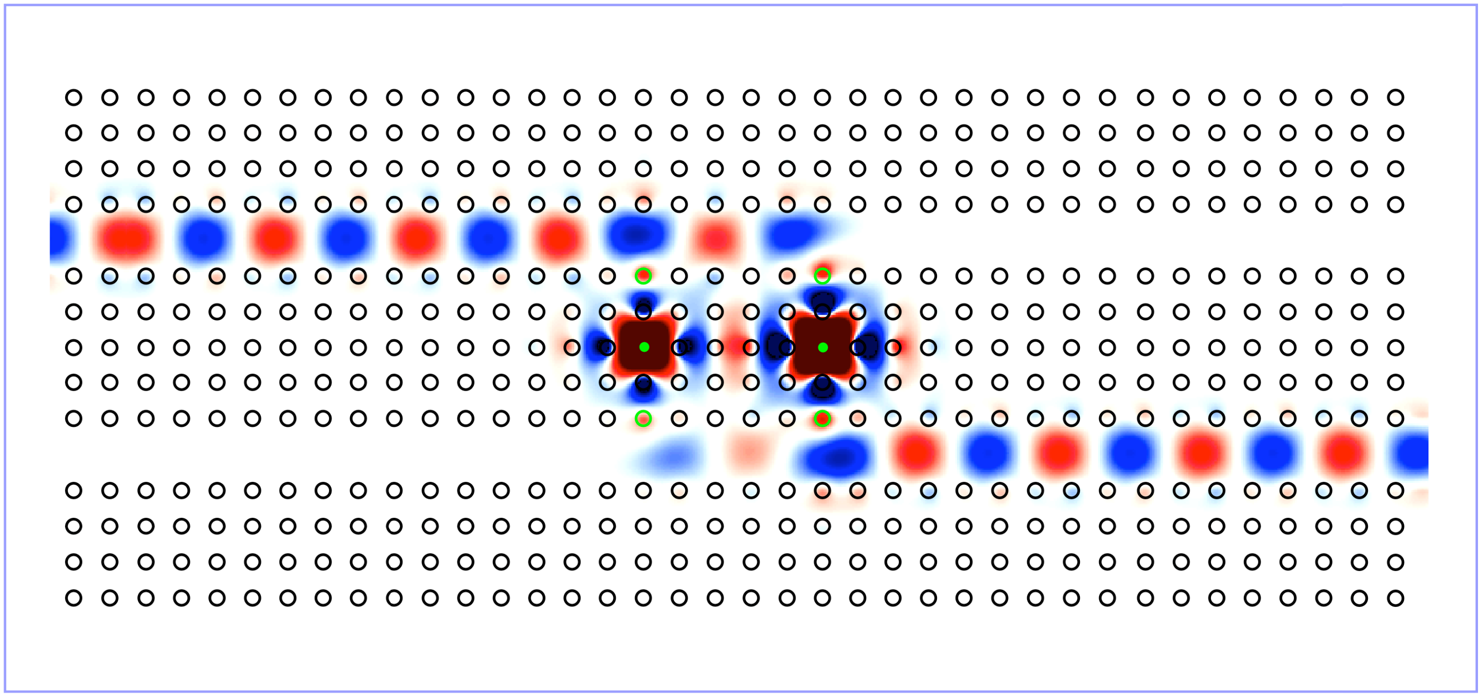


Criteria for Perfect Channel-dropping

Two resonant modes with even and odd symmetry

Modes must be degenerate

Modes must have equal decay rates



[S. Fan *et al.*, *Phys. Rev. Lett.* **80**, 960 (1998)]

Enough passive, linear devices…

Photonic crystal cavities:

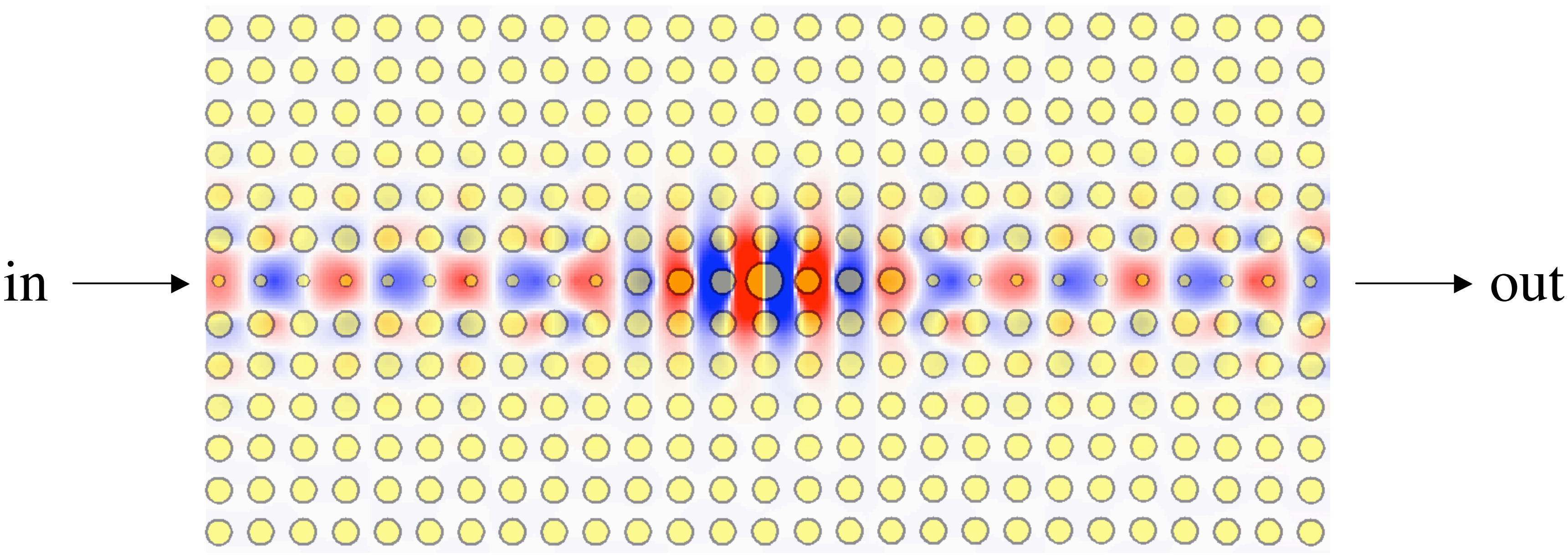
tight confinement (~ $\lambda/2$ diameter)

+ long lifetime (high Q independent of size)

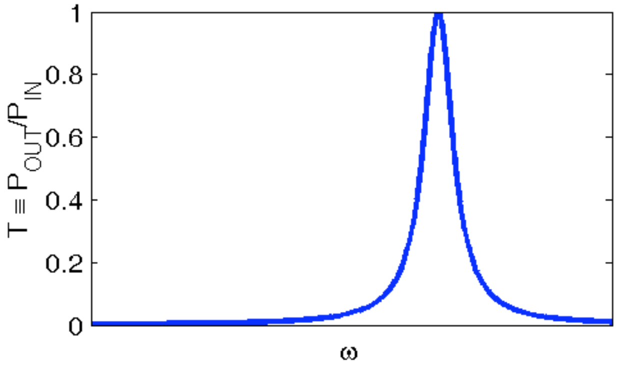
= enhanced nonlinear effects

e.g. Kerr nonlinearity, Δn ~ intensity

A ~~Linear~~ *Nonlinear* Filter

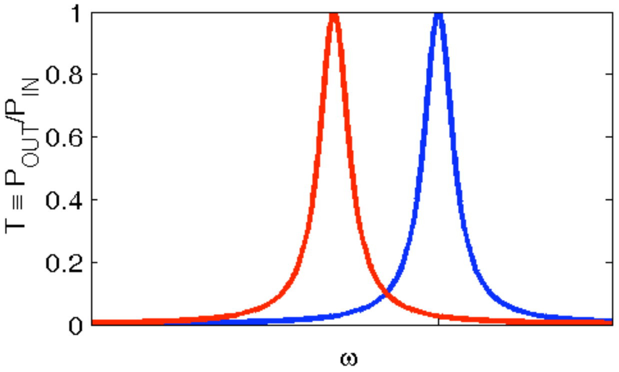


Linear response:
Lorenzian Transmisson



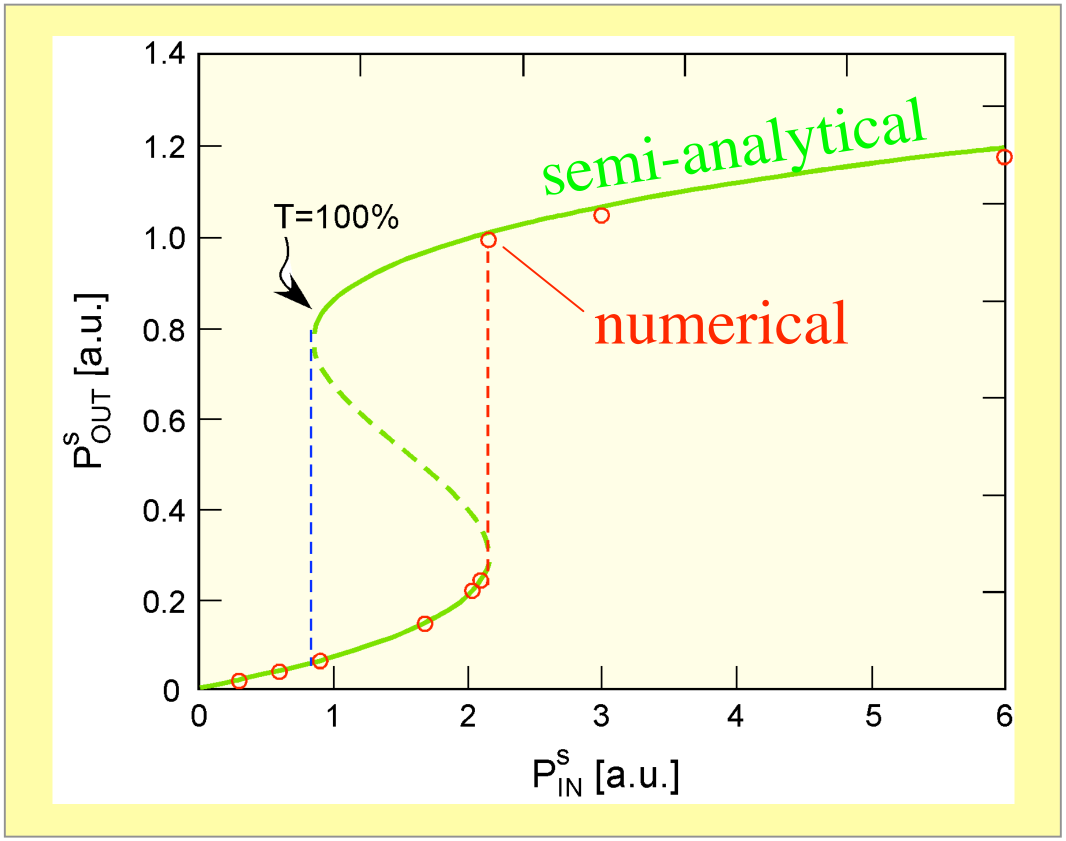
+ nonlinear
index shift

shifted peak



A ~~Linear~~ *Nonlinear* “Transistor”

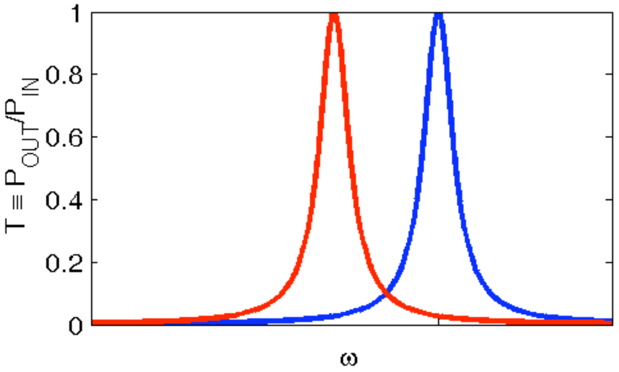
[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]



Logic gates, switching,
rectifiers, amplifiers,
isolators, …

+ feedback

shifted peak



Bistable (hysteresis) response

Power threshold is near optimal
(~mW for Si and telecom bandwidth)

Enough passive, linear devices…

Photonic crystal cavities:

tight confinement (~ $\lambda/2$ diameter)

+ long lifetime (high Q independent of size)

= enhanced nonlinear effects

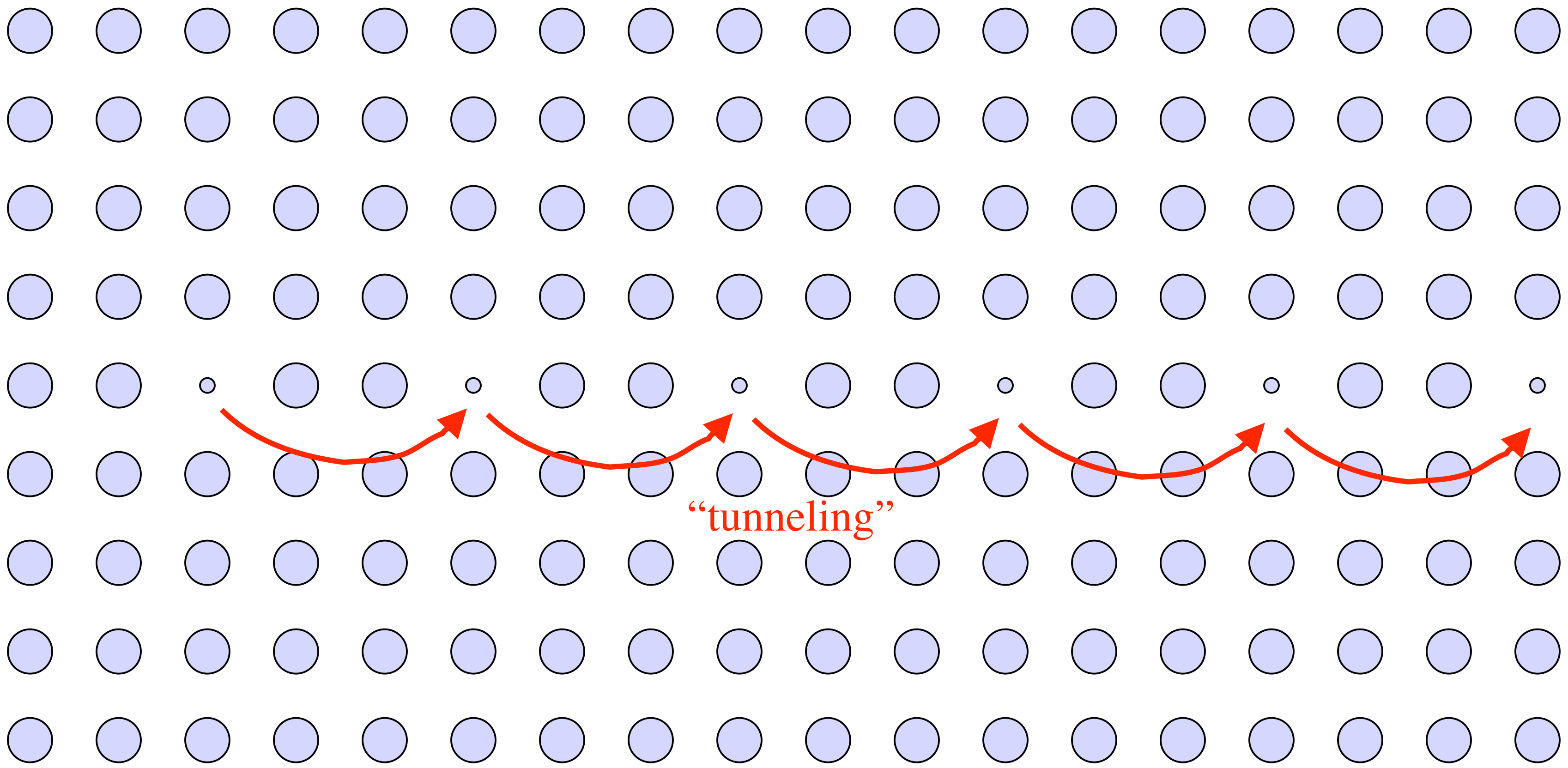
Photonic crystal waveguides:

tight confinement (~ $\lambda/2$ diameter)

+ slow light (e.g. near band edge)

= enhanced nonlinear effects

Cavities + Cavities = Waveguide

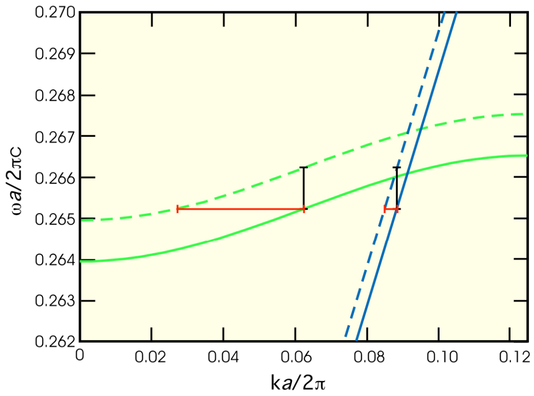


coupled-cavity waveguide (CCW/CROW): slow light + zero dispersion

[A. Yariv *et al*., *Opt. Lett*. **24**, 711 (1999)]

Enhancing tunability with slow light

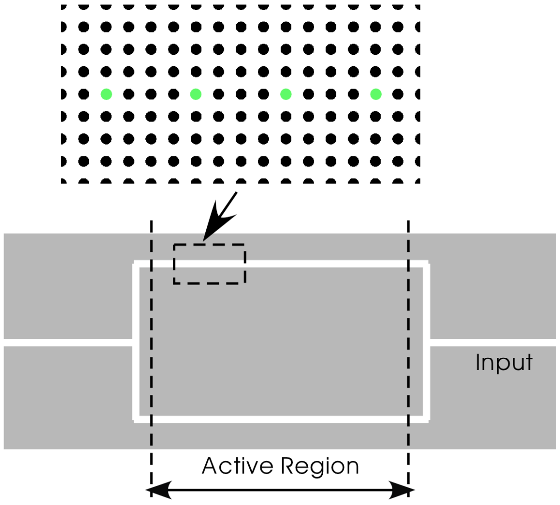
Photonic Crystal Slow-Light Enhancement of Non-linear Phase Sensitivity

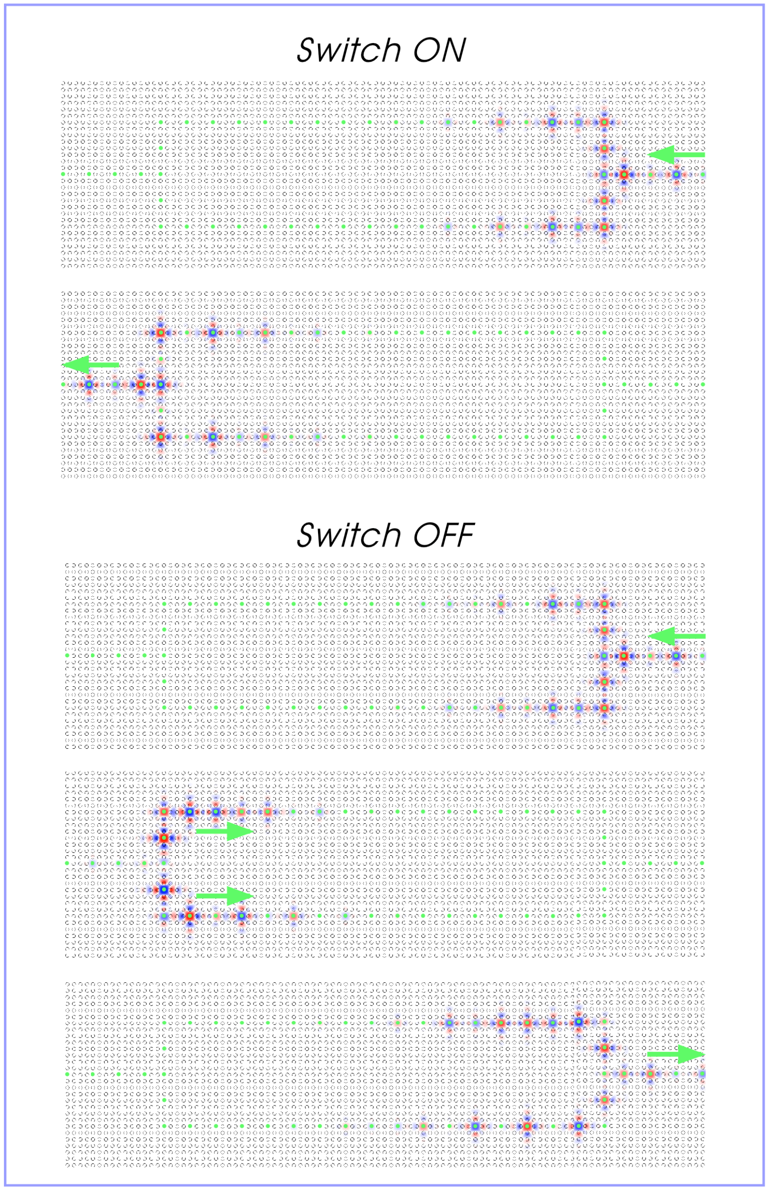


$$\delta\phi \sim \frac{\delta n}{n}\frac{\omega L}{v_g}$$

$$\frac{L}{\lambda_{air}} \sim \frac{n}{\delta n}\frac{v_g}{c}$$

Photonic Crystal Mach Zehnder Switch





[M. Soljacic *et al.*, *J. Opt. Soc. Am. B* **19**, 2052 (2002)]

periodicity:
light is slowed, but not reflected

Slow Light Enhances Everything

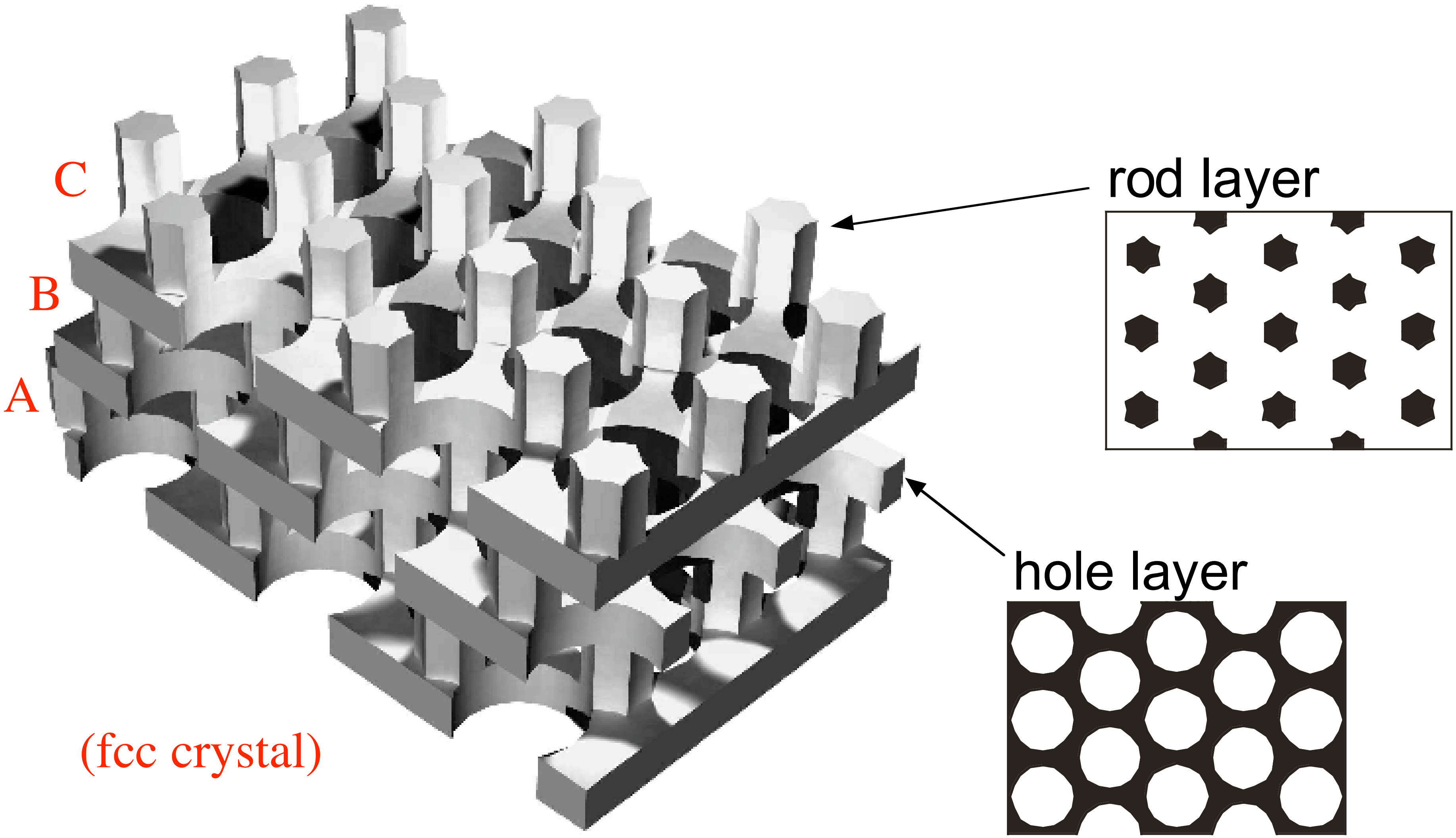
Get a factor of $1/v_g$ (or more) enhancement of:

Nonlinearity, gain (e.g. DBR lasers),
magneto-optic effects, loss…

Whoops!

…but device
length decreases by v_g too

Uh oh, we live in 3d…



C

B

A

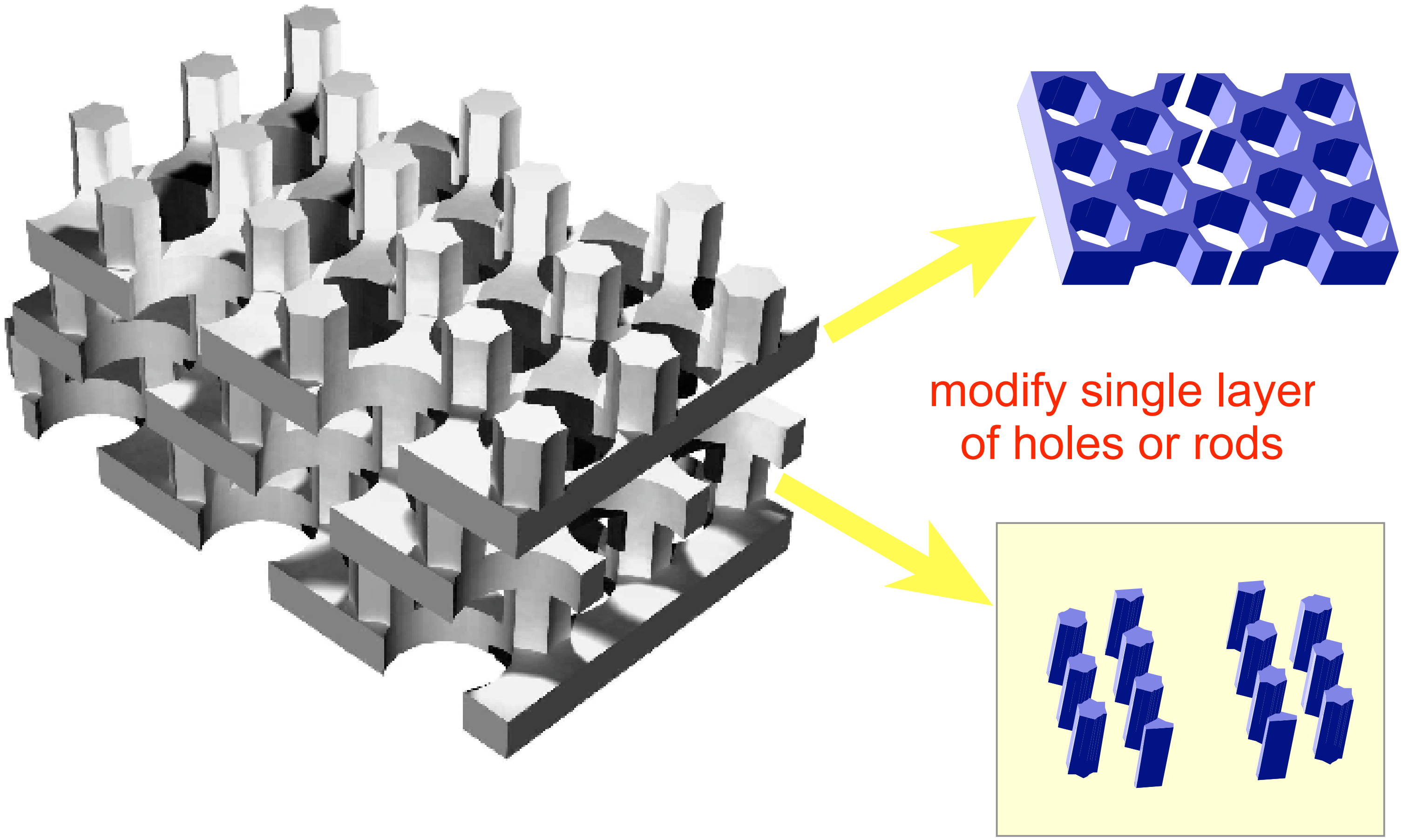
rod layer

hole layer

(fcc crystal)

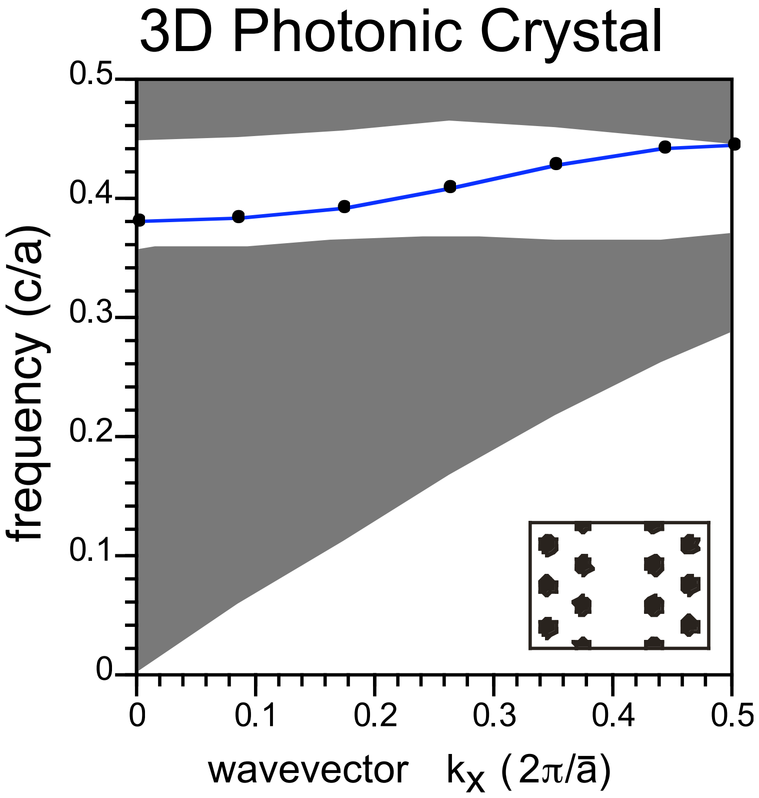
2d-like defects in 3d

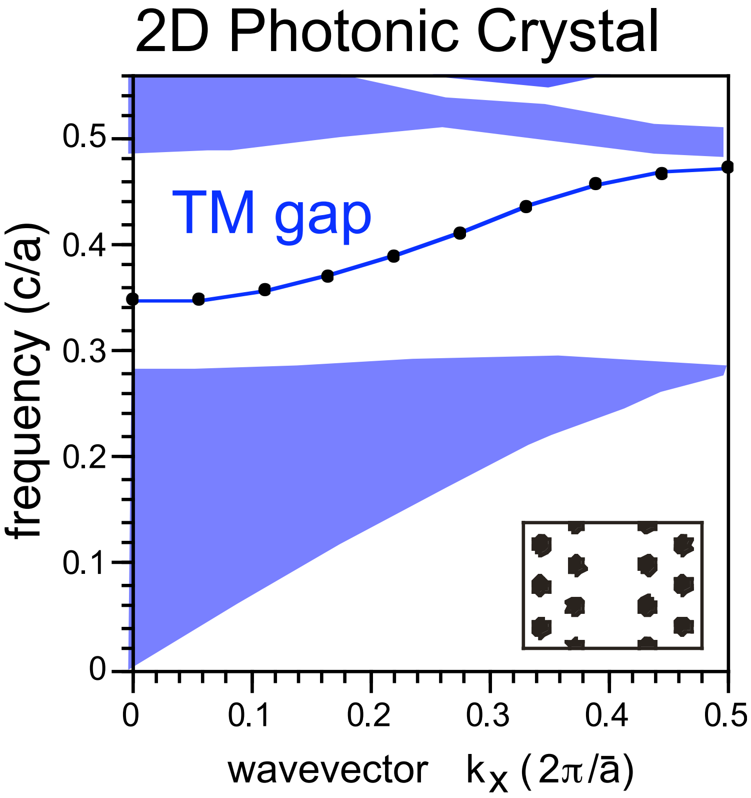
[M. L. Povinelli *et al.*, *Phys. Rev. B* **64**, 075313 (2001)]



modify single layer
of holes or rods

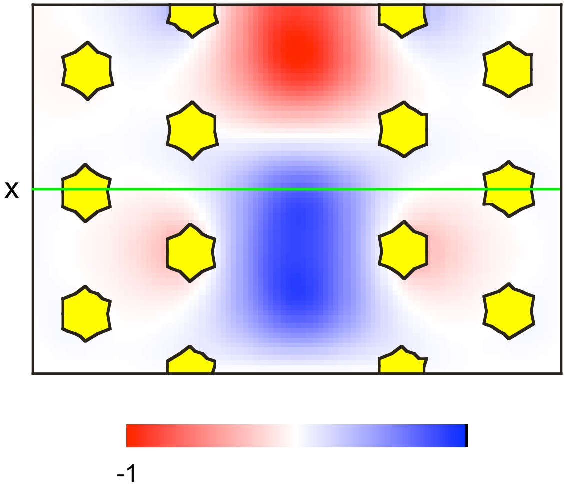
3d projected band diagram

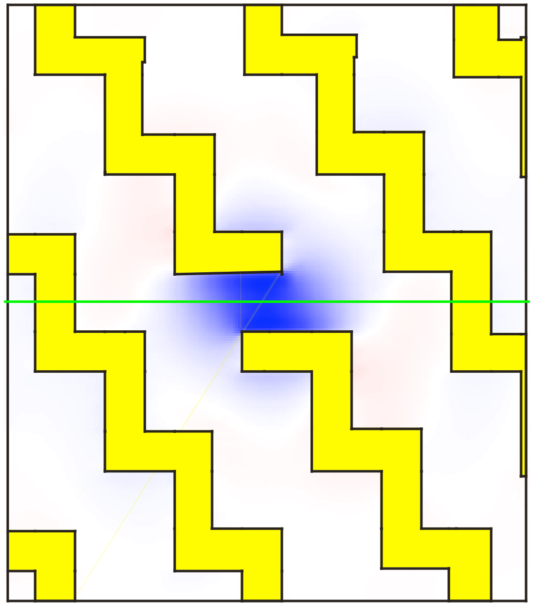

3D Photonic Crystal


2D Photonic Crystal

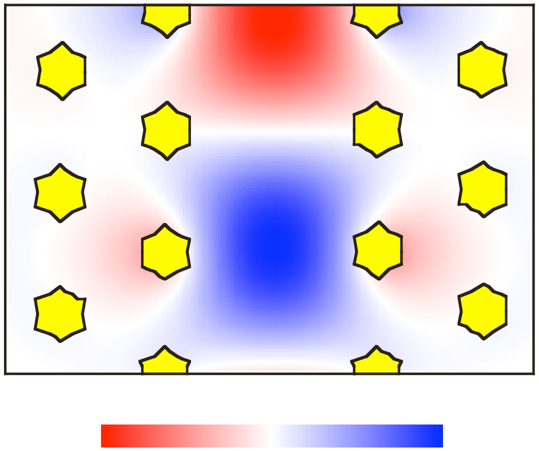
2d-like waveguide mode

3D Photonic Crystal

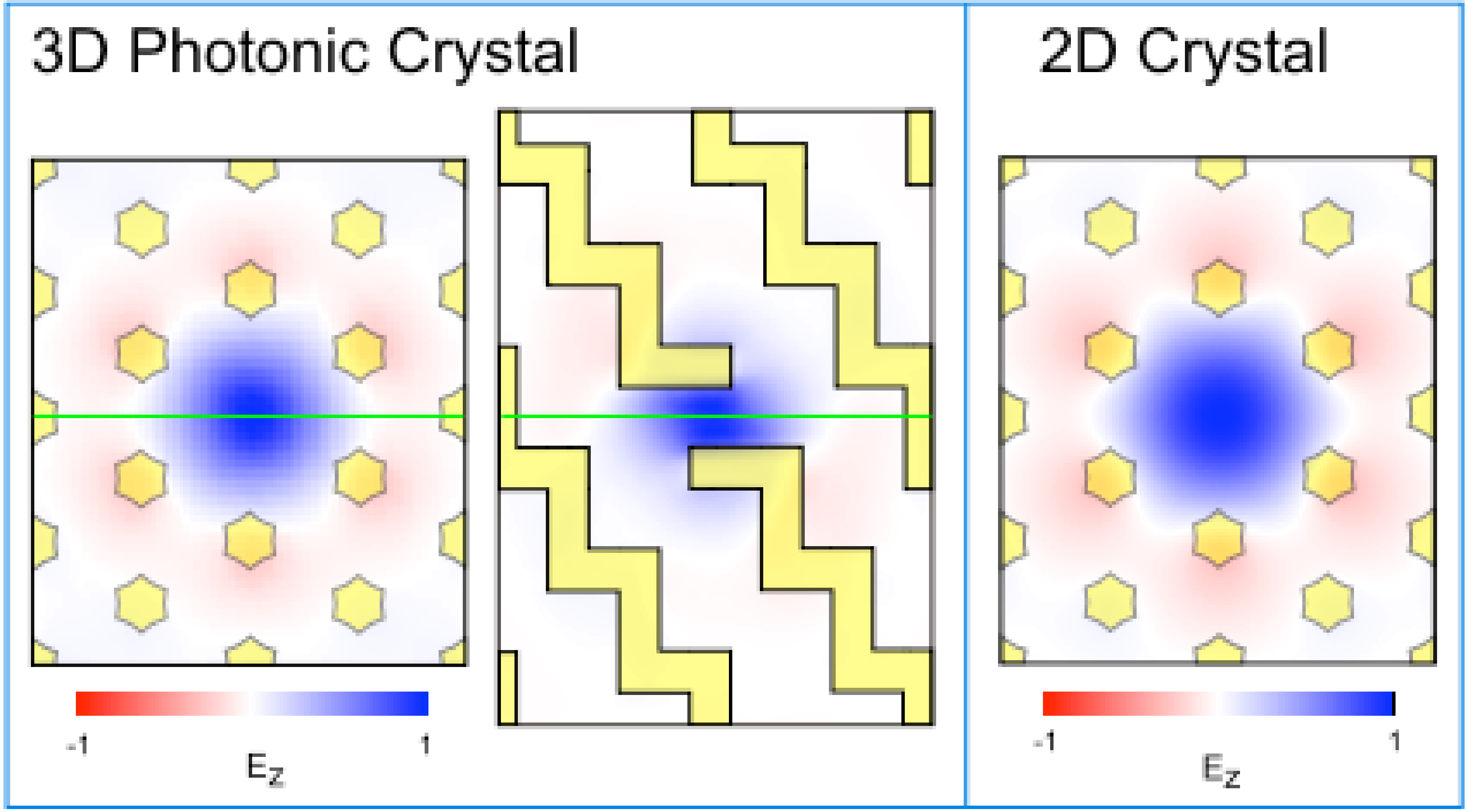




2D Photonic Crystal



2d-like cavity mode



The Upshot

To design an interesting device, you need only:

symmetry

+ single-mode (usually)

+ resonance

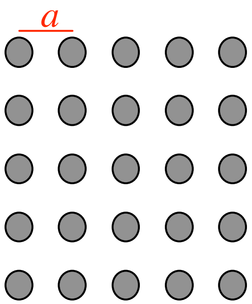
+ (ideally) a band gap to forbid losses

Oh, and a full Maxwell simulator to get Q parameters, *etcetera*.

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Perturbations, tuning, and disorder

Review: Bloch Basics



Waves in periodic media can have:

- propagation with no scattering (conserved **k**)
- photonic band gaps (with proper ε function)

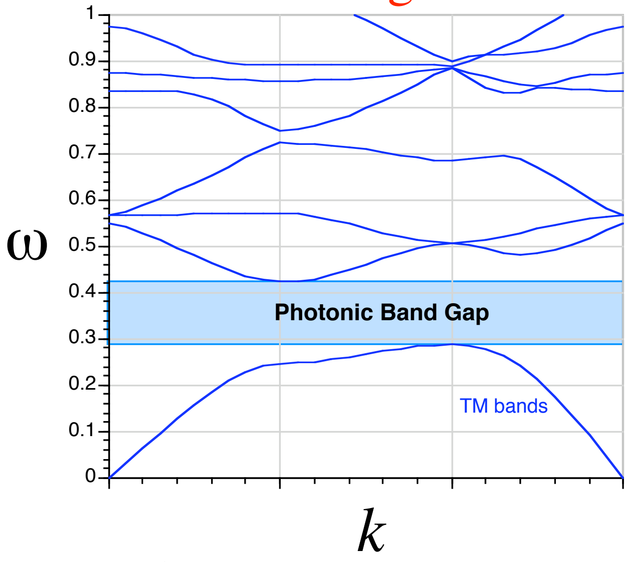
Eigenproblem gives simple insight:

Bloch form: $$\vec{H} = e^{i(\vec{k}\cdot\vec{x}-\omega t)}\vec{H}_{\vec{k}}(\vec{x})$$

$$\underbrace{\left[(\vec{\nabla}+i\vec{k})\times\frac{1}{\varepsilon}(\vec{\nabla}+i\vec{k})\times\right]}_{\hat{\Theta}_{\vec{k}}}\vec{H}_{\vec{k}} = \left(\frac{\omega_n(\vec{k})}{c}\right)^2\vec{H}_{\vec{k}}$$

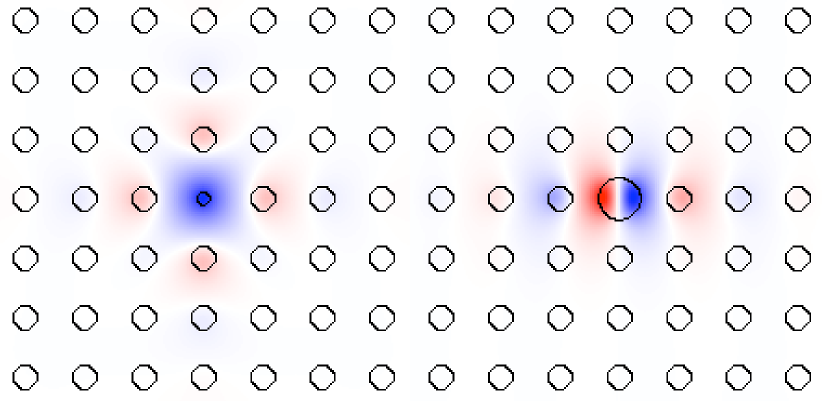
Hermitian –> complete, orthogonal, variational theorem, *etc*.

band diagram

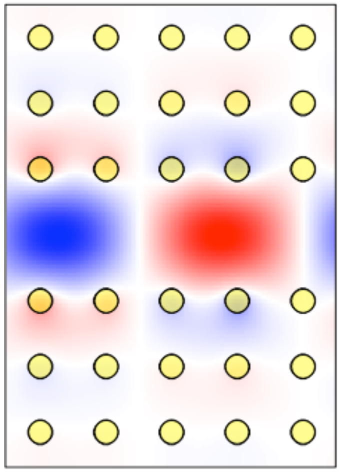


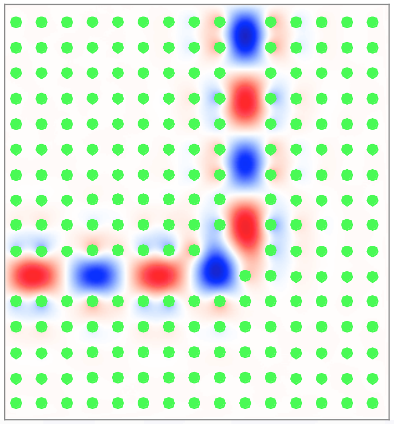
Review: Defects and Devices

Point defects = Cavities



Line defects = Waveguides



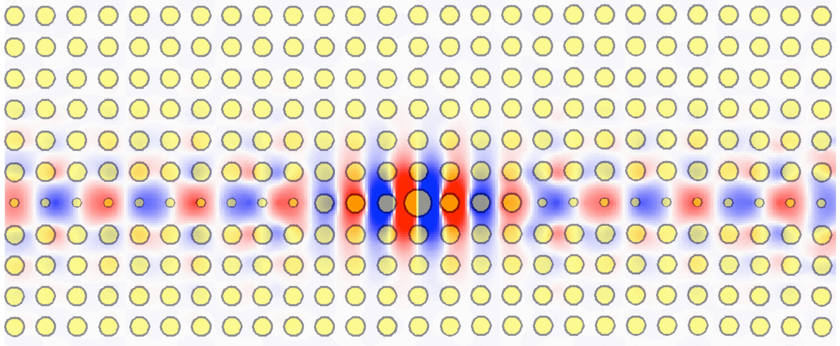


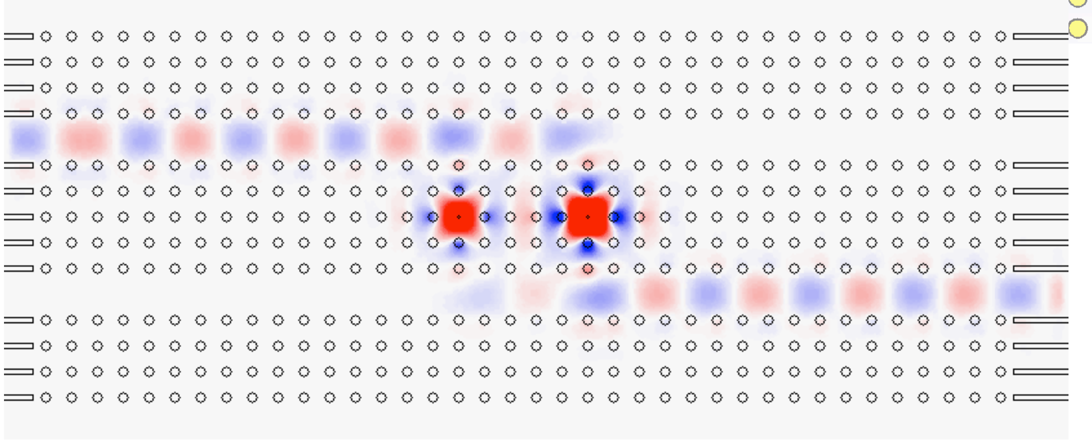
Waveguides

+

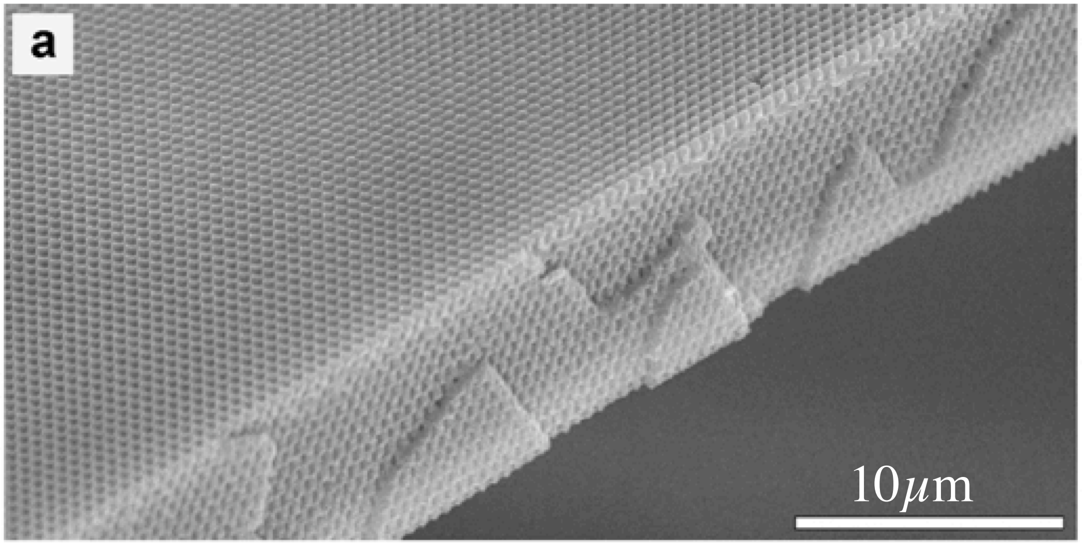
Resonant

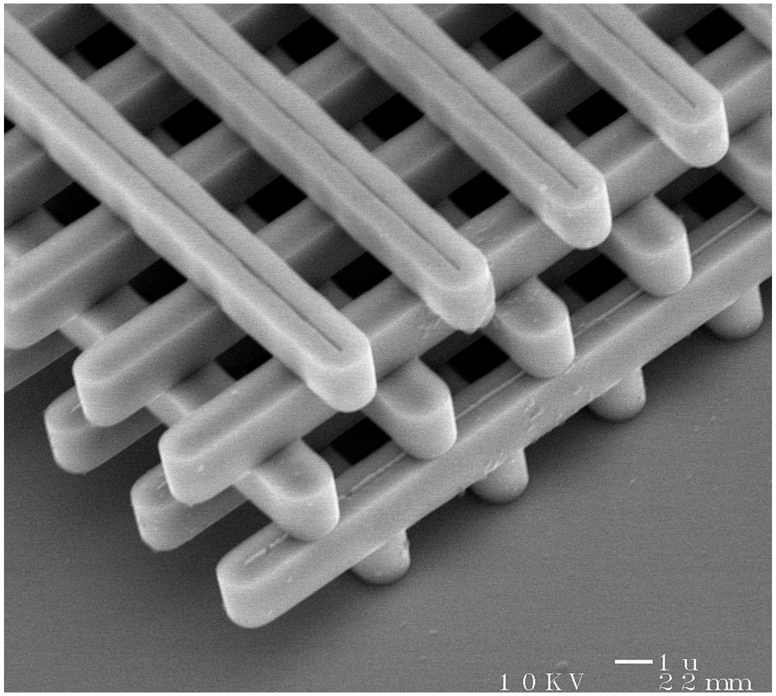
Cavities

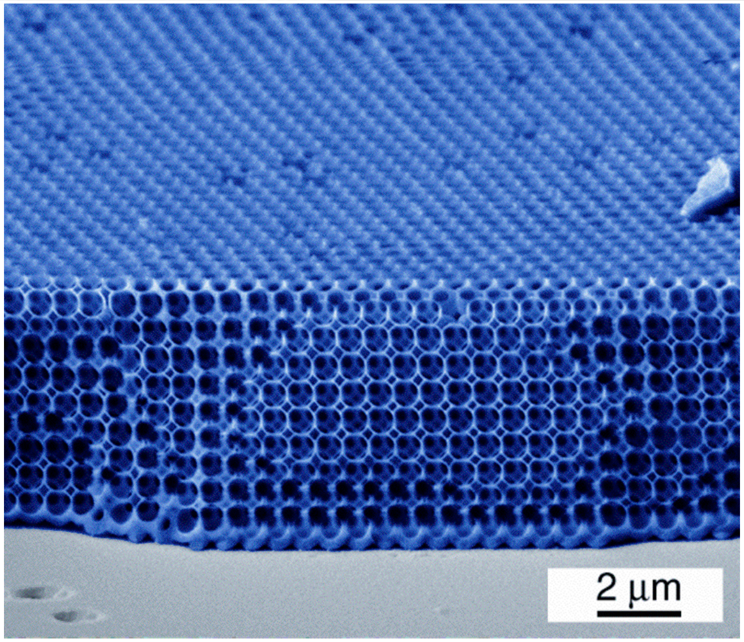




Review: 3d Crystals and Fabrication







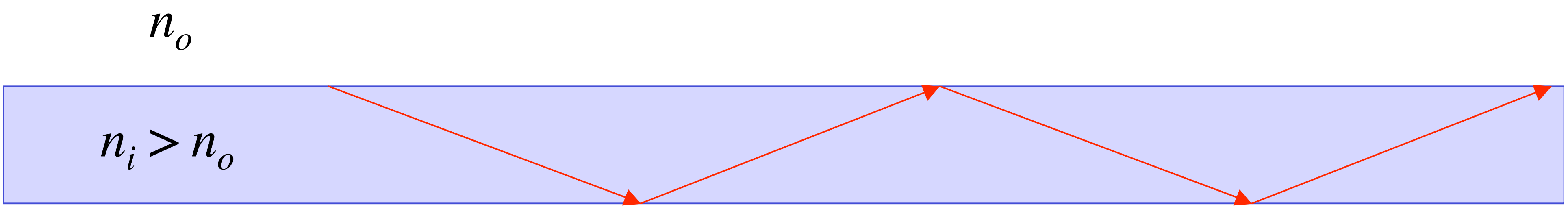
Much progress
in making complex structures

…

incorporation of defects & devices
still in early stages

How *else* can we confine light?

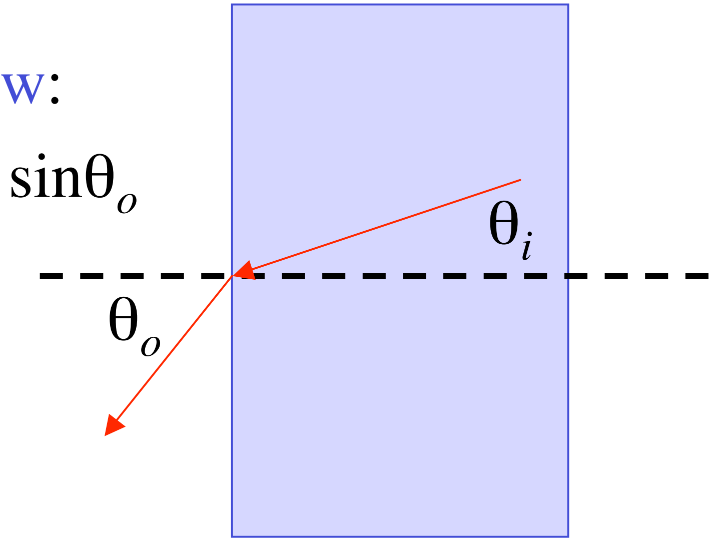
Total Internal Reflection



rays at shallow angles $> \theta_c$
are totally reflected

Snell's Law:

$$n_i \sin\theta_i = n_o \sin\theta_o$$

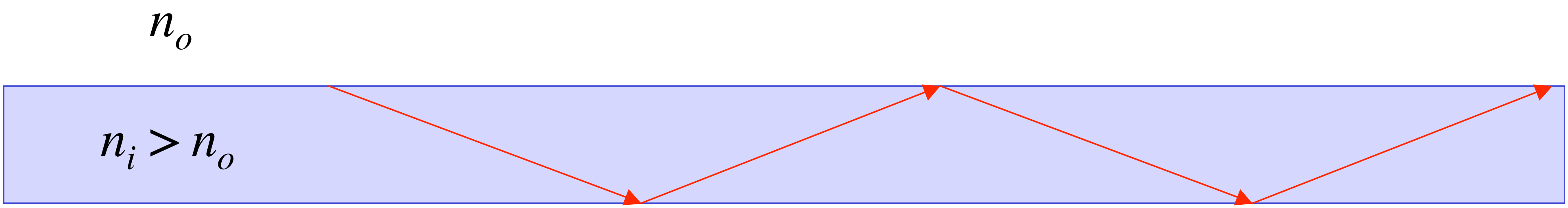


$$\sin\theta_c = n_o / n_i$$

< 1, so θ_c is real

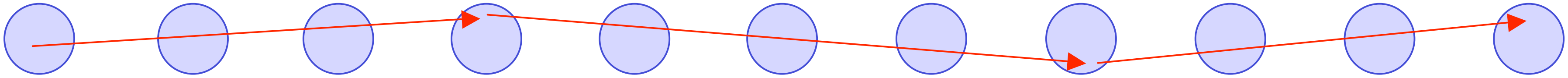
i.e. TIR can only guide
within higher index
unlike a band gap

Total Internal Reflection?



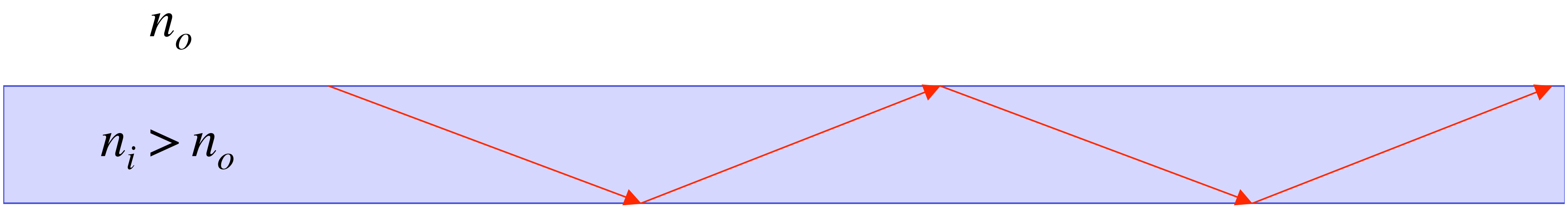
rays at shallow angles > θ_c
are totally reflected

So, for example,
a discontiguous structure can’t possibly guide by TIR…

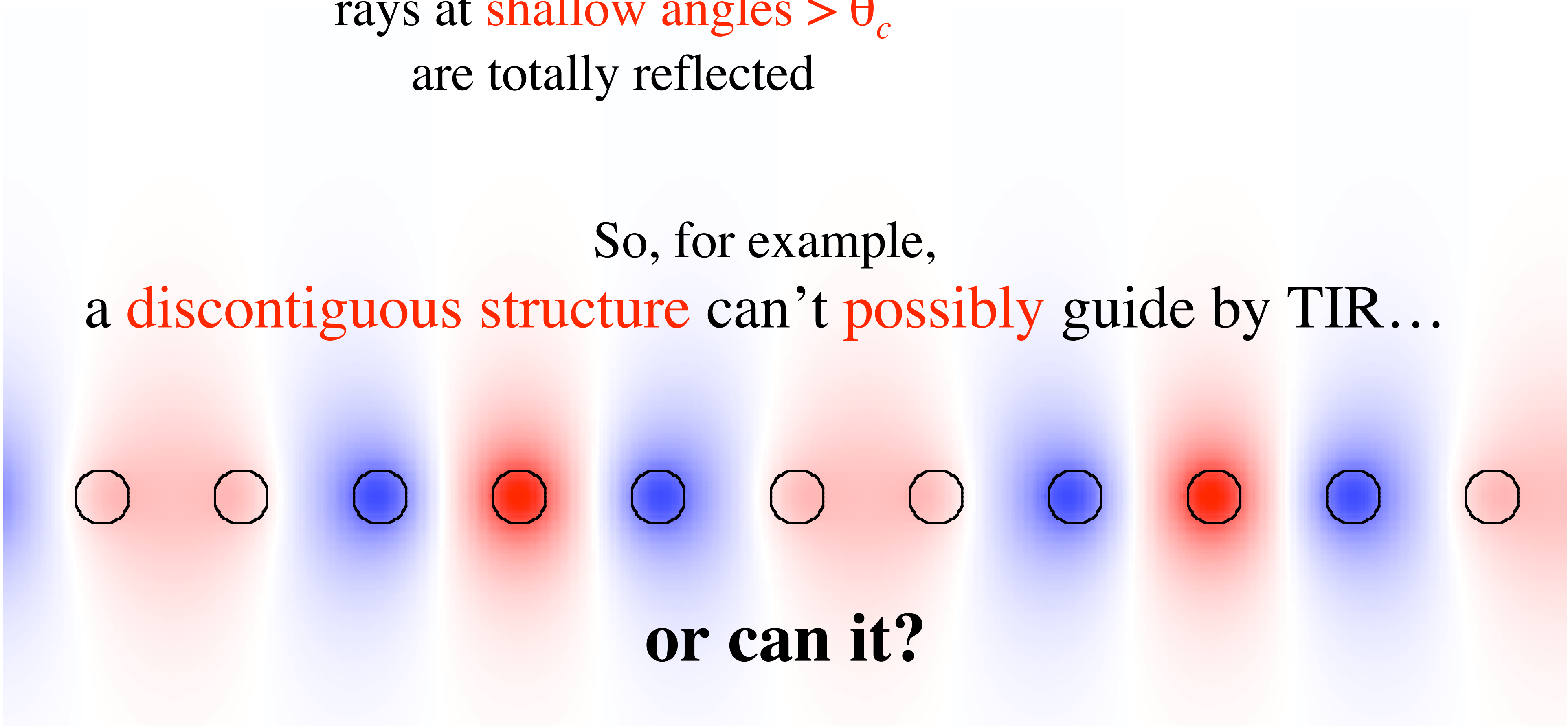


the rays can’t stay inside!

Total Internal Reflection?



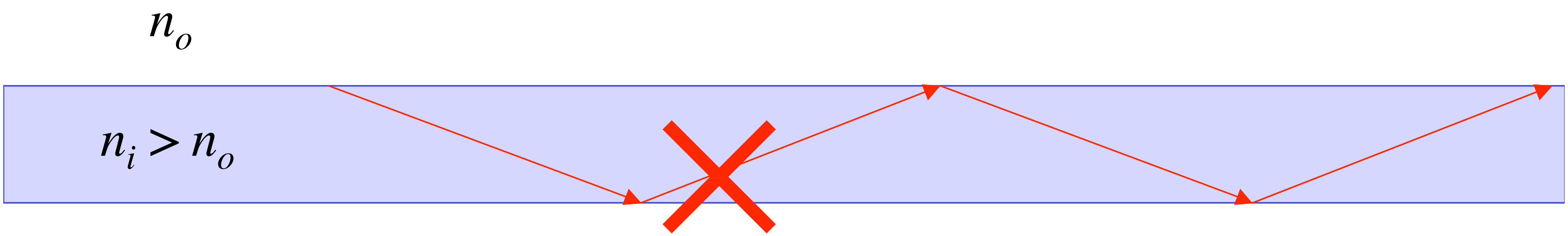
rays at shallow angles $> \theta_c$
are totally reflected



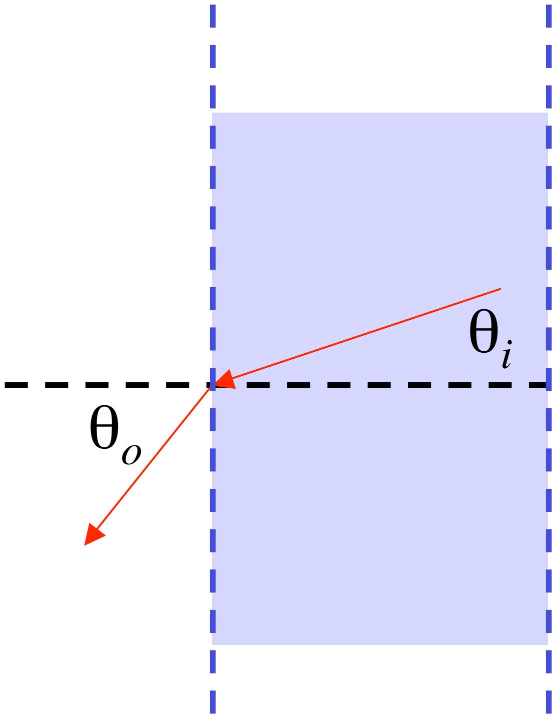
So, for example,
a discontiguous structure can’t possibly guide by TIR…

or can it?

Total Internal Reflection Redux



ray-optics picture is invalid on λ scale
(neglects coherence, near field…)



translational symmetry

$k_{\parallel}$

↓

conserved!

Snell's Law is really
conservation of $k_{\parallel}$ and ω:

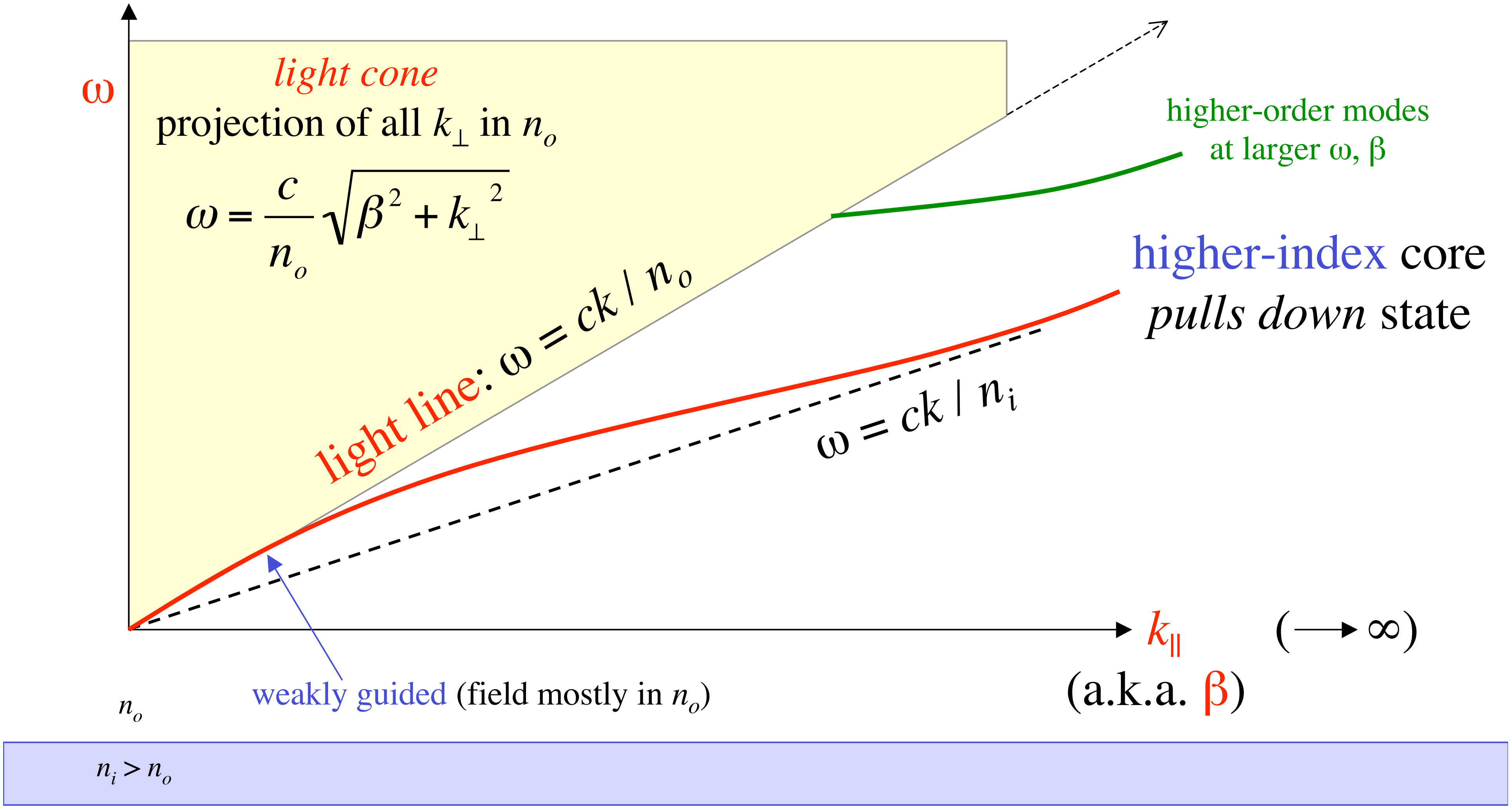
$$|k_i| \sin\theta_i = |k_o| \sin\theta_o$$

$$|k| = n\omega/c$$

(wavevector) (frequency)

Waveguide Dispersion Relations

i.e. projected band diagrams



ω

light cone

projection of all $k_\perp$ in n_o

$$\omega = \frac{c}{n_o}\sqrt{\beta^2 + k_\perp{}^2}$$

light line: $\omega = ck / n_o$

$\omega = ck / n_i$

higher-order modes
at larger ω, β

higher-index core
pulls down state

$k_{||}$ $(\longrightarrow \infty)$

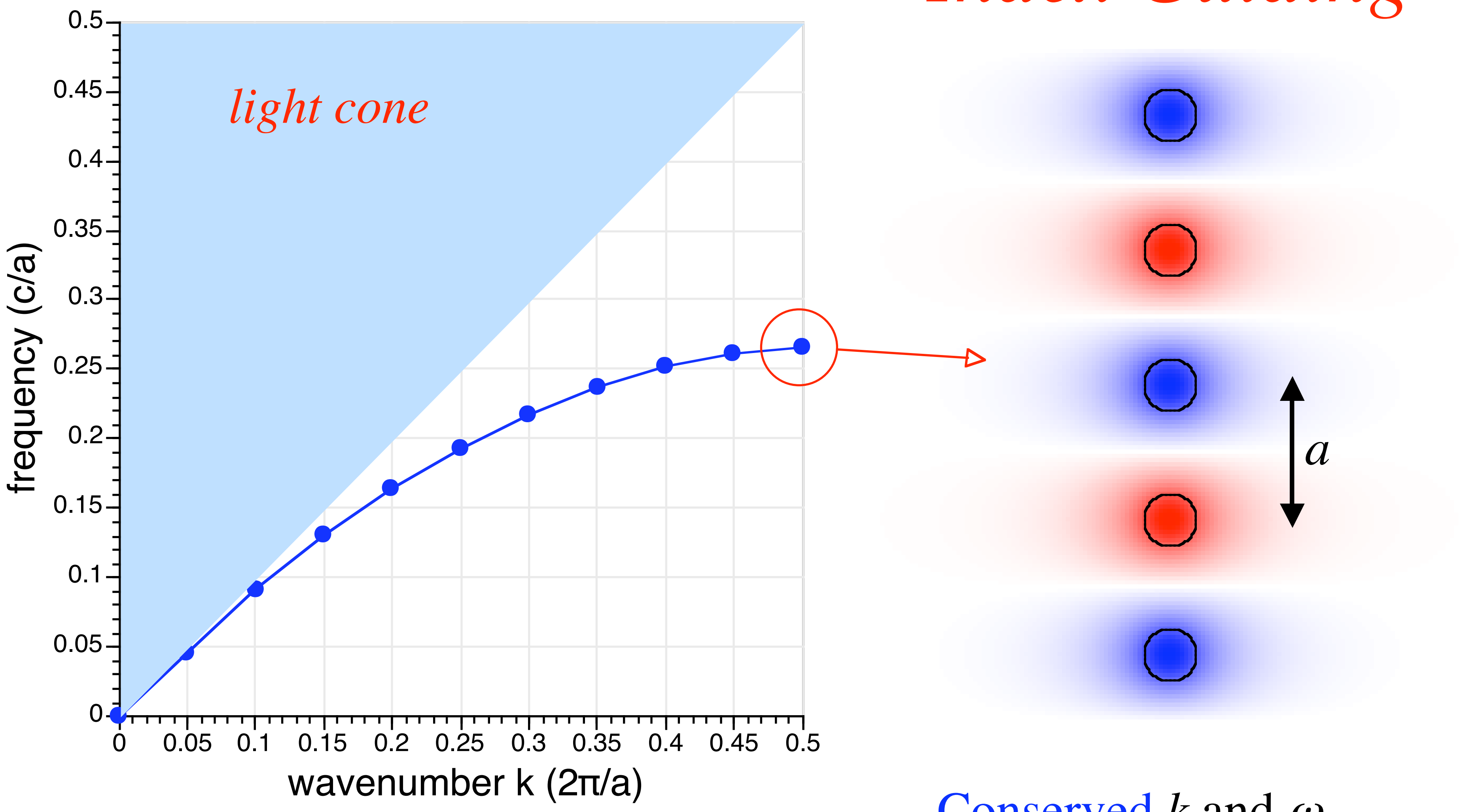
(a.k.a. β)

weakly guided (field mostly in n_o)

n_o

$n_i > n_o$

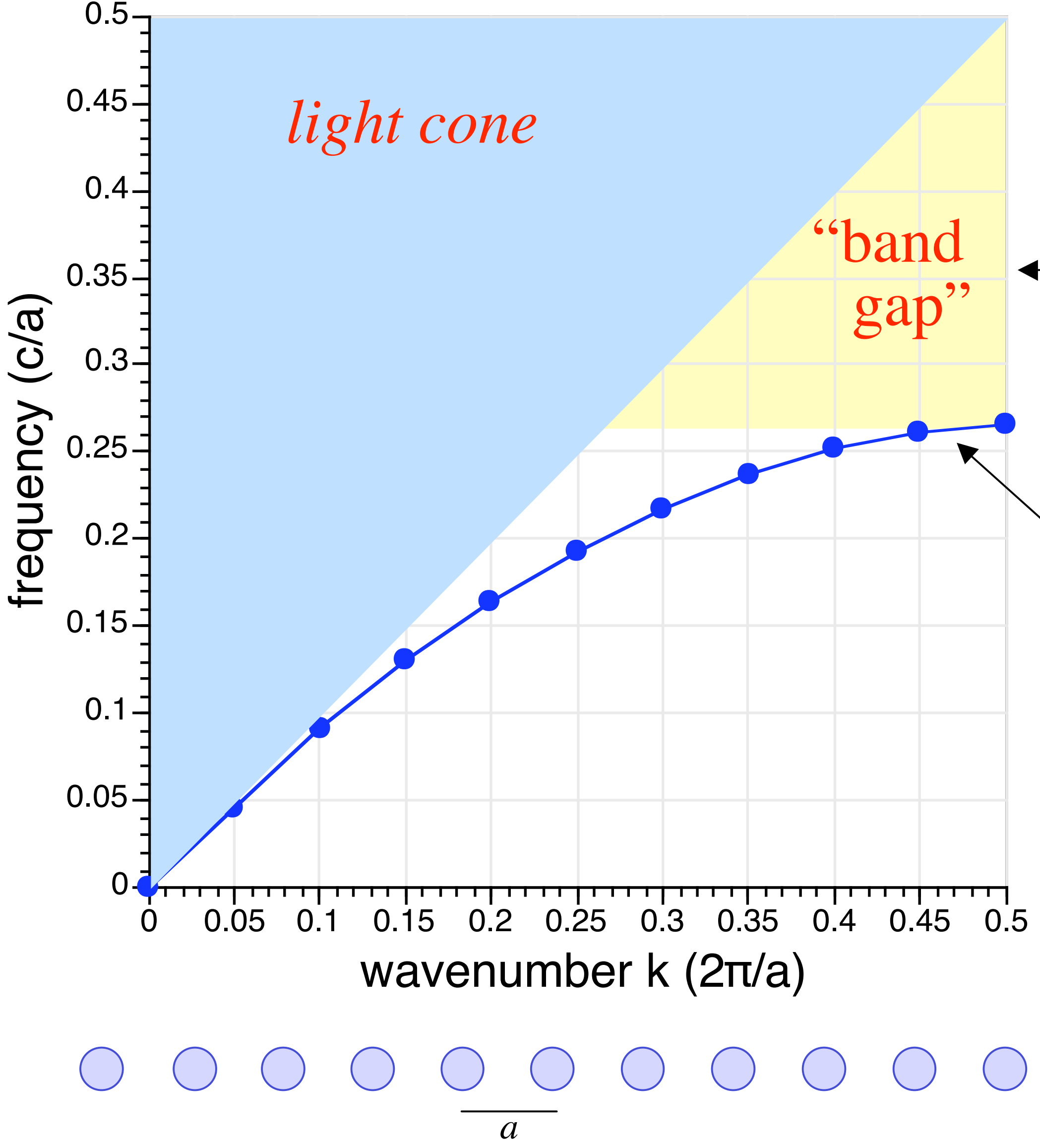
Strange ~~Total Internal Reflection~~ *Index Guiding*



Conserved k and ω
+ higher index to pull down state
= localized/guided mode.

A Hybrid Photonic Crystal:

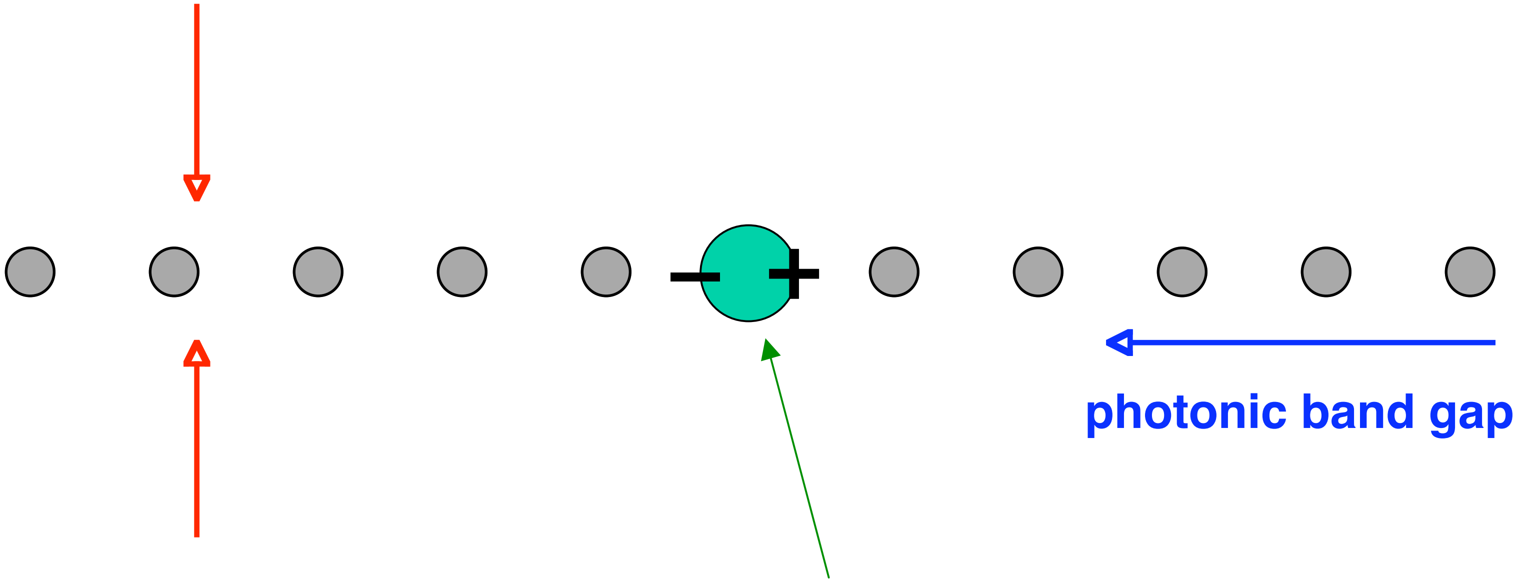
1d band gap + index guiding



range of frequencies in which there are no guided modes

slow-light band edge

A Resonant Cavity

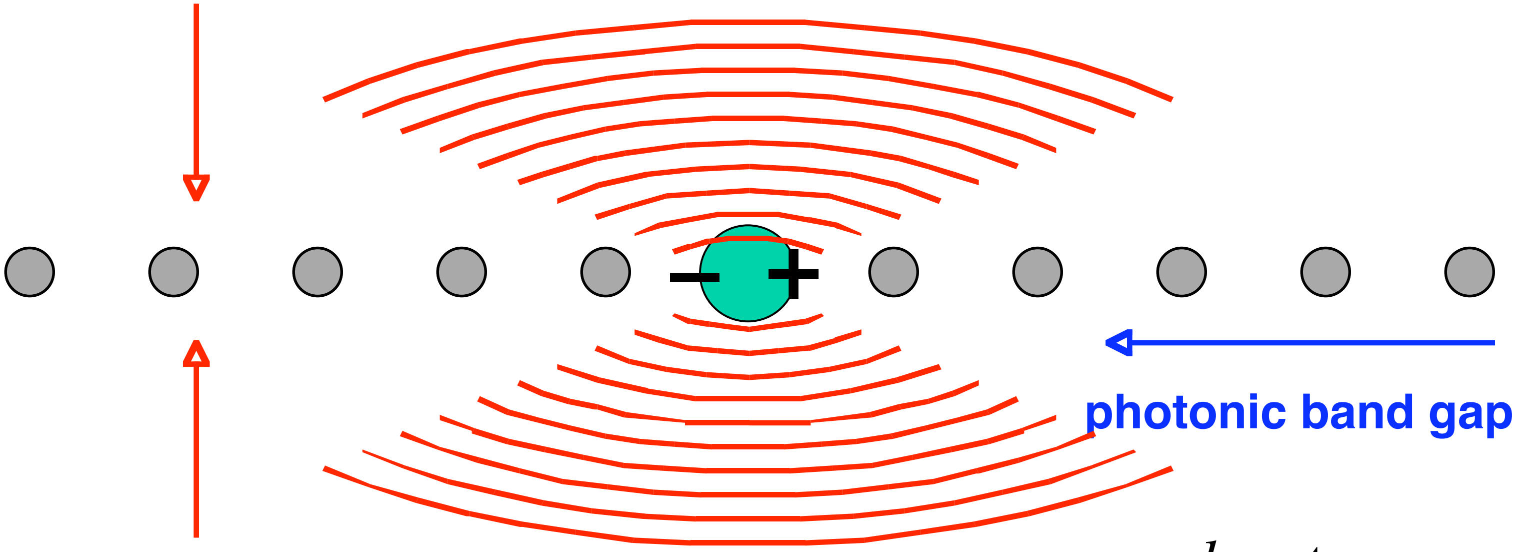


index-confined

photonic band gap

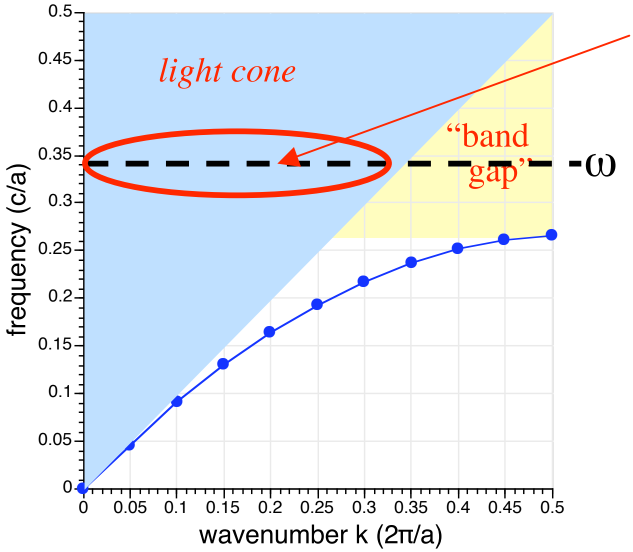
increased rod radius
pulls down “dipole” mode
(non-degenerate)

A Resonant Cavity



index-confined

photonic band gap



k not conserved

so coupling to light cone:

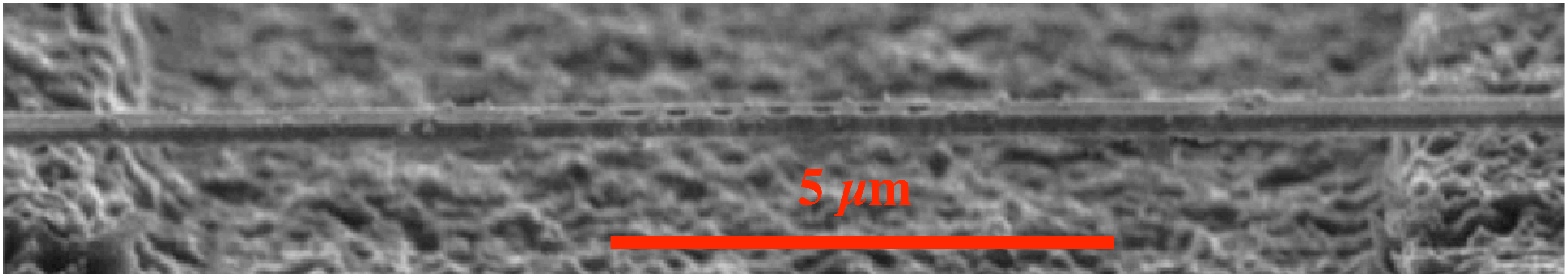
radiation

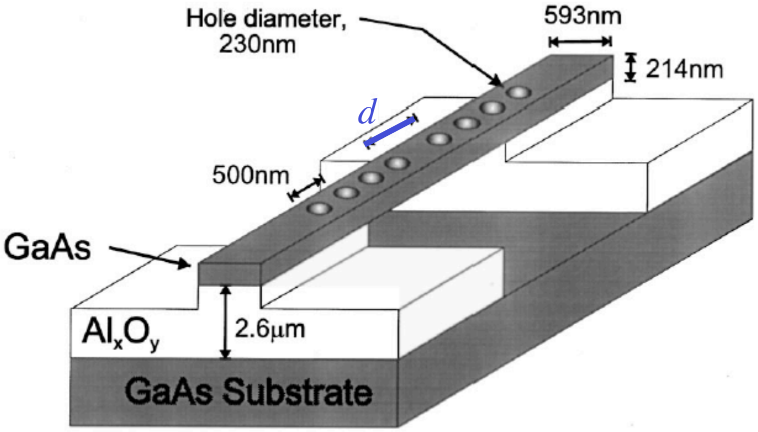
The trick is to keep the radiation small…

(more on this later)

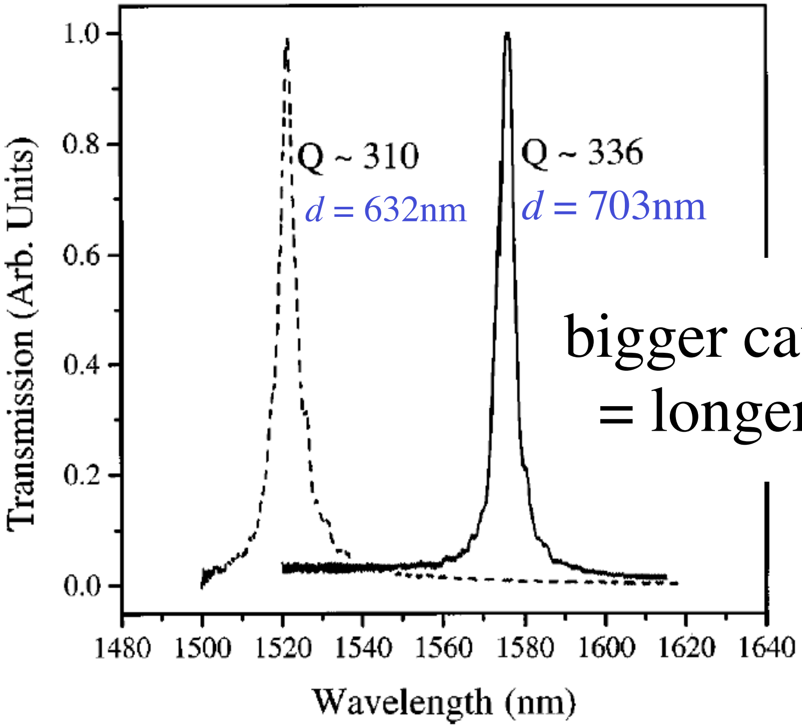
Meanwhile, back in reality…

Air-bridge Resonator: 1d gap + 2d index guiding





[D. J. Ripin *et al.*, *J. Appl. Phys.* **87**, 1578 (2000)]

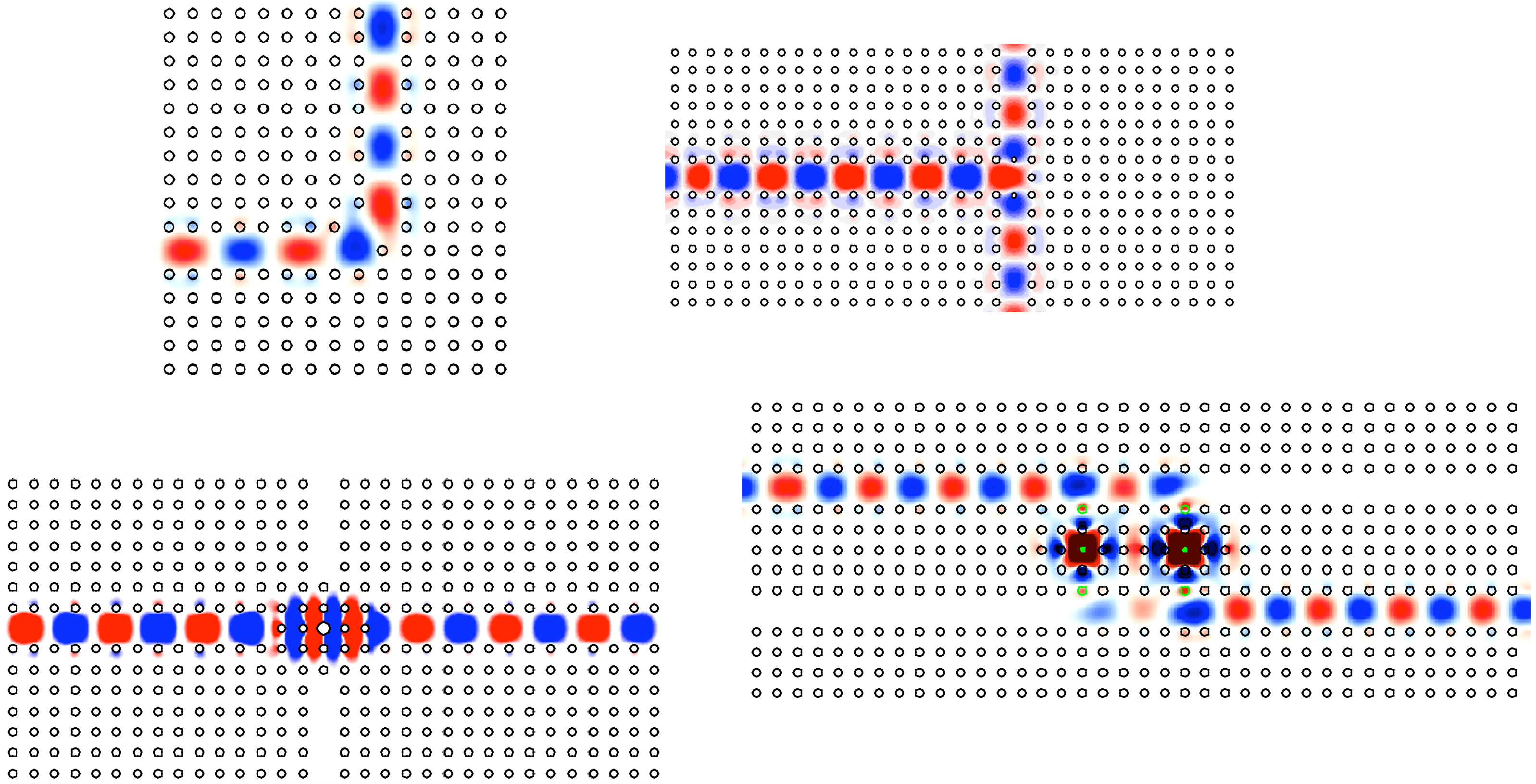


bigger cavity
= longer λ

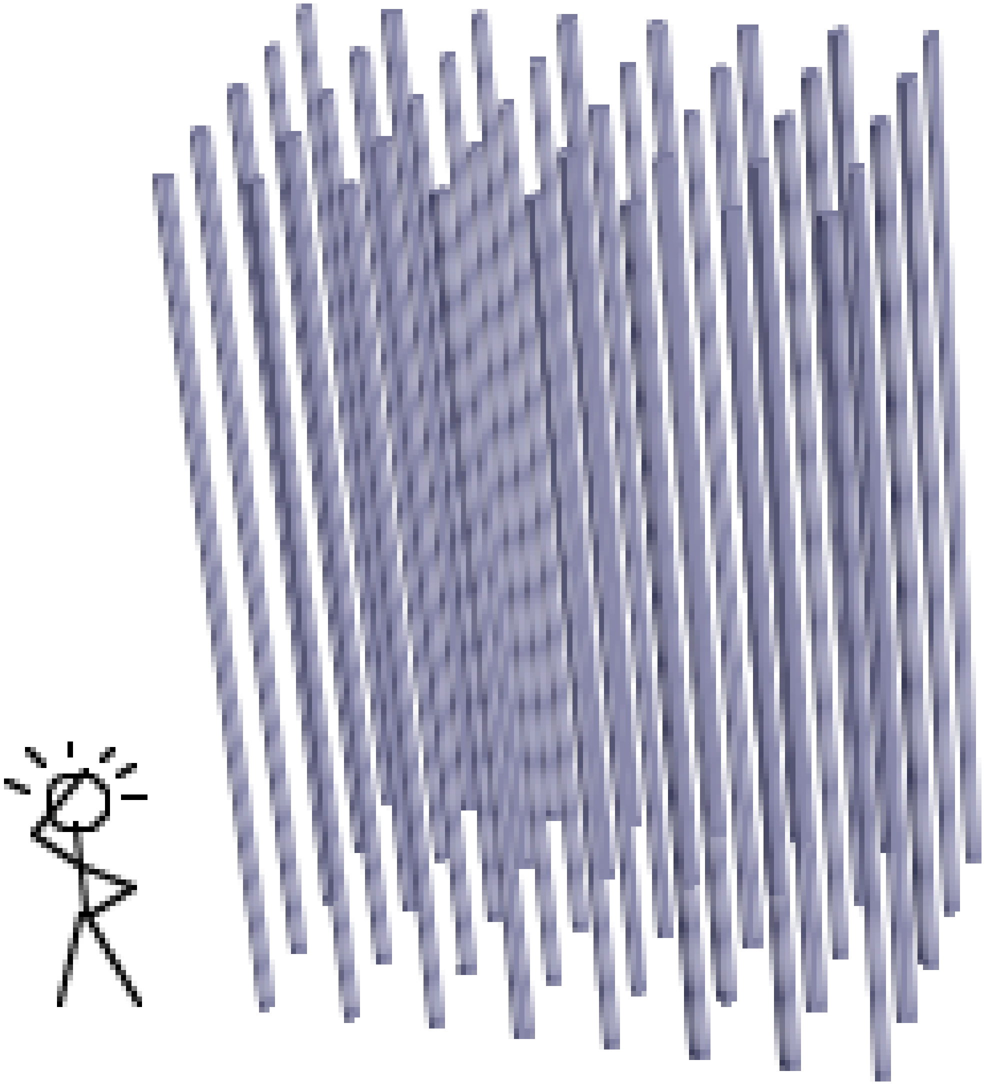
Time for Two Dimensions…

2d is all we really need for many interesting devices

…darn *z* direction!



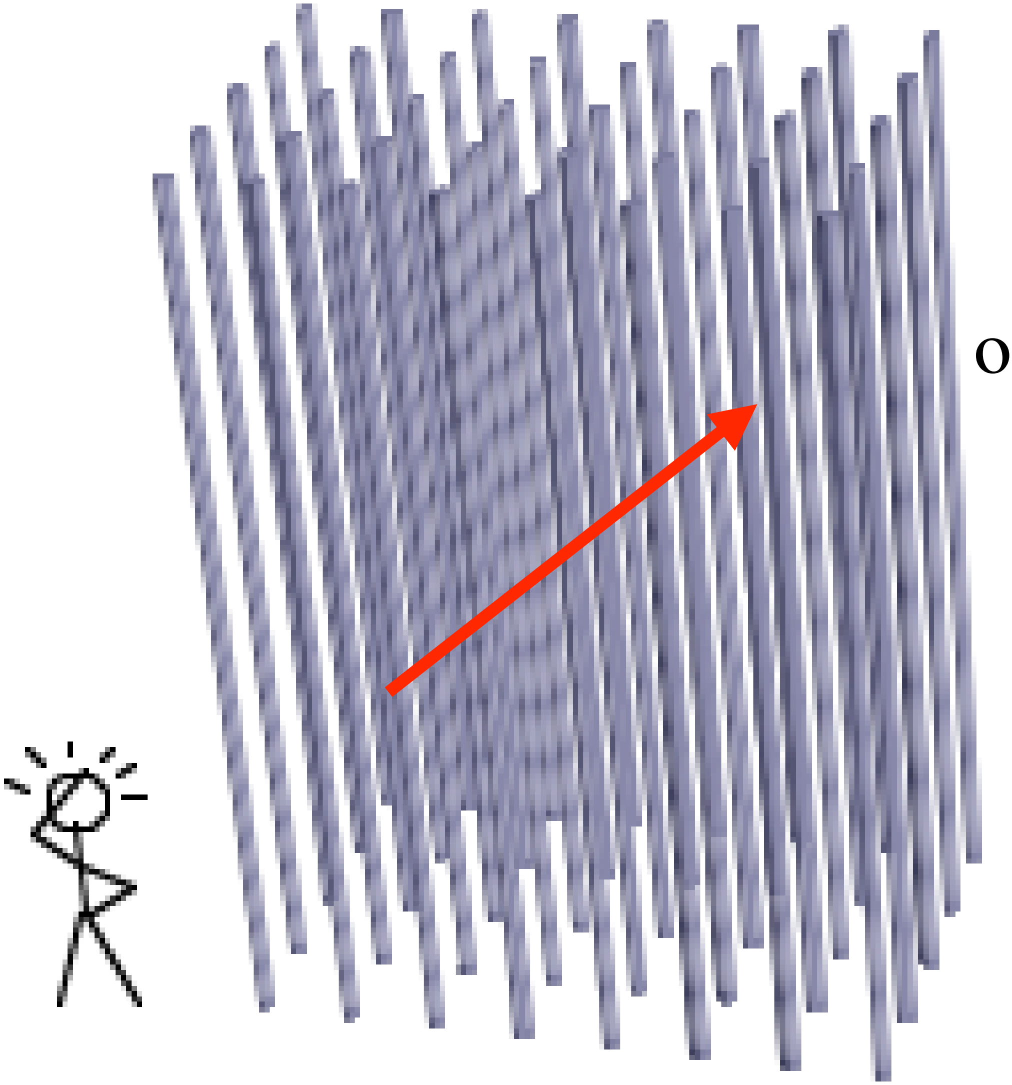
How do we make a 2d bandgap?



Most obvious solution?

make
2d pattern
really tall

How do we make a 2d bandgap?



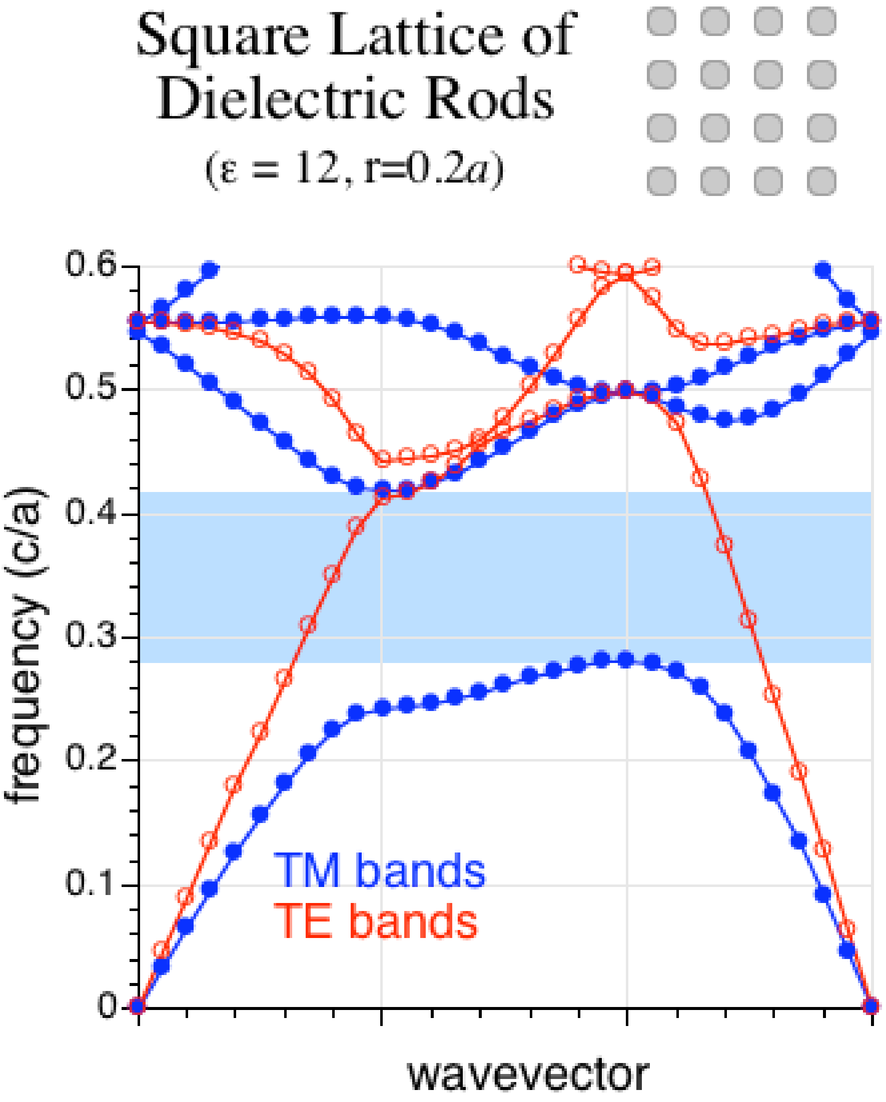
If height is finite,
we must couple to
out-of-plane wavevectors…

↑ k_z not conserved

A 2d band diagram in 3d

Let's start with the 2d band diagram.

This is what we'd like to have in 3d, too!



Square Lattice of Dielectric Rods

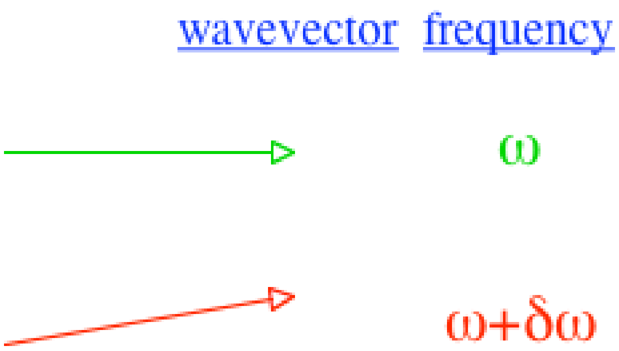
(ε = 12, r=0.2a)

A 2d band diagram in 3d

Let's start with the 2d band diagram.

This is what we'd like to have in 3d, too!

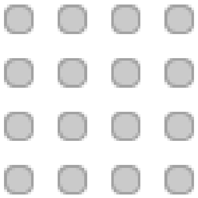
No! When we include **out-of-plane** propagation, we get:

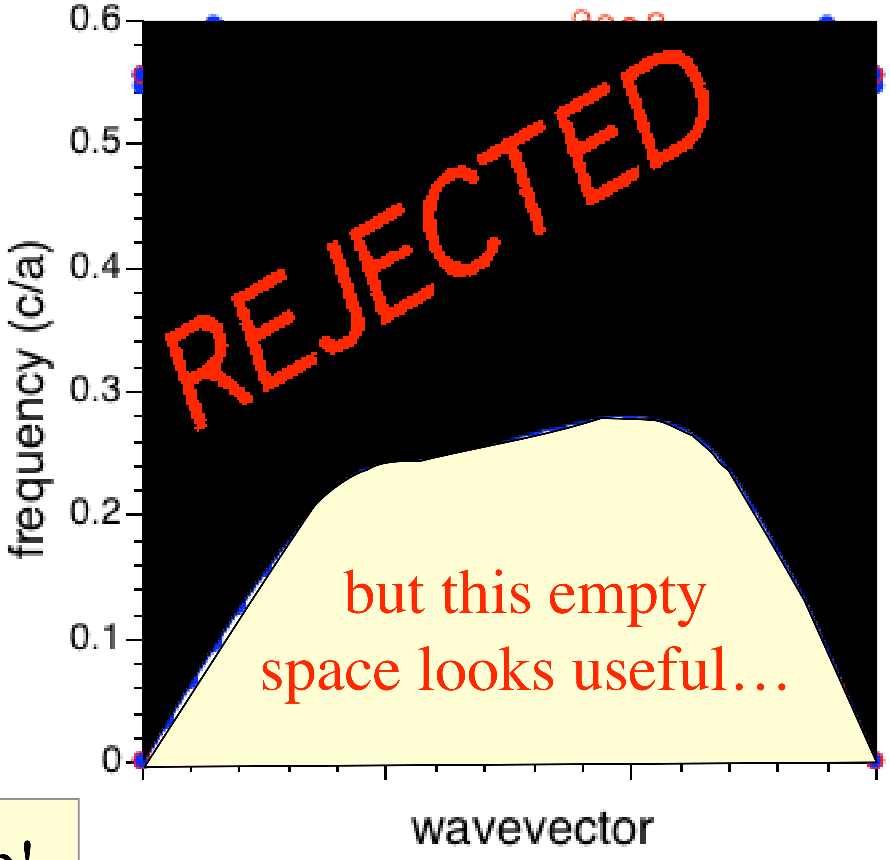


projected band diagram fills gap!

Square Lattice of Dielectric Rods

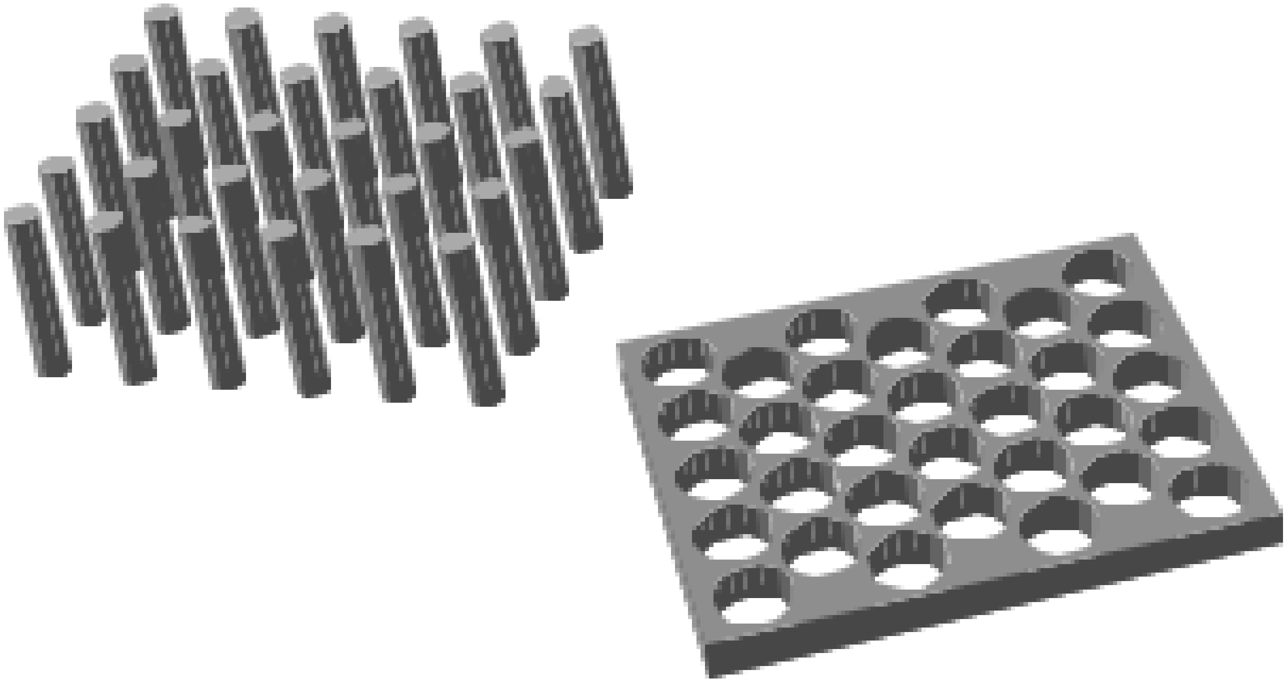
(ε = 12, r=0.2*a*)





but this empty space looks useful…

Photonic-Crystal Slabs

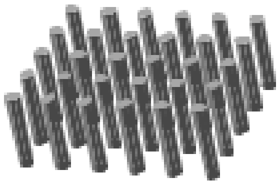


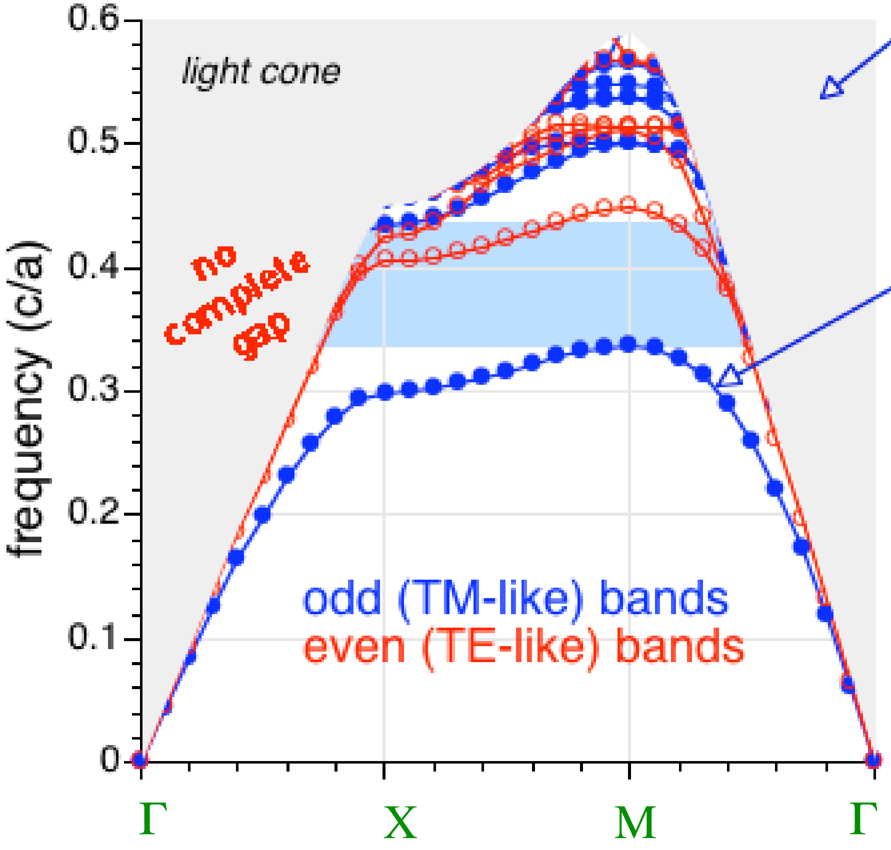
2d photonic bandgap + vertical index guiding

[S. G. Johnson and J. D. Joannopoulos, *Photonic Crystals: The Road from Theory to Practice*]

Rod-Slab Projected Band Diagram

Square Lattice of
Dielectric Rods
(ε = 12, r=0.2a, h=2a)



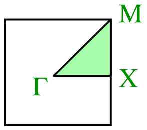


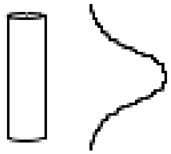
The Light Cone:
All possible states
propagating in the air

The Guided Modes:
Cannot couple to
the light cone…
—> confined to the slab

Thickness is critical.

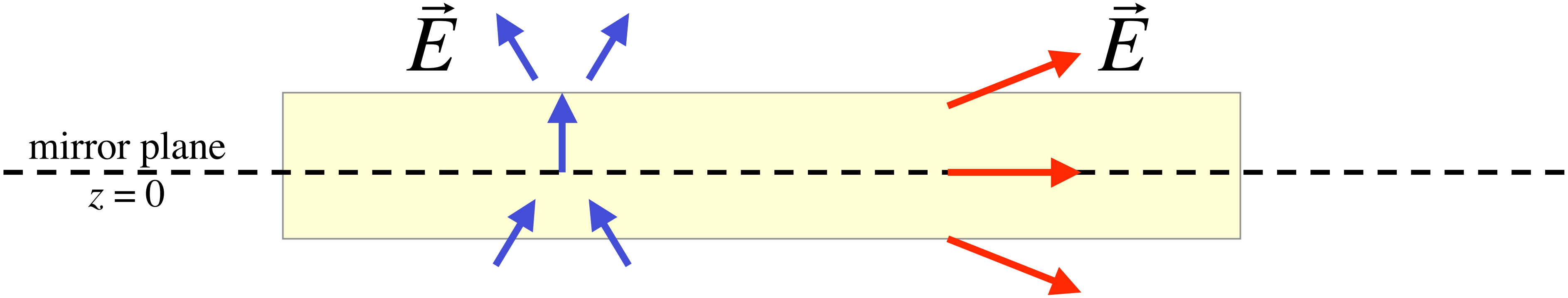
Should be about λ/2
(to have a gap
& be single-mode)





Symmetry in a Slab

2d: TM and TE modes

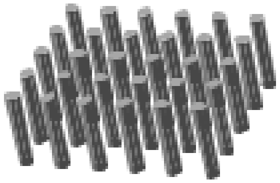


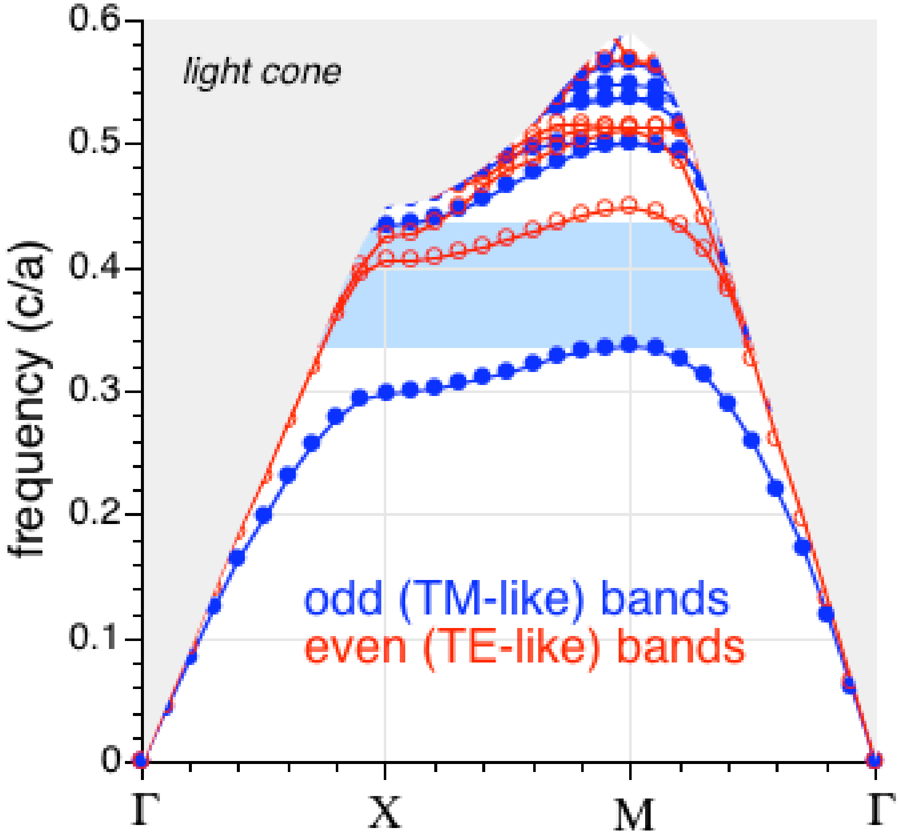
slab: odd (TM-like) and even (TE-like) modes

Like in 2d, there may only be a band gap in one symmetry/polarization

Slab Gaps

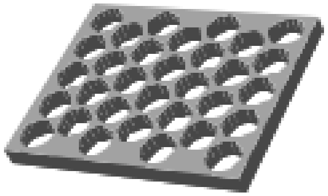
Square Lattice of Dielectric Rods
(ε = 12, r=0.2a, h=2a)

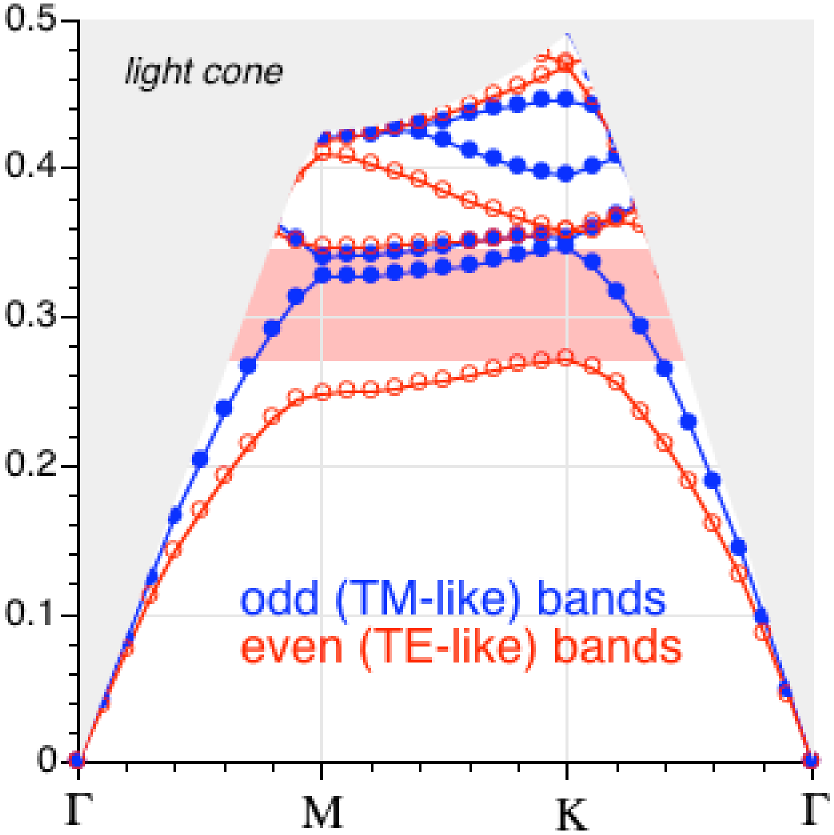




TM-like gap

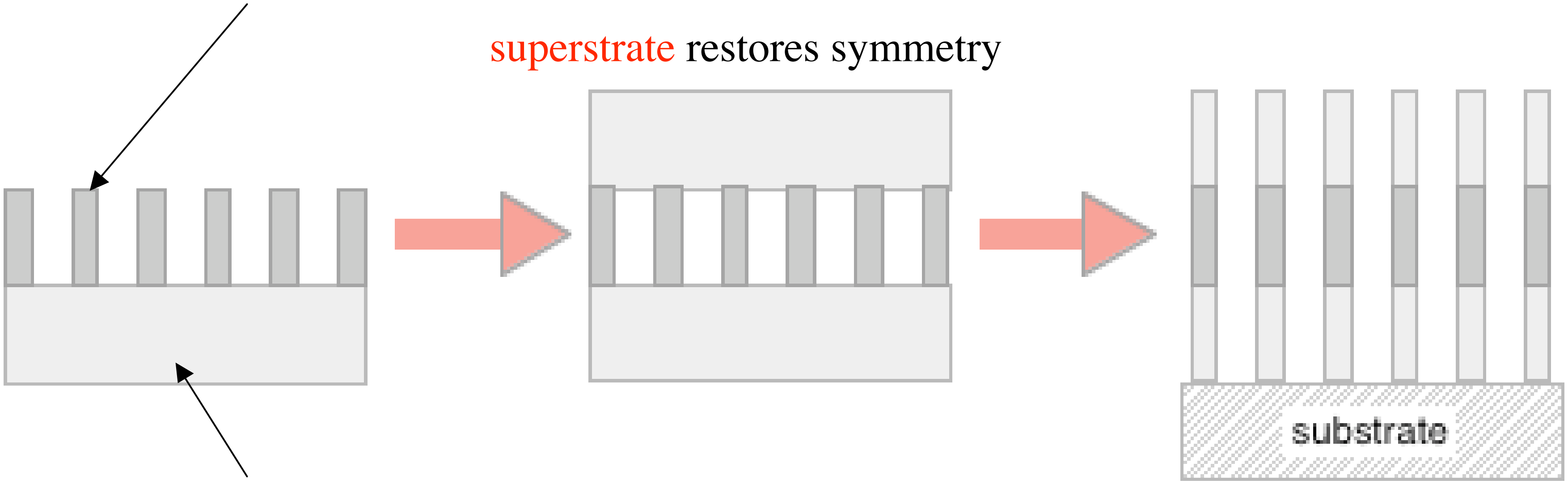
Triangular Lattice of Air Holes
(ε = 12, r=0.3a, h=0.5a)





TE-like gap

Substrates, for the Gravity-Impaired



(rods or holes)

superstrate restores symmetry

substrate breaks symmetry:
some even/odd mixing "kills" gap

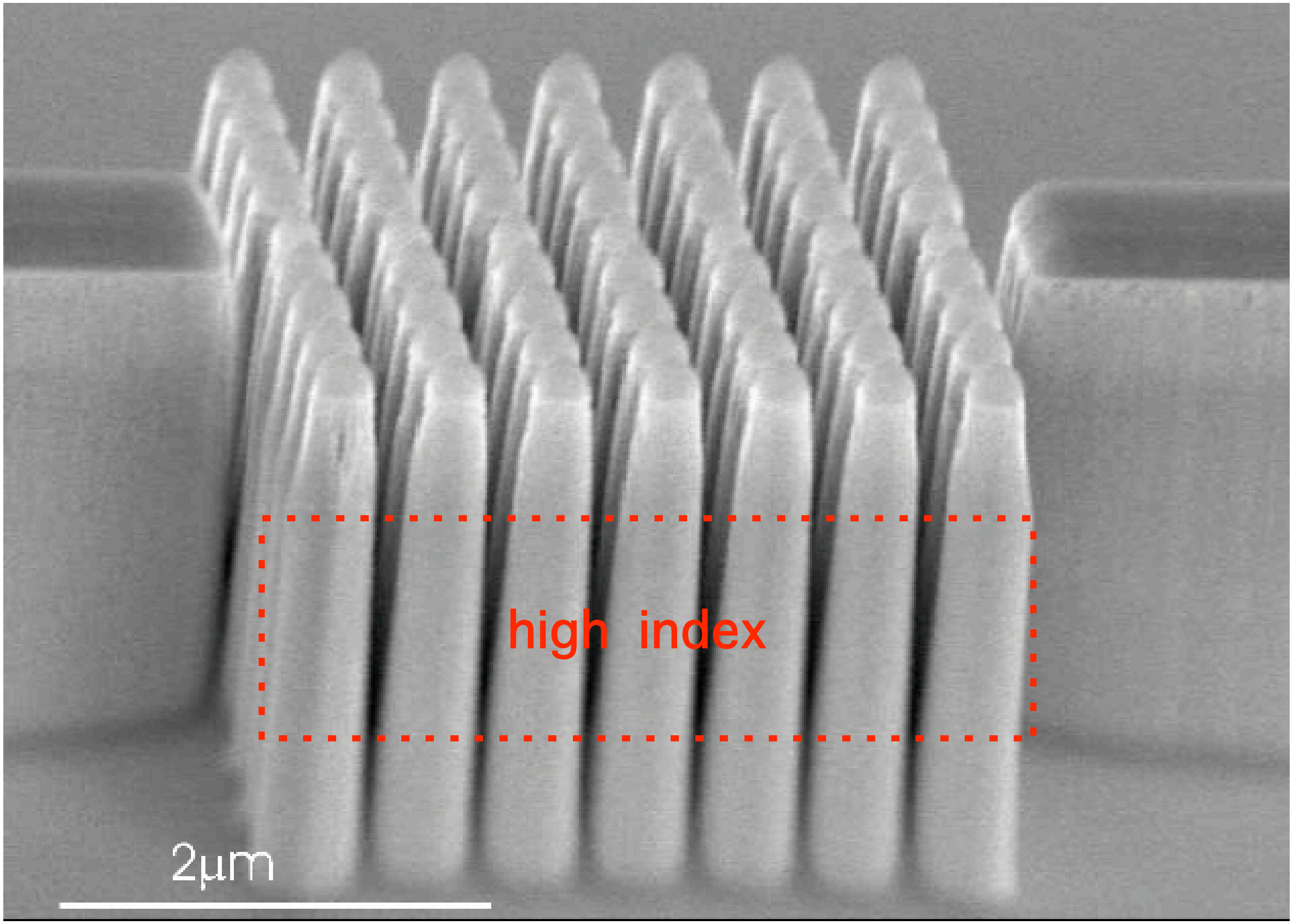
BUT

with strong confinement
(high index contrast)
mixing can be weak

"extruded" substrate
= stronger confinement

(less mixing even
without superstrate

Extruded Rod Substrate



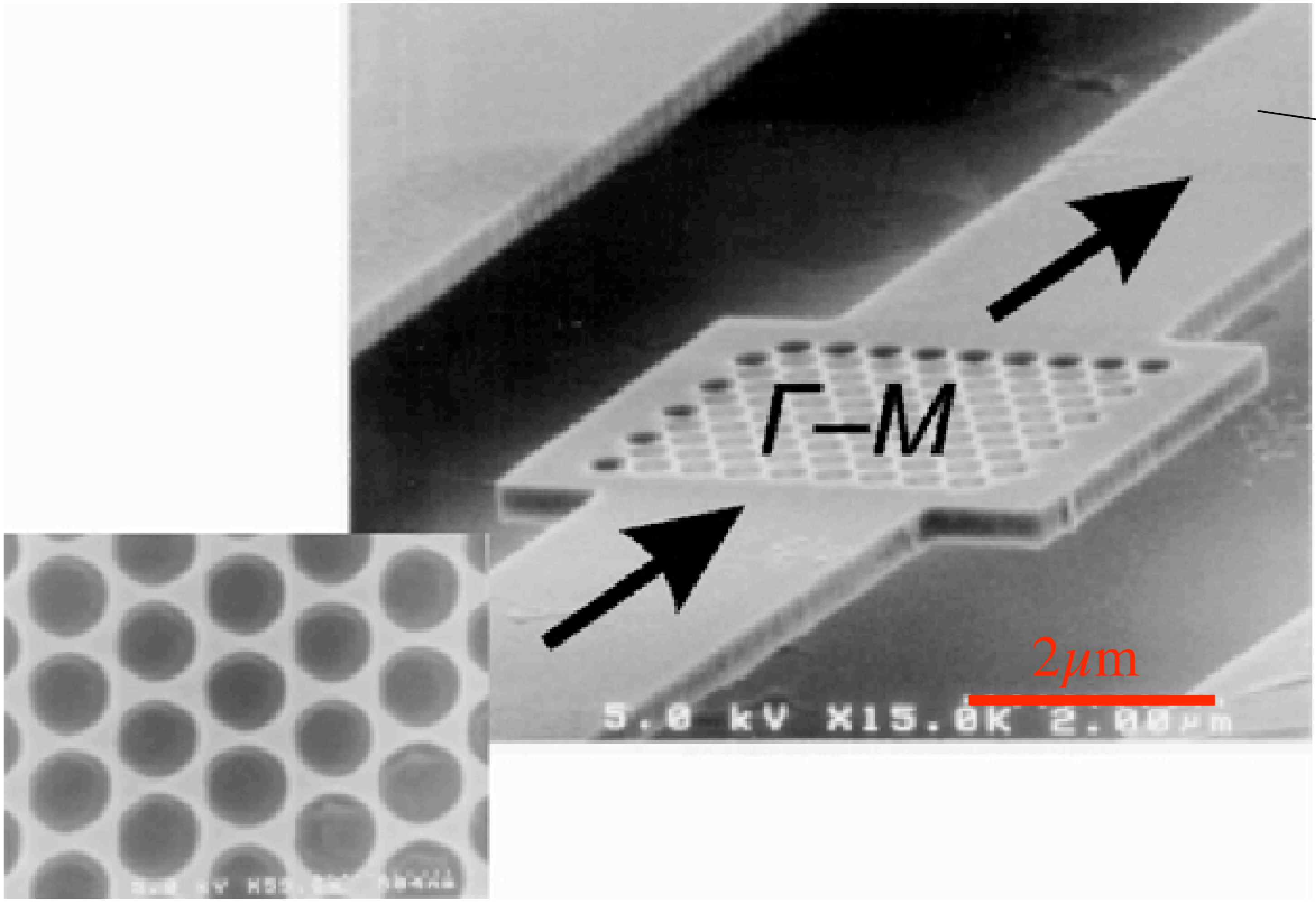
S. Assefa, L. A. Kolodziejski

(GaAs on AlO_x)

[S. Assefa *et al.*, *APL* **85**, 6110 (2004).]

Air-membrane Slabs

who needs a substrate?

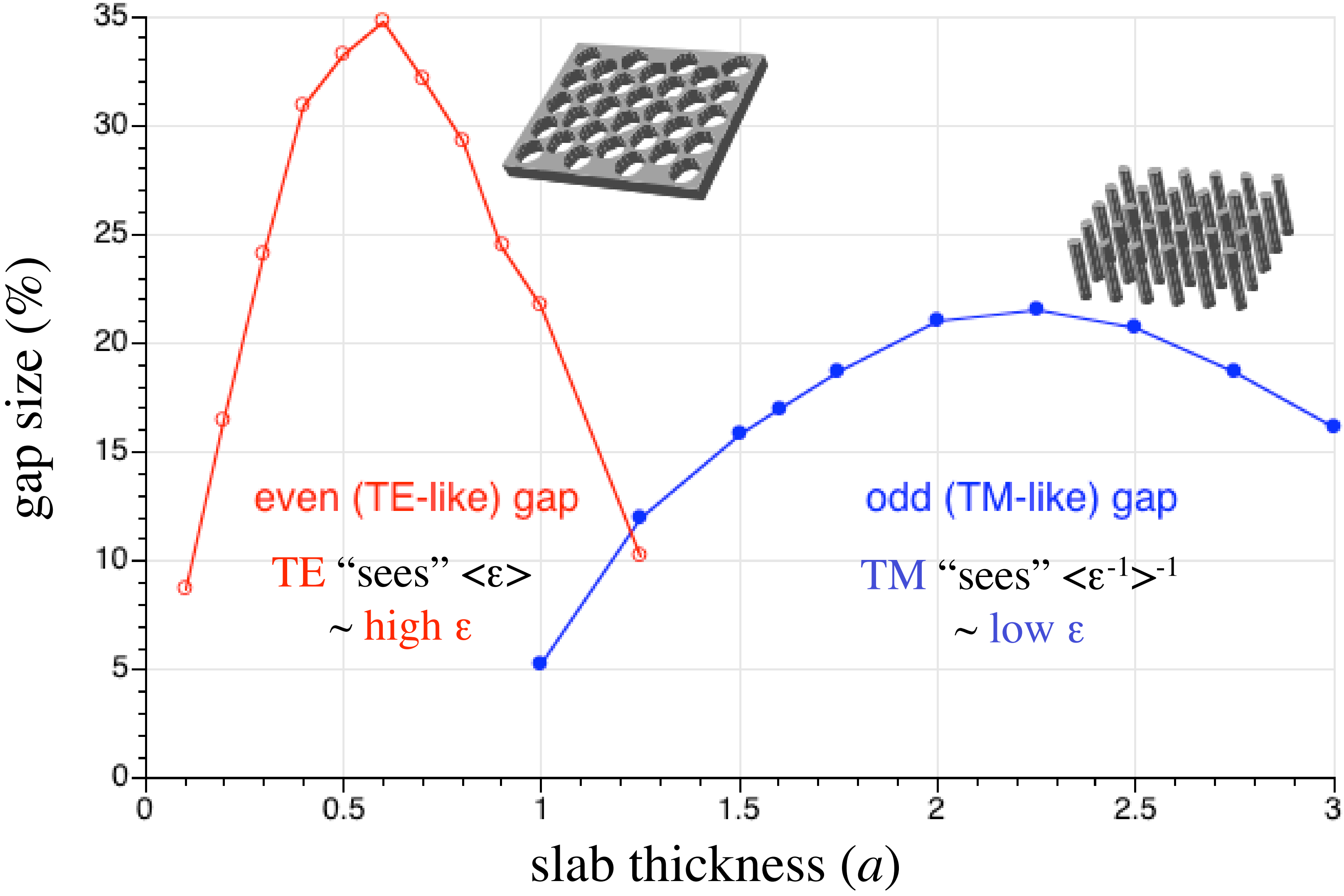


[N. Carlsson *et al.*, *Opt*. *Quantum Elec*. **34**, 123 (2002)]

Optimal Slab Thickness

~ λ/2, but λ/2 in what material?

effective medium theory: effective ε depends on polarization

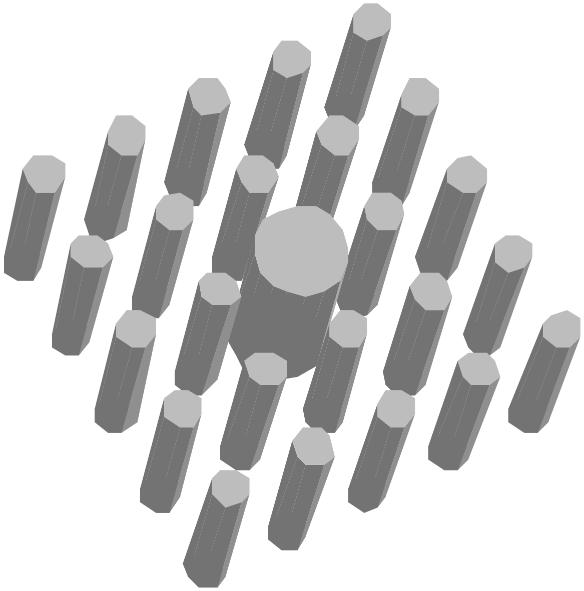


TE “sees” <ε>
~ high ε

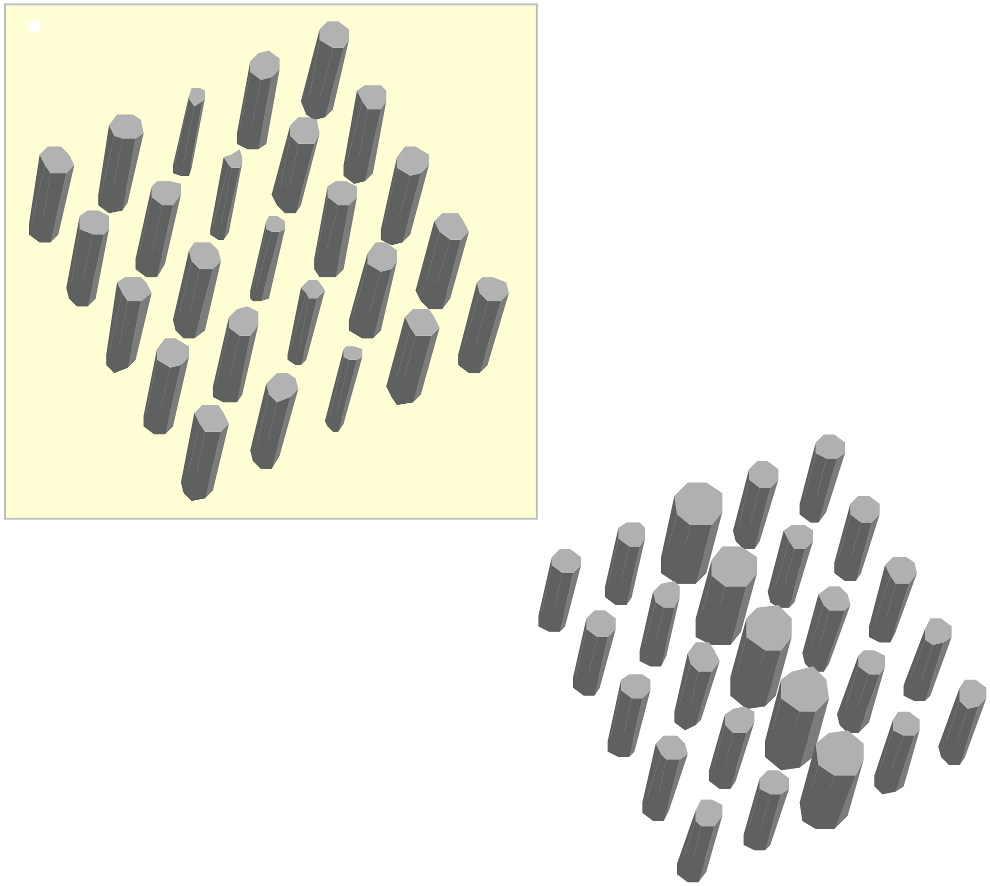
TM “sees” $<\varepsilon^{-1}>^{-1}$
~ low ε

Photonic-Crystal Building Blocks

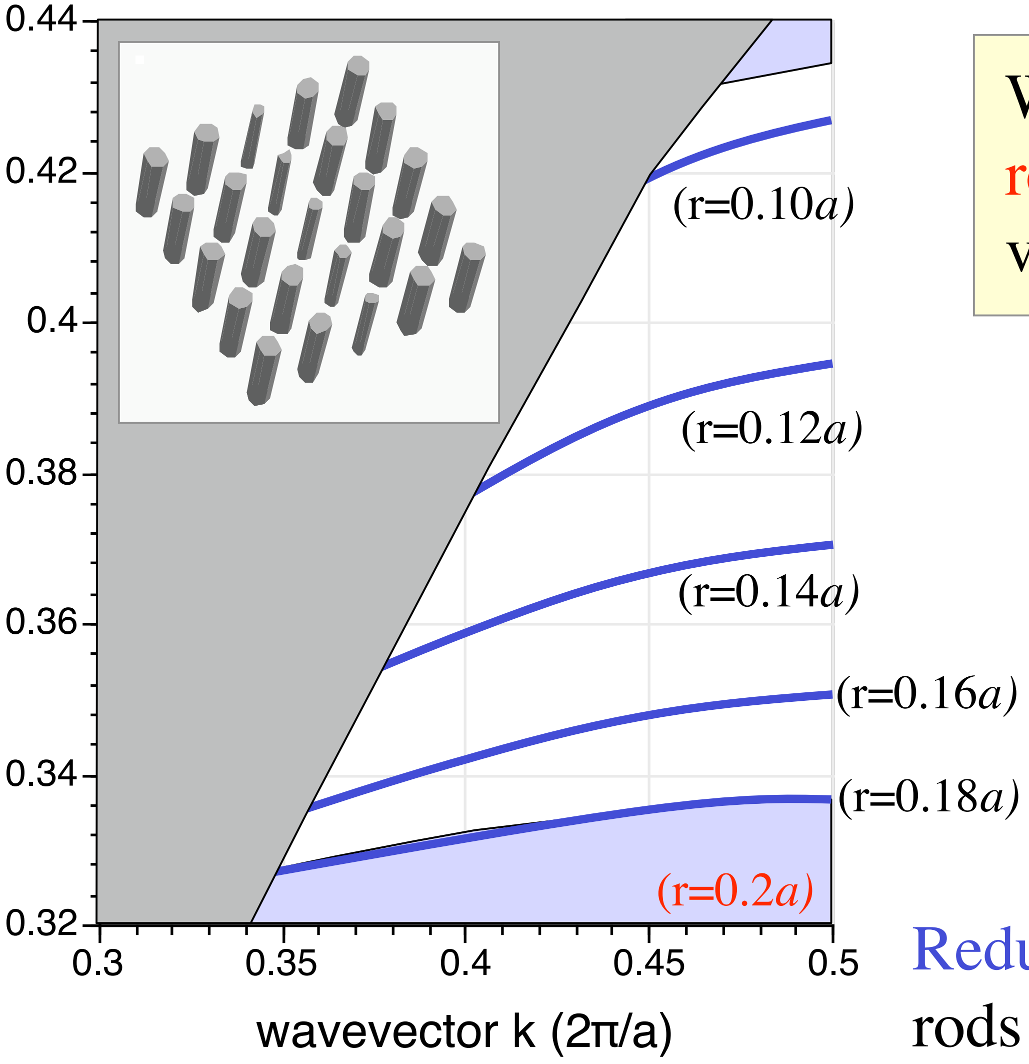
point defects
(cavities)



line defects
(waveguides)



A Reduced-Index Waveguide

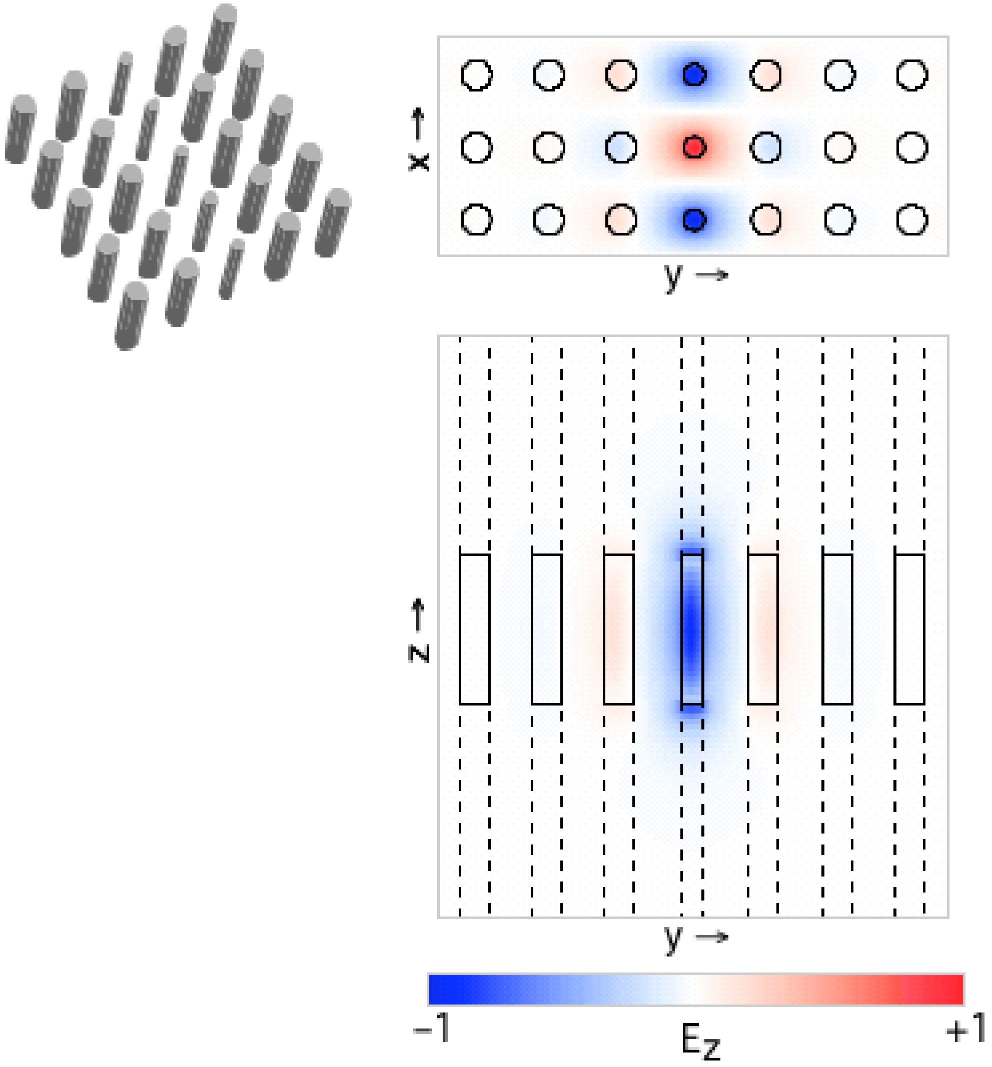


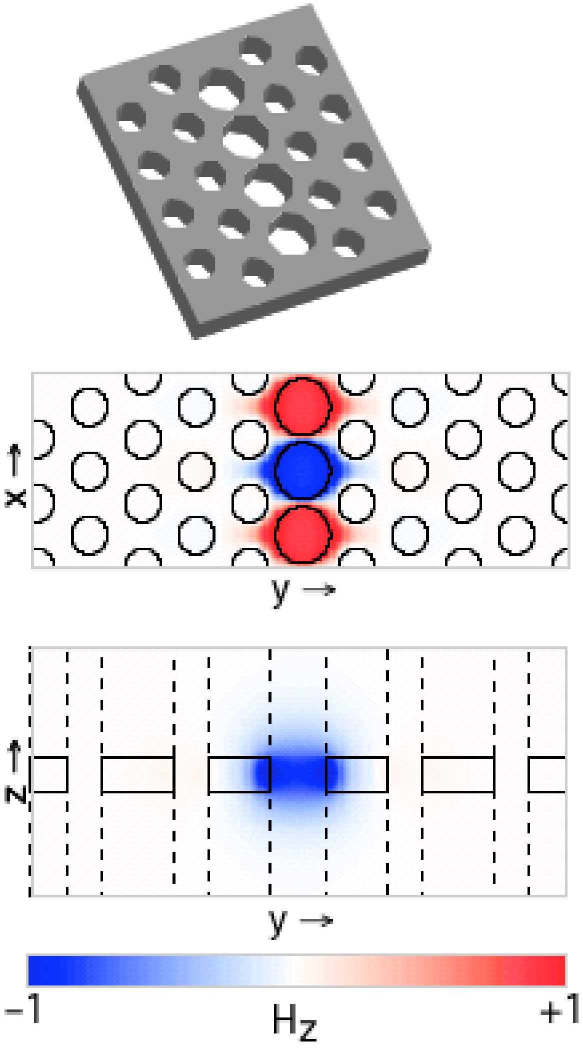
We *cannot* completely remove the rods—no vertical confinement!

Still have conserved wavevector—under the light cone, no radiation

Reduce the radius of a row of rods to "trap" a waveguide mode in the gap.

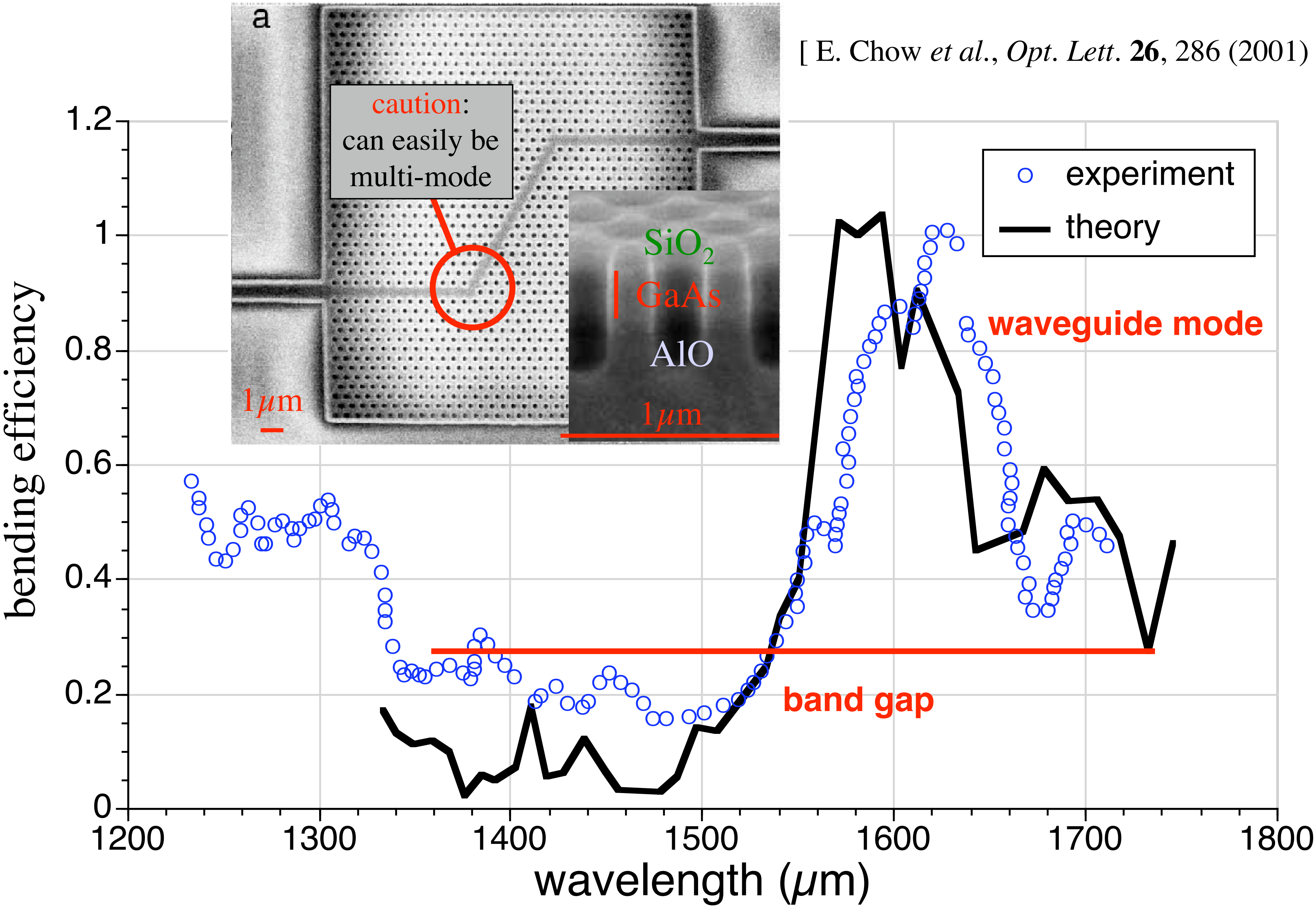
Reduced-Index Waveguide Modes





Experimental Waveguide & Bend

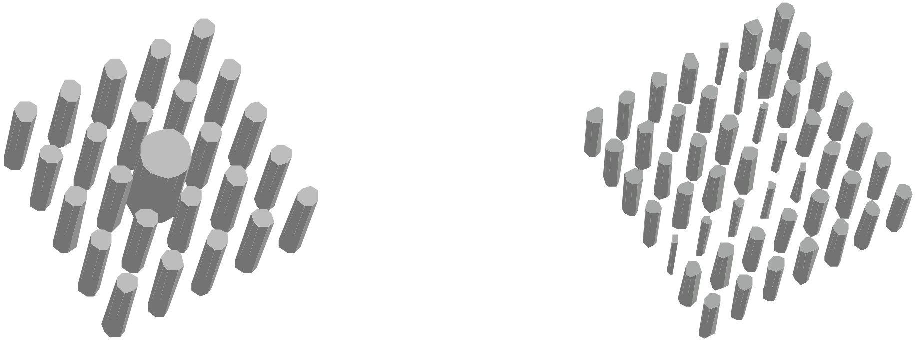
[E. Chow *et al.*, *Opt. Lett.* **26**, 286 (2001)]

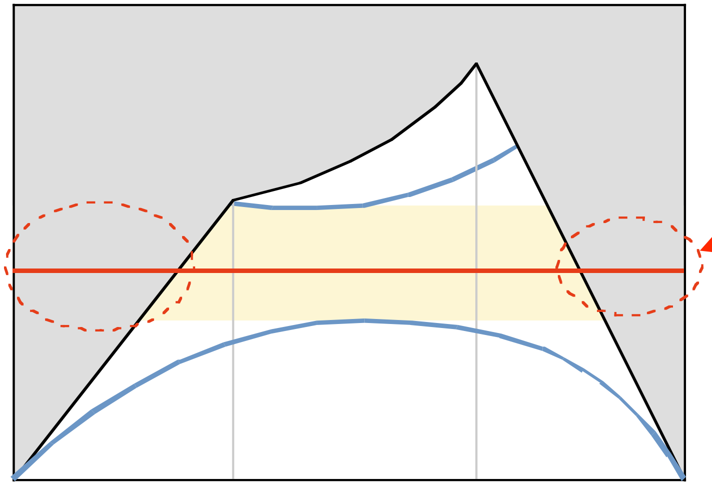


Inevitable Radiation Losses

whenever translational symmetry is broken

e.g. at cavities, waveguide bends, disorder…





ω
(conserved)

coupling to light cone
= radiation losses

k is *no longer* conserved!

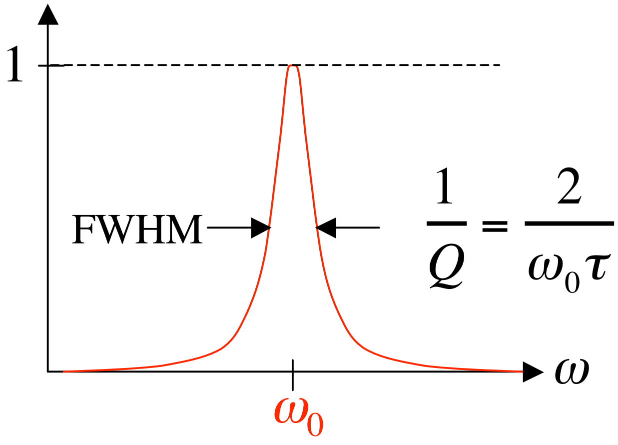
Dimensionless Losses: *Q*

quality factor Q = # optical periods for energy to decay by $\exp(-2\pi)$

$$\text{energy} \sim \exp(-\omega t/Q)$$

in frequency domain: $1/Q$ = bandwidth

from last time:
(coupling-of-
modes-in-time)



$$\frac{1}{Q} = \frac{2}{\omega_0 \tau}$$

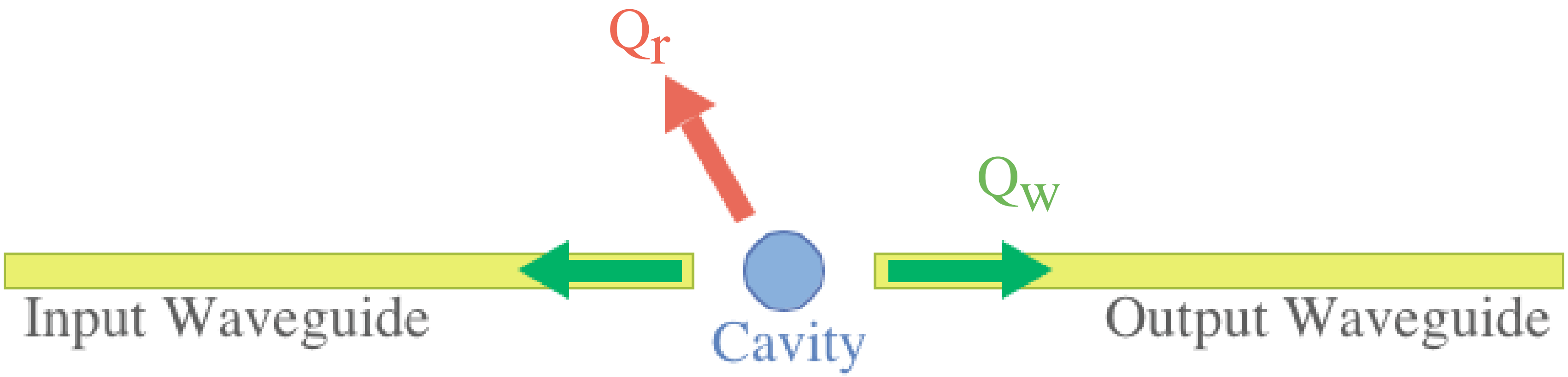
T = Lorentzian filter

$$= \frac{\frac{4}{\tau^2}}{(\omega - \omega_0)^2 + \frac{4}{\tau^2}}$$

…quality factor Q

All Is Not Lost

A simple model device (filters, bends, …):



$$1/Q = 1/Q_r + 1/Q_w$$

Q = lifetime/period
= frequency/bandwidth

We want: $Q_r >> Q_w$

1 – transmission ~ $2Q / Q_r$

worst case: high-Q (narrow-band) cavities

Semi-analytical losses

A low-loss
strategy:

Make field inside
defect small
= delocalize mode

Make defect weak
= delocalize mode

$$\underbrace{\vec{E}(\vec{x})}_{\text{far-field (radiation)}} = \int_{defect} \underbrace{\overleftrightarrow{G}_{\omega}(\vec{x},\vec{x}')}_{\text{Green's function (defect-free system)}} \cdot \overbrace{\underbrace{\vec{E}(\vec{x}')}_{\text{near-field (cavity mode)}}}^{\text{Make field inside defect small = delocalize mode}} \cdot \overbrace{\underbrace{\Delta\varepsilon(\vec{x}')}_{\text{defect}}}^{\text{Make defect weak = delocalize mode}}$$

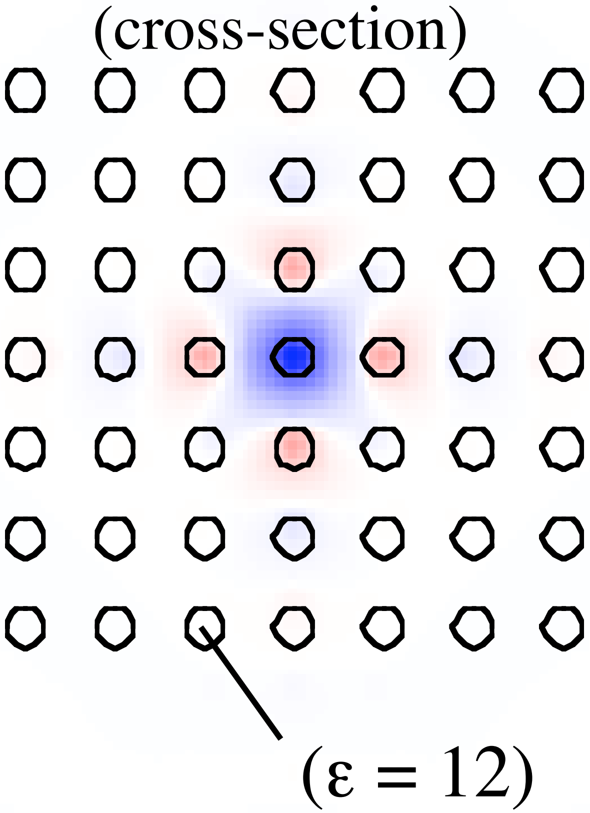
far-field
(radiation)

Green's function
(defect-free
system)

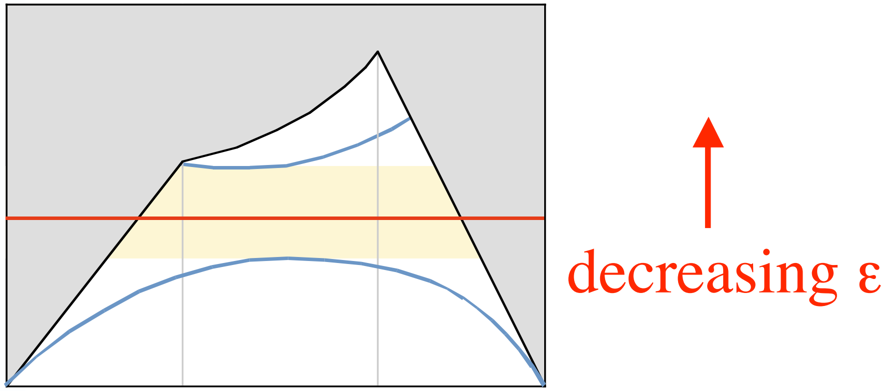
near-field
(cavity mode)

defect

Monopole Cavity in a Slab

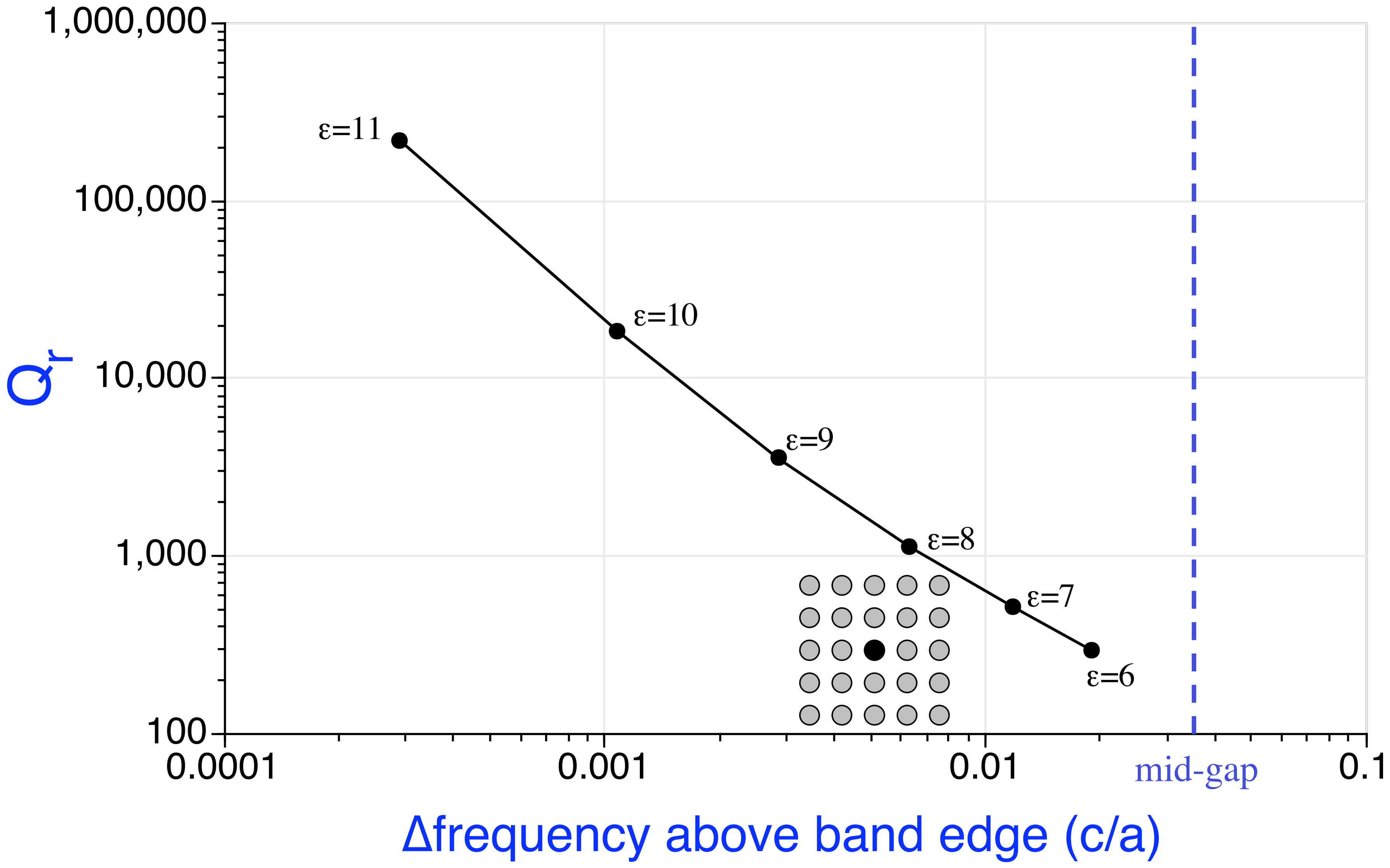


Lower the ε of a single rod: push up a monopole (singlet) state.



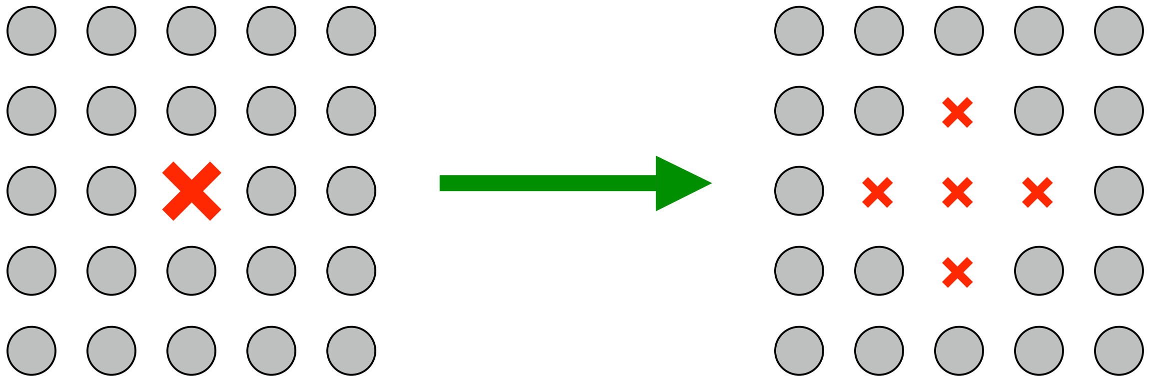
Use small Δε: delocalized in-plane, & high-Q (we hope)

Delocalized Monopole Q



[S. G. Johnson *et al*., *Computing in Sci. and Eng*. **3**, 38 (2001).]

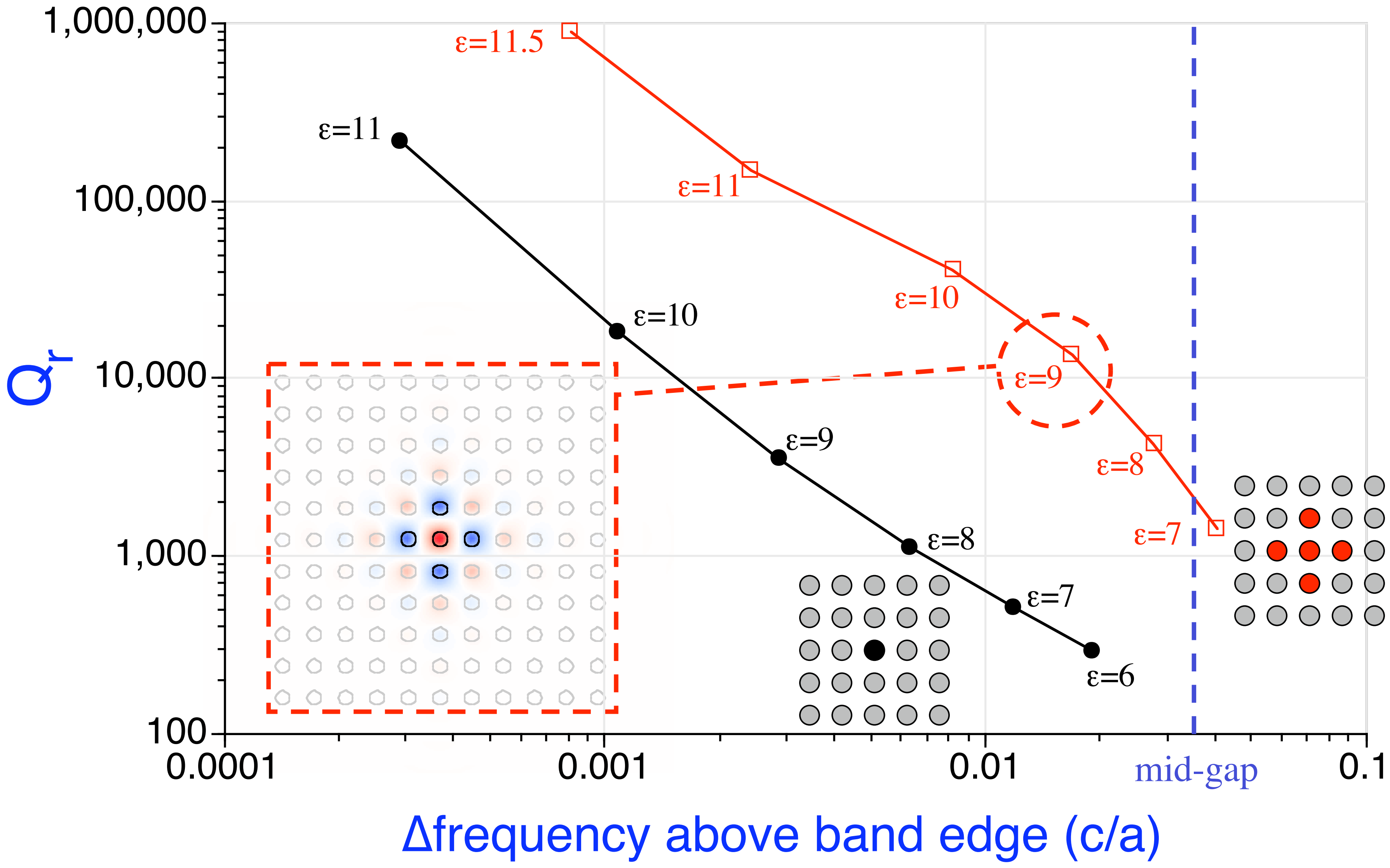
Super-defects



Weaker defect with more unit cells.

More delocalized
at the same point in the gap
(*i.e.* at same bulk decay rate)

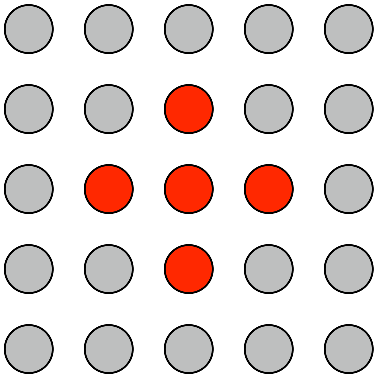
Super-Defect vs. Single-Defect Q



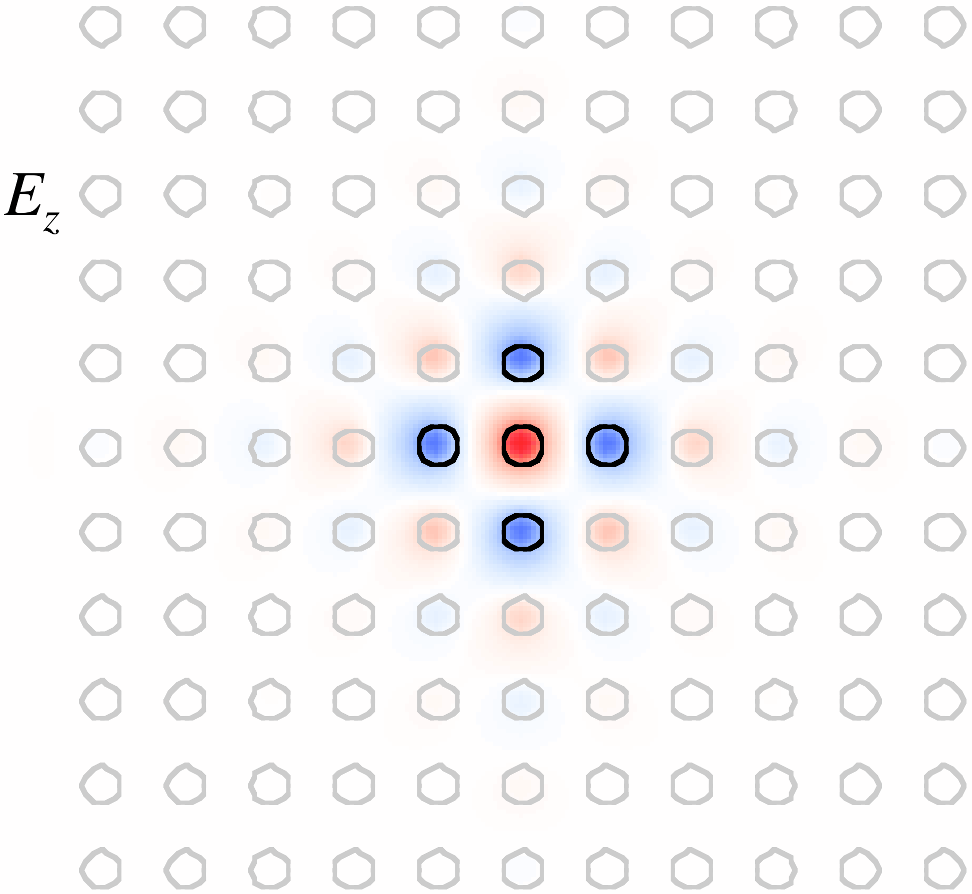
[S. G. Johnson *et al.*, *Computing in Sci. and Eng*. **3**, 38 (2001).]

Super-Defect State

(cross-section)


(super defect)

$\Delta\varepsilon = -3$, $Q_{rad} = 13{,}000$



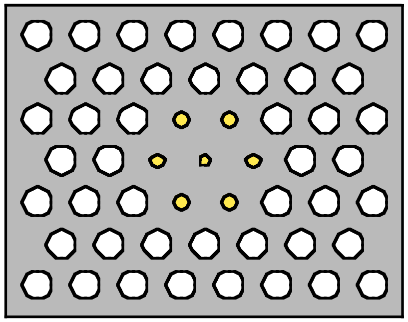
still ~localized: *In-plane* $Q_{||}$ is > 50,000 for only 4 bulk periods

Hole Slab

ε=11.56

period *a*, radius 0.3*a*

thickness 0.5*a*

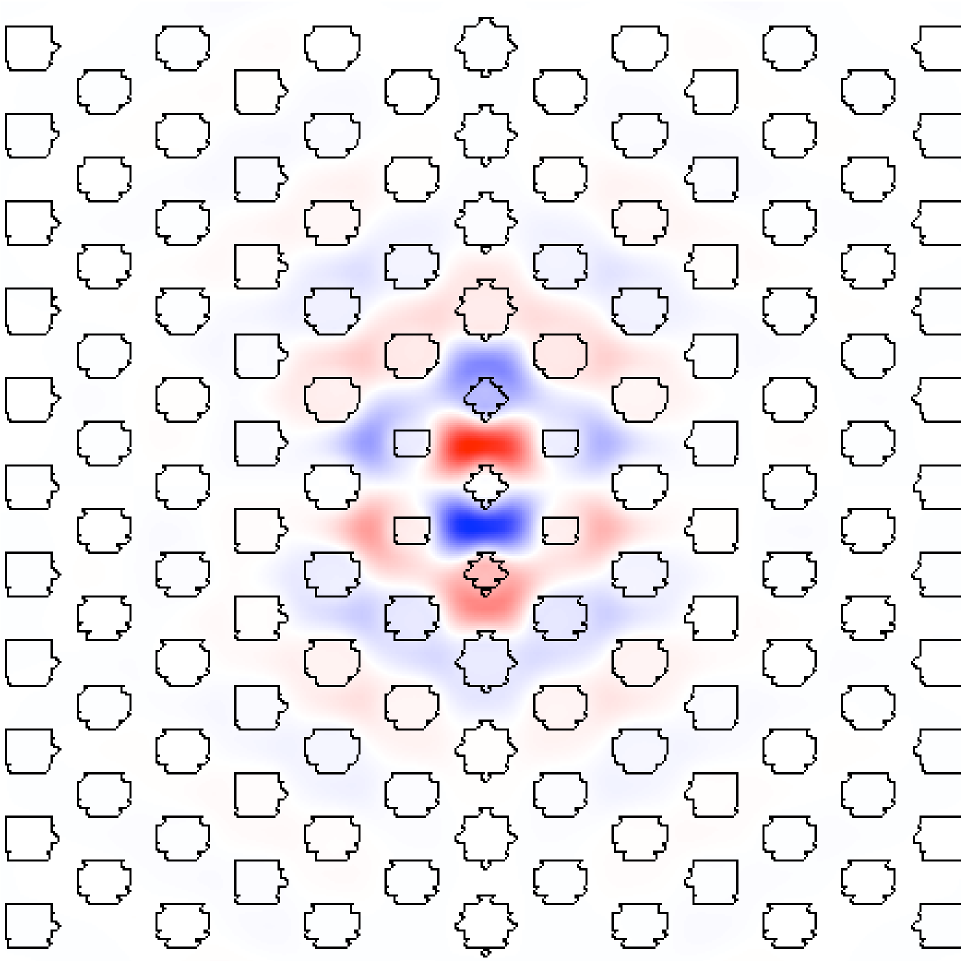


Reduce radius of

7 holes to 0.2*a*

Q = 2500

near mid-gap (Δfreq = 0.03)



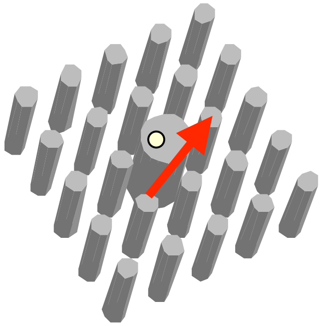
Very robust to roughness

(note pixellization, *a* = 10 pixels).

How do we compute Q?

(via 3d FDTD [finite-difference time-domain] simulation)

①



excite cavity with dipole source

(broad bandwidth, *e.g.* Gaussian pulse)

… monitor field at some point ○

…extract frequencies, decay rates via fancy signal processing (not just FFT/fit)

[V. A. Mandelshtam, *J. Chem. Phys.* **107**, 6756 (1997)]

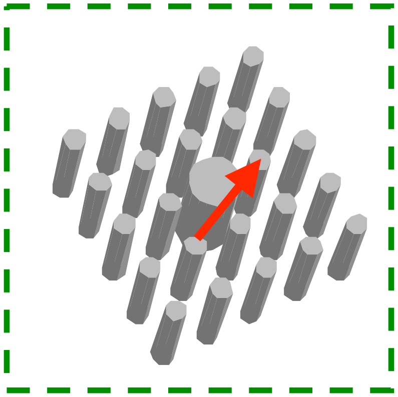
Pro: no *a priori* knowledge, get all ω's and Q's at once

Con: no separate Q_w/Q_r,
mixed-up field pattern if multiple resonances

How do we compute Q?

(via 3d FDTD [finite-difference time-domain] simulation)

②



excite cavity with
narrow-band dipole source
(*e.g.* temporally broad Gaussian pulse)

— source is at ω_0 resonance,
which must already be known (via ①)

…measure outgoing power P and energy U

$$\boxed{Q = \omega_0 \, U / P}$$

Pro: separate Q_w/Q_r, also get field pattern when multimode

Con: requires separate run ① to get ω_0,
long-time source for closely-spaced resonances

Can we increase Q
without delocalizing?

Semi-analytical losses

Another low-loss strategy:

$$\underbrace{\vec{E}(\vec{x})}_{\text{far-field (radiation)}} = \int_{defect} \overbrace{\underbrace{\overleftrightarrow{G}_{\omega}(\vec{x},\vec{x}\,')}_{\text{Green's function (defect-free system)}} \cdot \underbrace{\vec{E}(\vec{x}\,')}_{\text{near-field (cavity mode)}} \cdot \underbrace{\Delta\varepsilon(\vec{x}\,')}_{\text{defect}}}^{\text{exploit cancellations from sign oscillations}}$$

Need a more compact representation

Cannot cancel infinitely many $\mathbf{E}(x)$ integrals

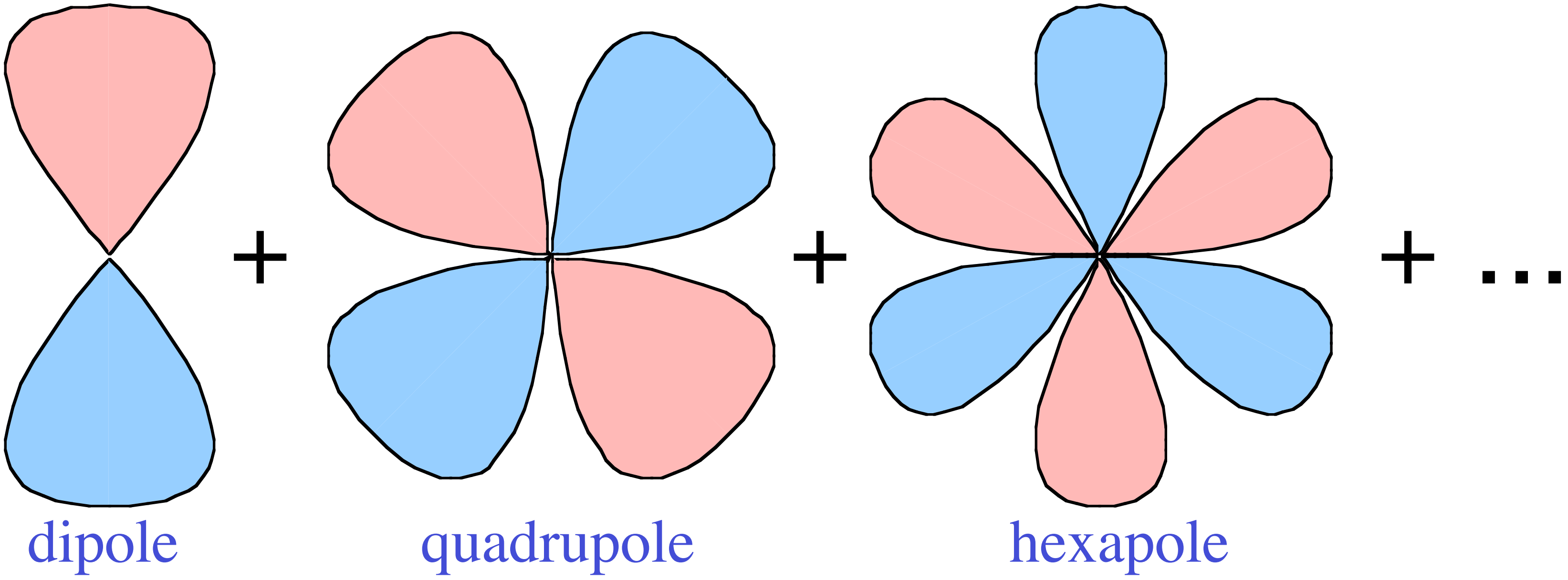
Radiation pattern from localized source…

— use multipole expansion
& cancel largest moment

Multipole Expansion

[Jackson, *Classical Electrodynamics*]

radiated field =



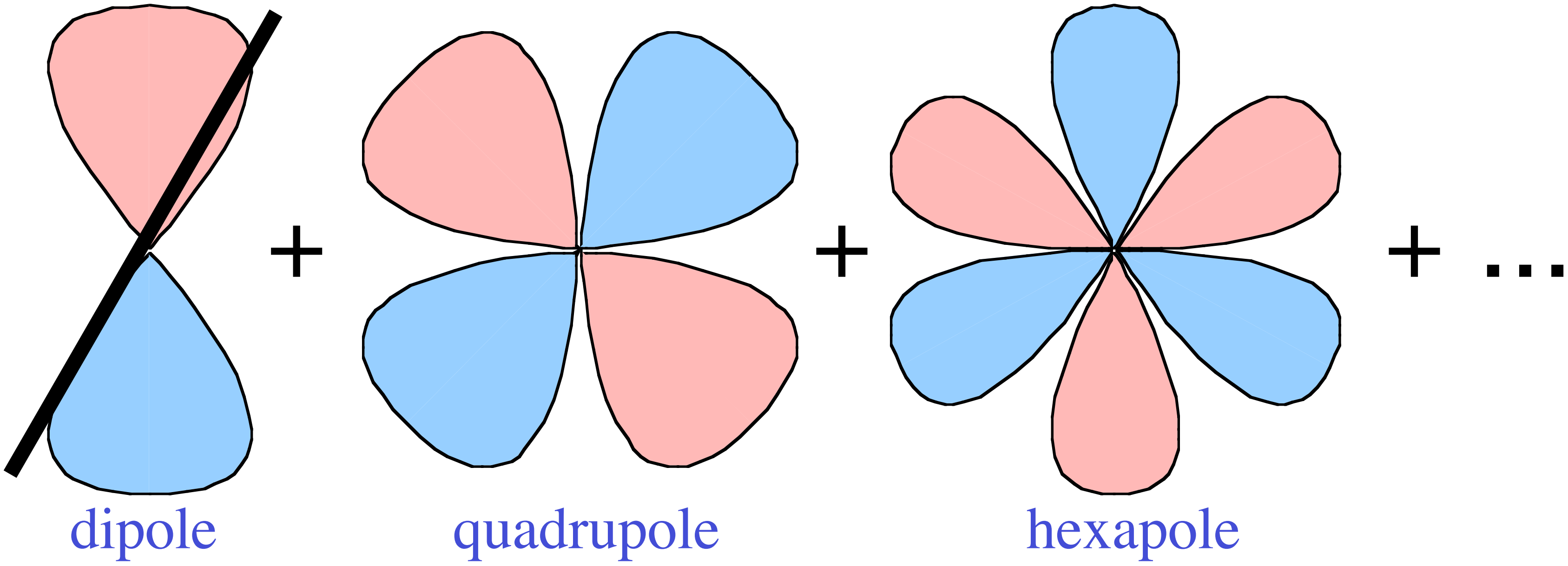
Each term’s strength = single integral over near field

…one term is cancellable by tuning one defect parameter

Multipole Expansion

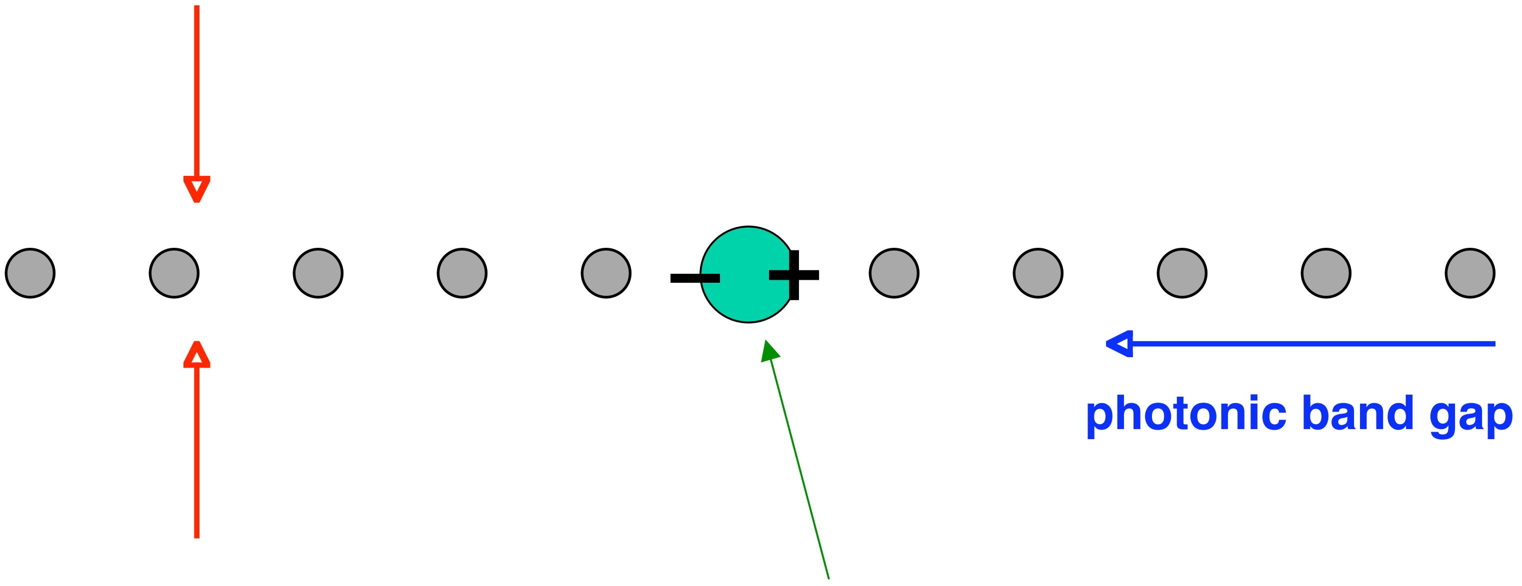
[Jackson, *Classical Electrodynamics*]

radiated field =



peak Q (cancellation) = transition to higher-order radiation

Multipoles in a 2d example



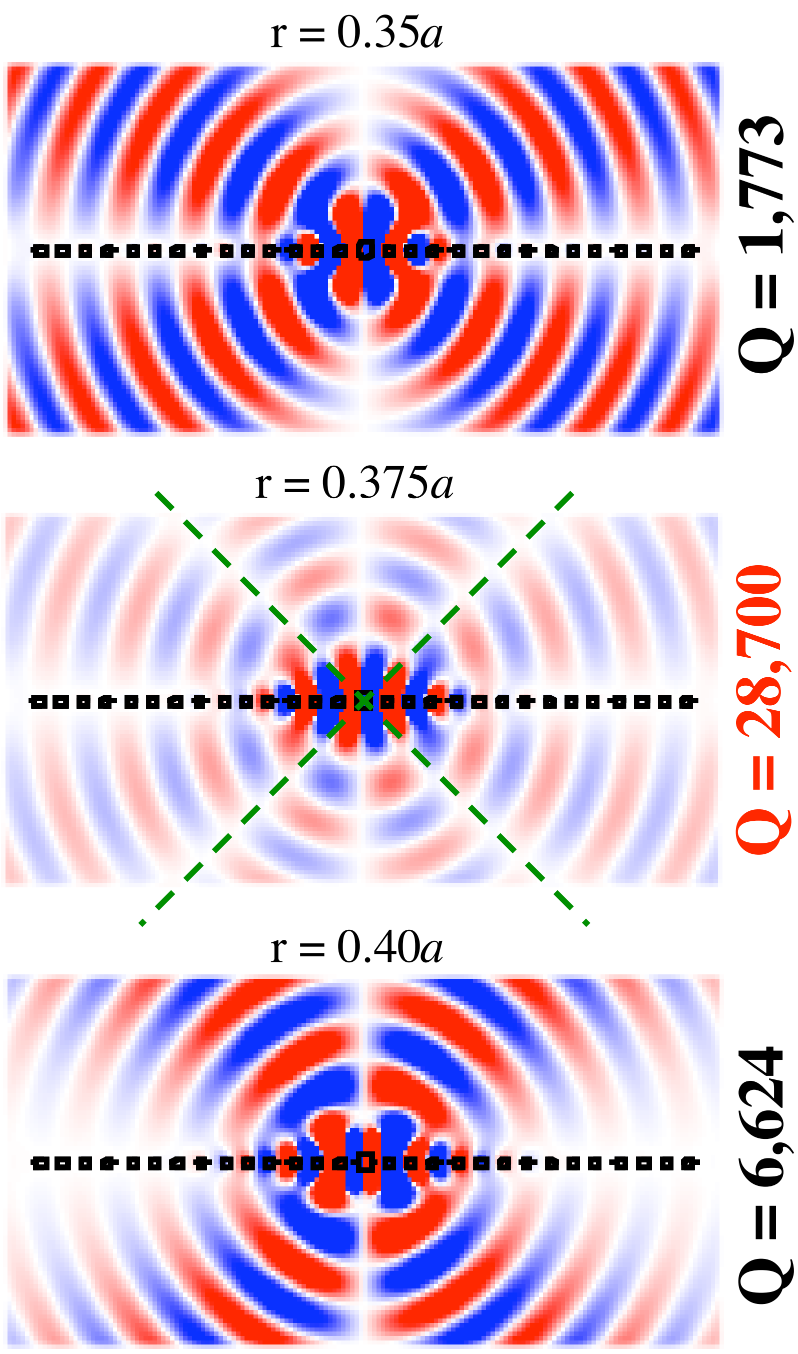
photonic band gap

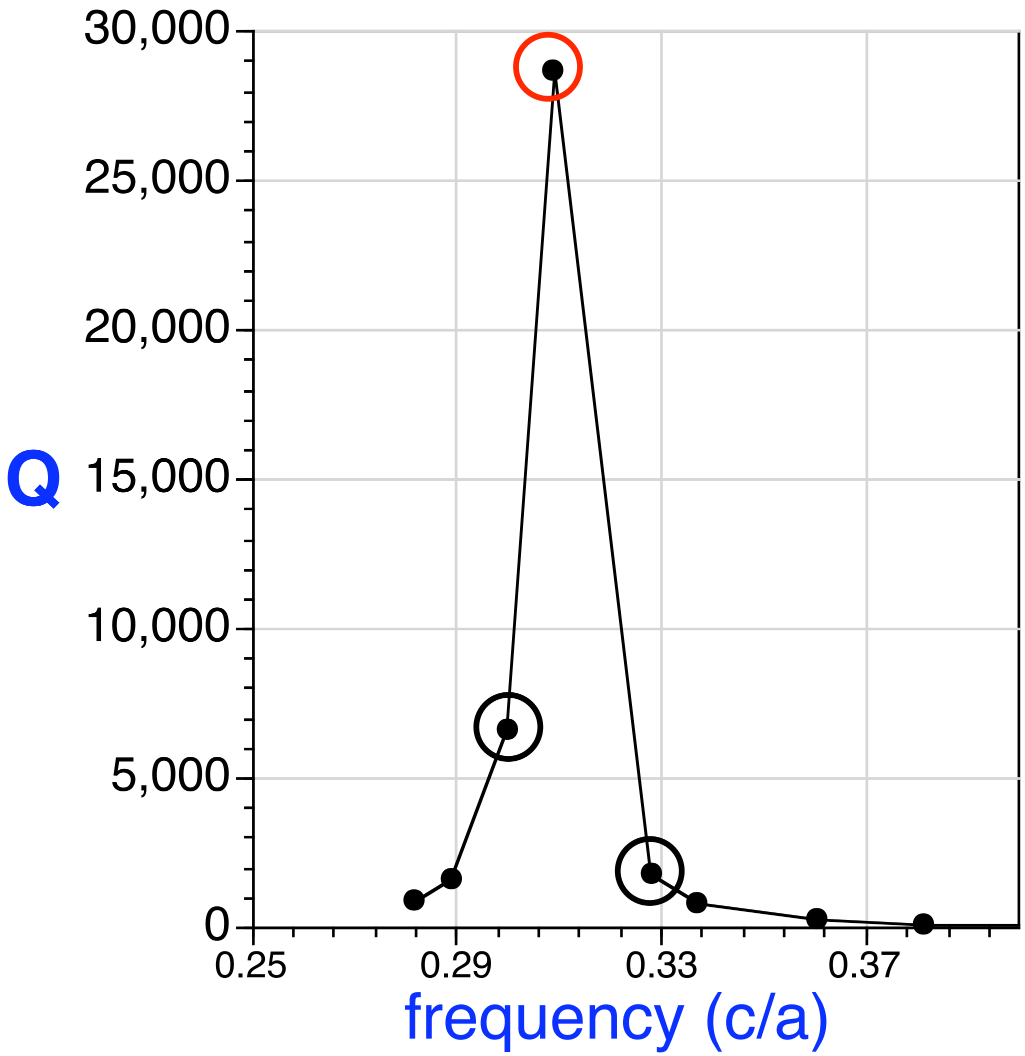
index-confined

increased rod radius
pulls down "dipole" mode
(non-degenerate)

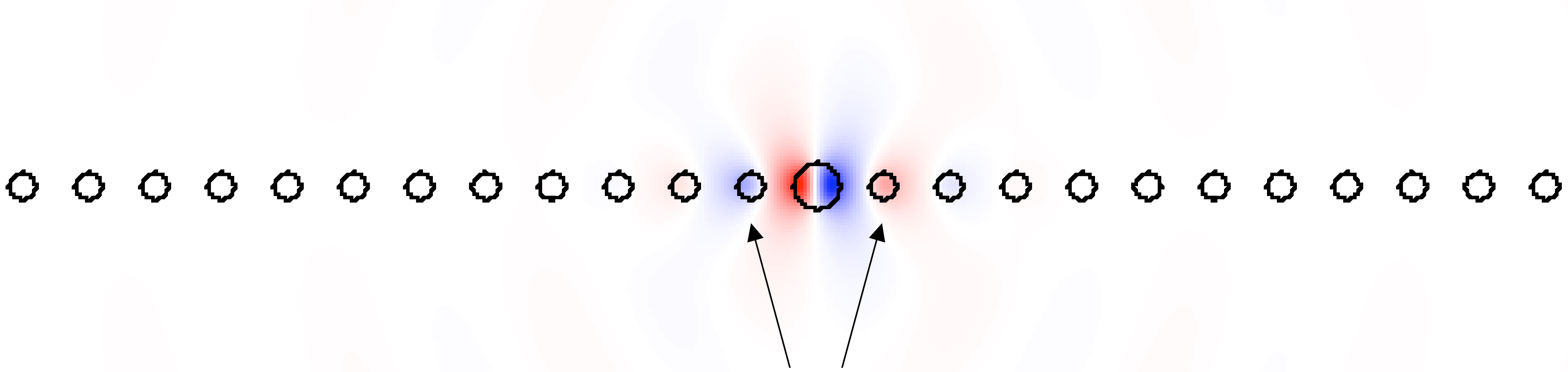
as we change the radius, ω sweeps across the gap

2d multipole cancellation





cancel a dipole by opposite dipoles…

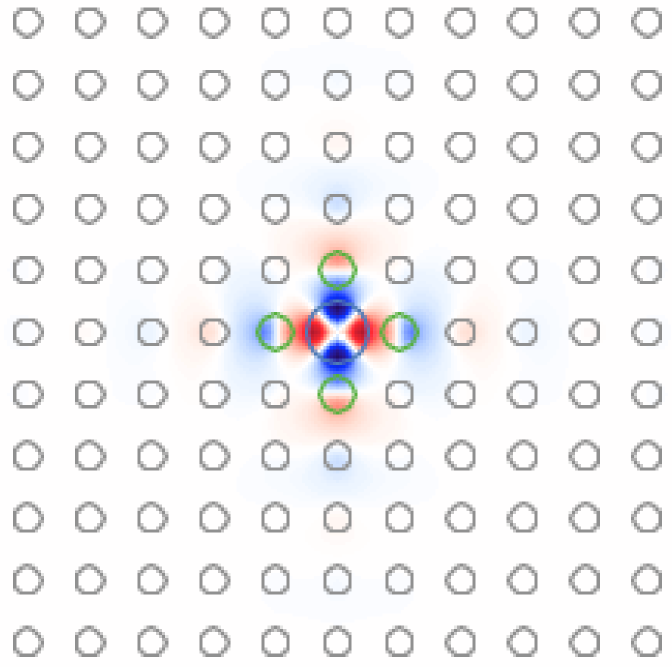


cancellation comes from
opposite-sign fields in adjacent rods

… changing radius changed balance of dipoles

3d multipole cancellation?

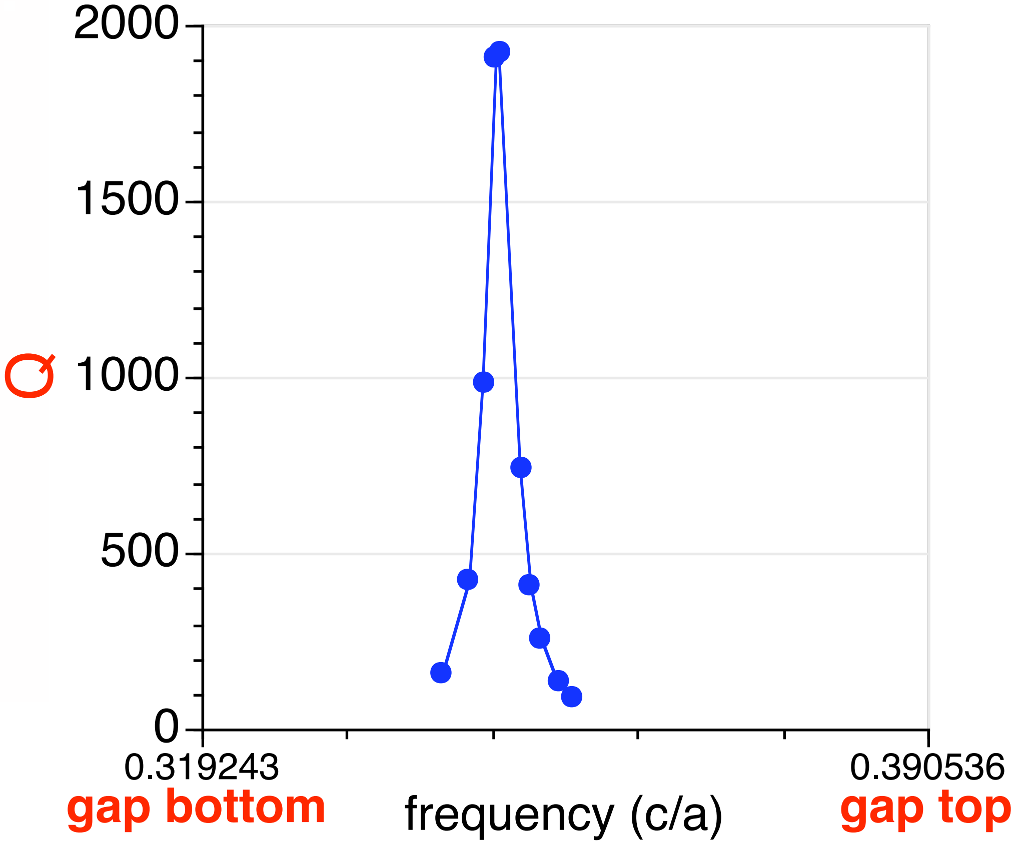
quadrupole mode



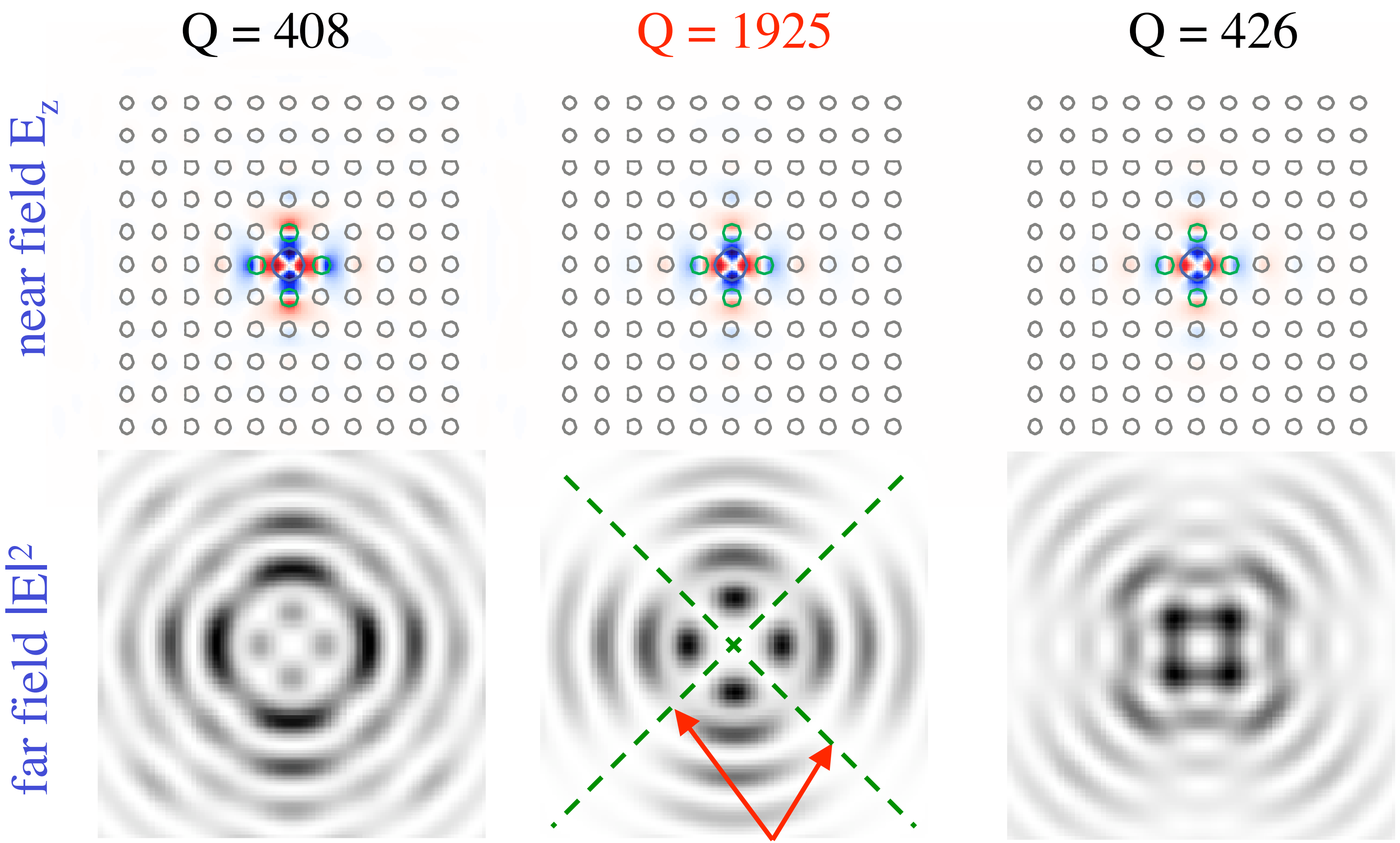
(E_z cross section)

enlarge center & adjacent rods

vary side-rod ε slightly
for continuous tuning
(balance central moment with opposite-sign side rods)



3d multipole cancellation



Q = 408

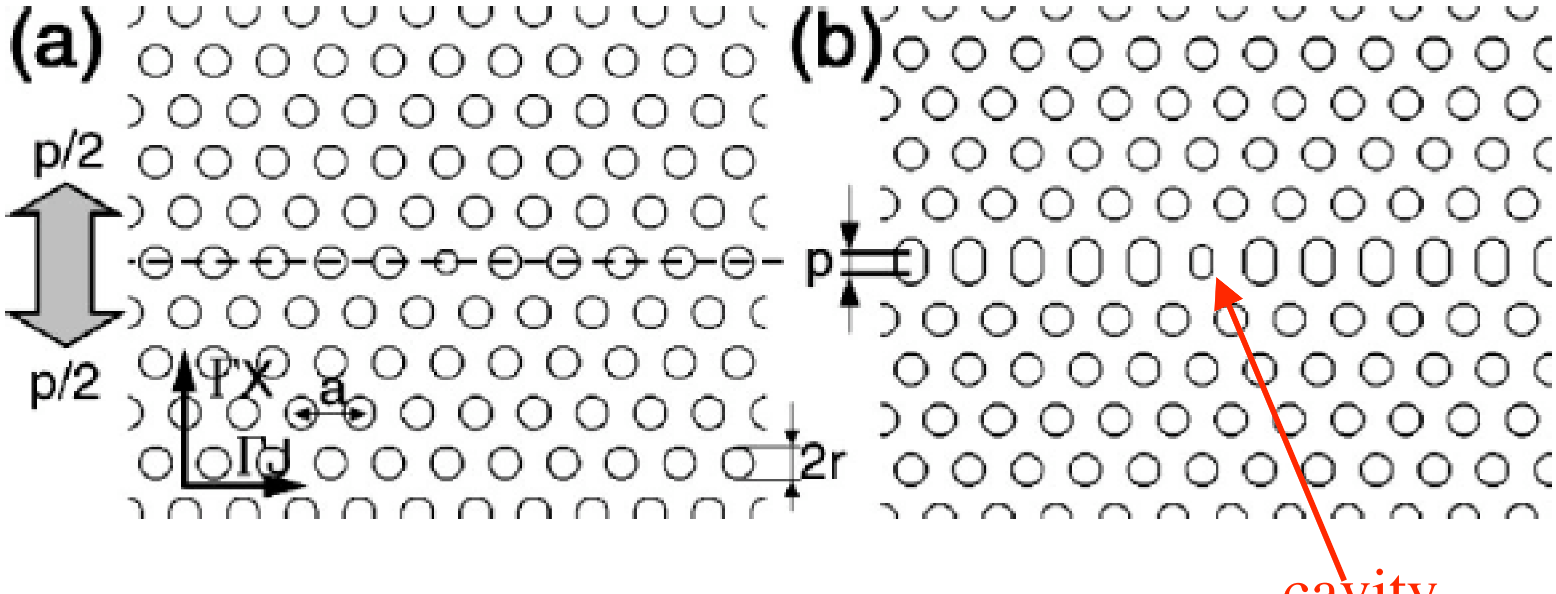
Q = 1925

Q = 426

nodal planes
(source of high Q)

An Experimental (Laser) Cavity

[M. Loncar *et al*., *Appl*. *Phys*. *Lett*. **81**, 2680 (2002)]



elongate row of holes

cavity

Elongation p is a tuning parameter for the cavity…

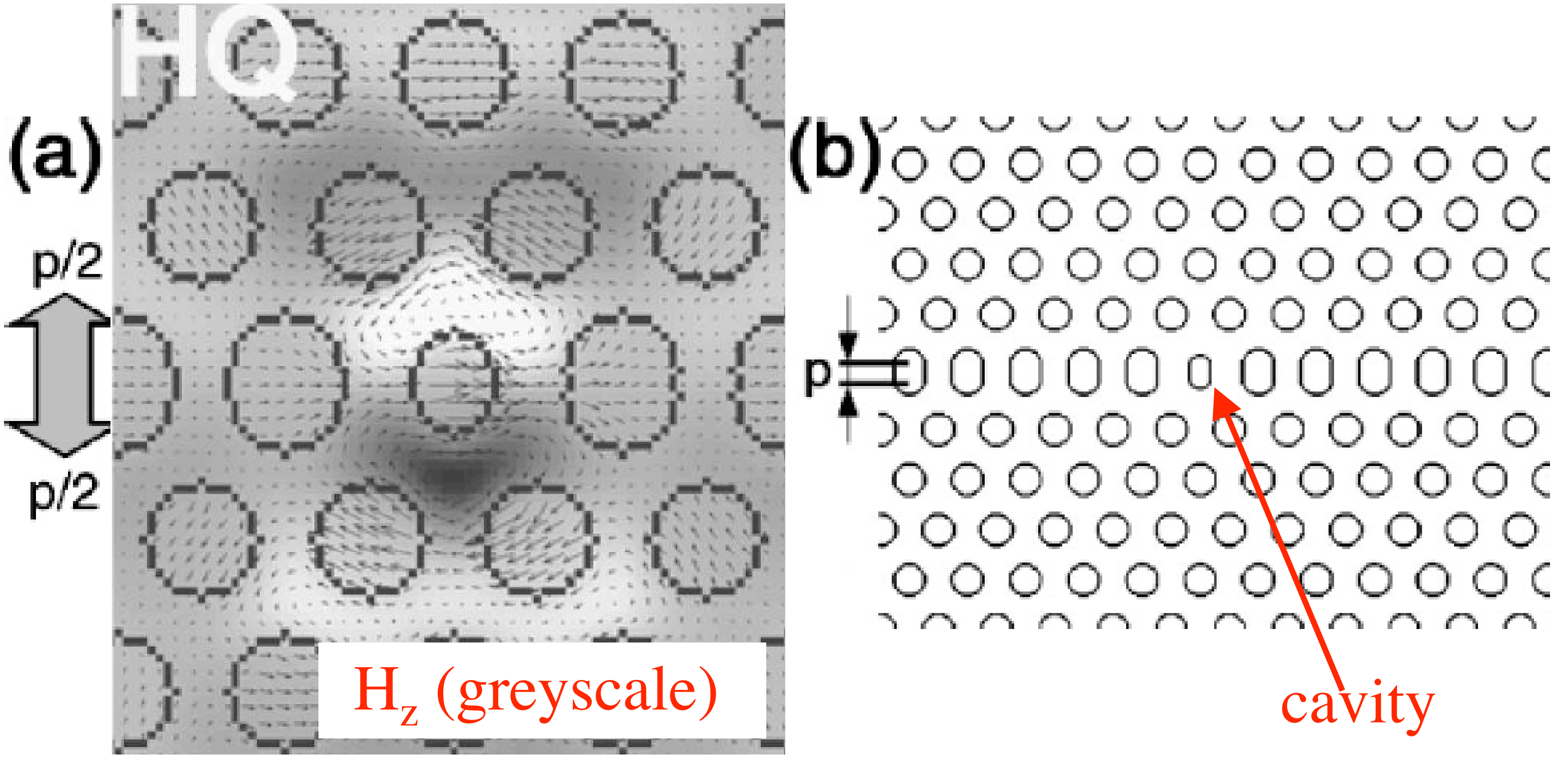
…in simulations, Q peaks sharply to ~10000 for $p = 0.1a$

(likely to be a multipole-cancellation effect)

* actually, there are two cavity modes; p breaks degeneracy

An Experimental (Laser) Cavity

[M. Loncar *et al.*, *Appl*. *Phys*. *Lett*. **81**, 2680 (2002)]



elongate row of holes

H_z (greyscale)

cavity

Elongation p is a tuning parameter for the cavity…

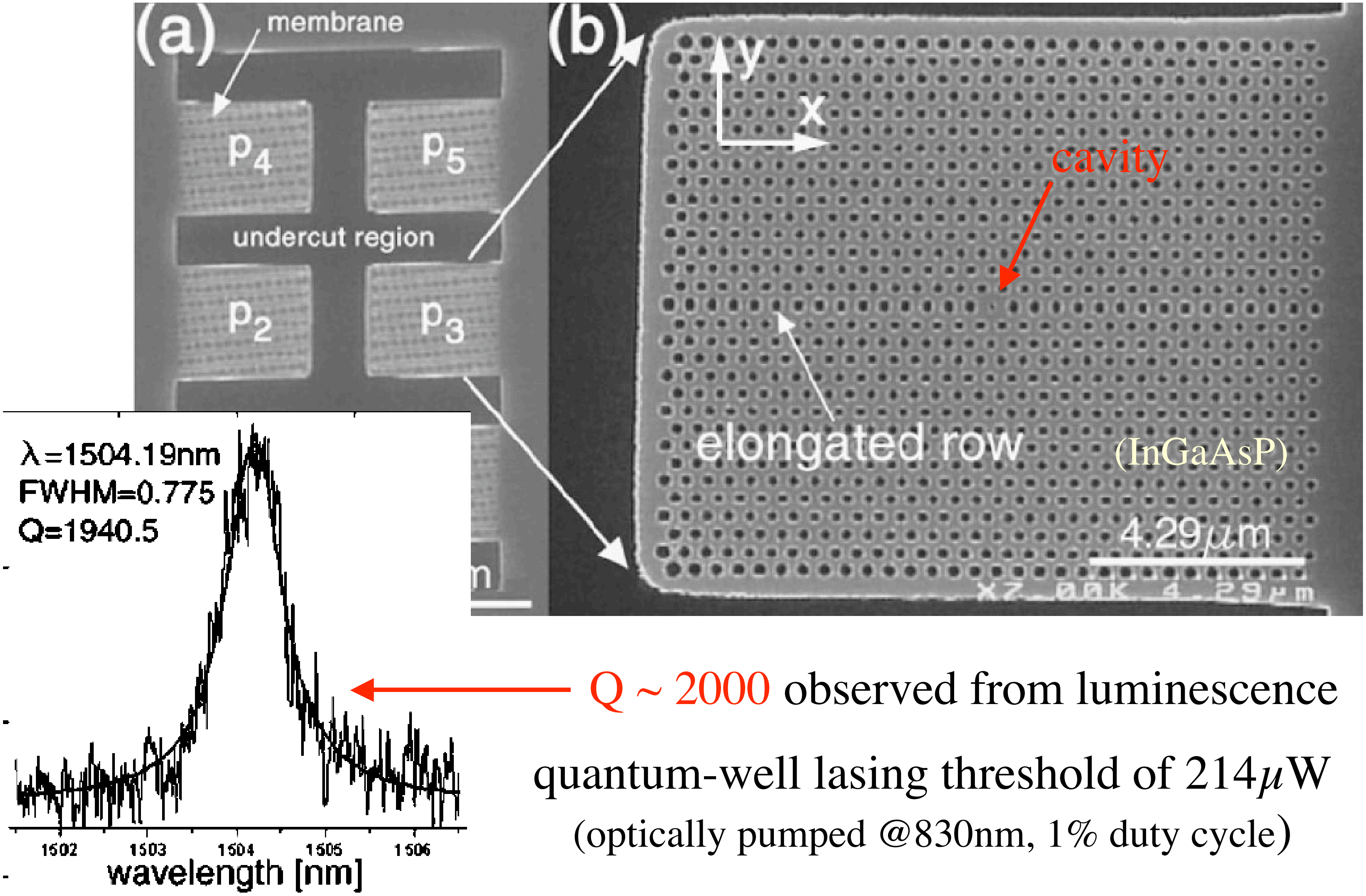
…in simulations, Q peaks sharply to ~10000 for $p = 0.1a$

(likely to be a multipole-cancellation effect)

* actually, there are two cavity modes; p breaks degeneracy

An Experimental (Laser) Cavity

[M. Loncar *et al.*, *Appl. Phys. Lett.* **81**, 2680 (2002)]



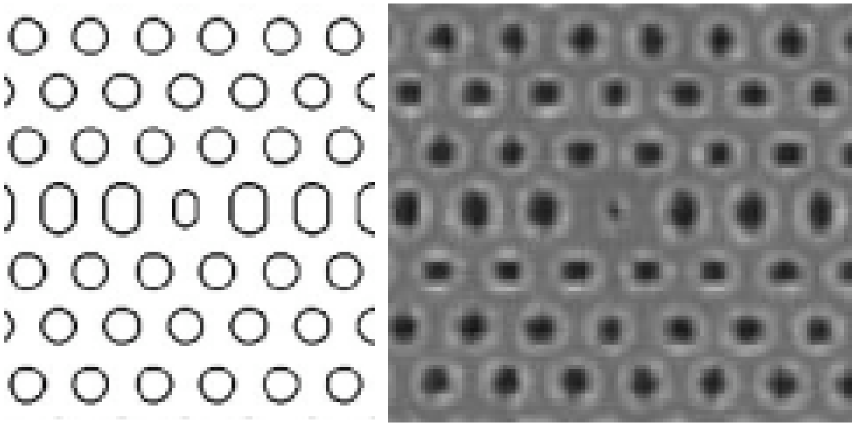
Q ~ 2000 observed from luminescence

quantum-well lasing threshold of 214μW

(optically pumped @830nm, 1% duty cycle)

Slab Cavities in Practice: Q vs. V

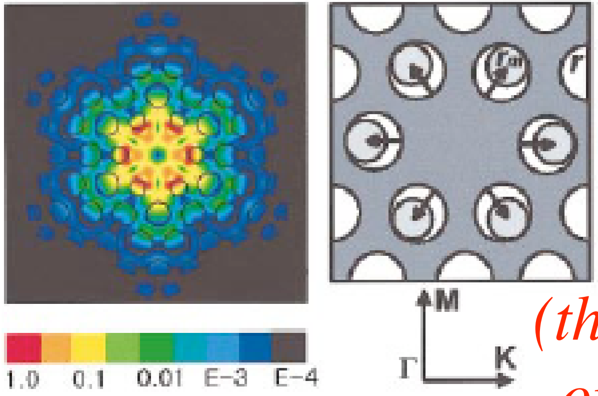
[Loncar, *APL* **81**, 2680 (2002)]



$Q \sim 10{,}000$ ($V \sim 4 \times$ optimum)

$= (\lambda/2n)^3$

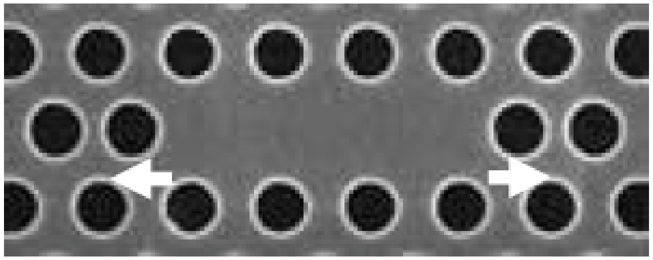
[Ryu, *Opt. Lett.* **28**, 2390 (2003)]



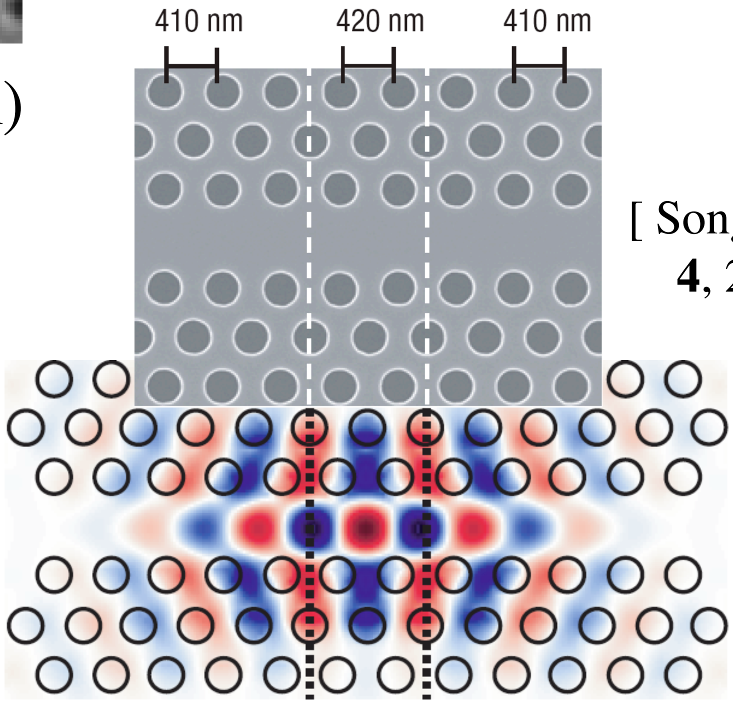
(theory only)

$Q \sim 10^6$ ($V \sim 11 \times$ optimum)

[Akahane, *Nature* **425**, 944 (2003)]



$Q \sim 45{,}000$ ($V \sim 6 \times$ optimum)



[Song, *Nature Mat.* **4**, 207 (2005)]

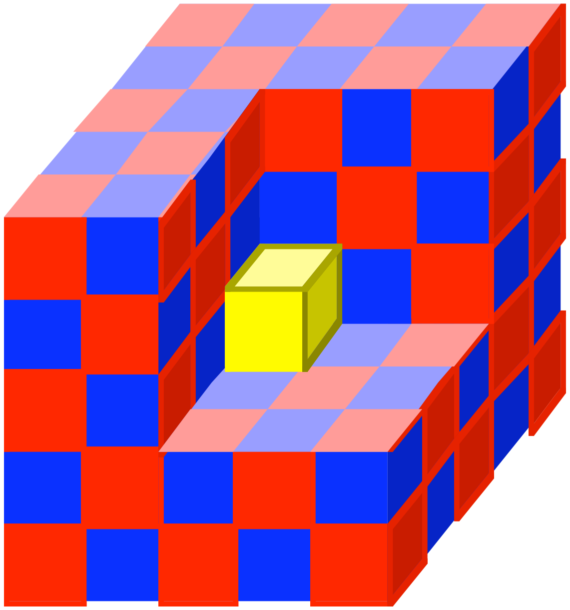
$Q \sim 600{,}000$ ($V \sim 10 \times$ optimum)

How can we get *arbitrary* Q with *finite* modal volume?

~~Only~~ one way:

a full 3d band gap

(or perfect metal)



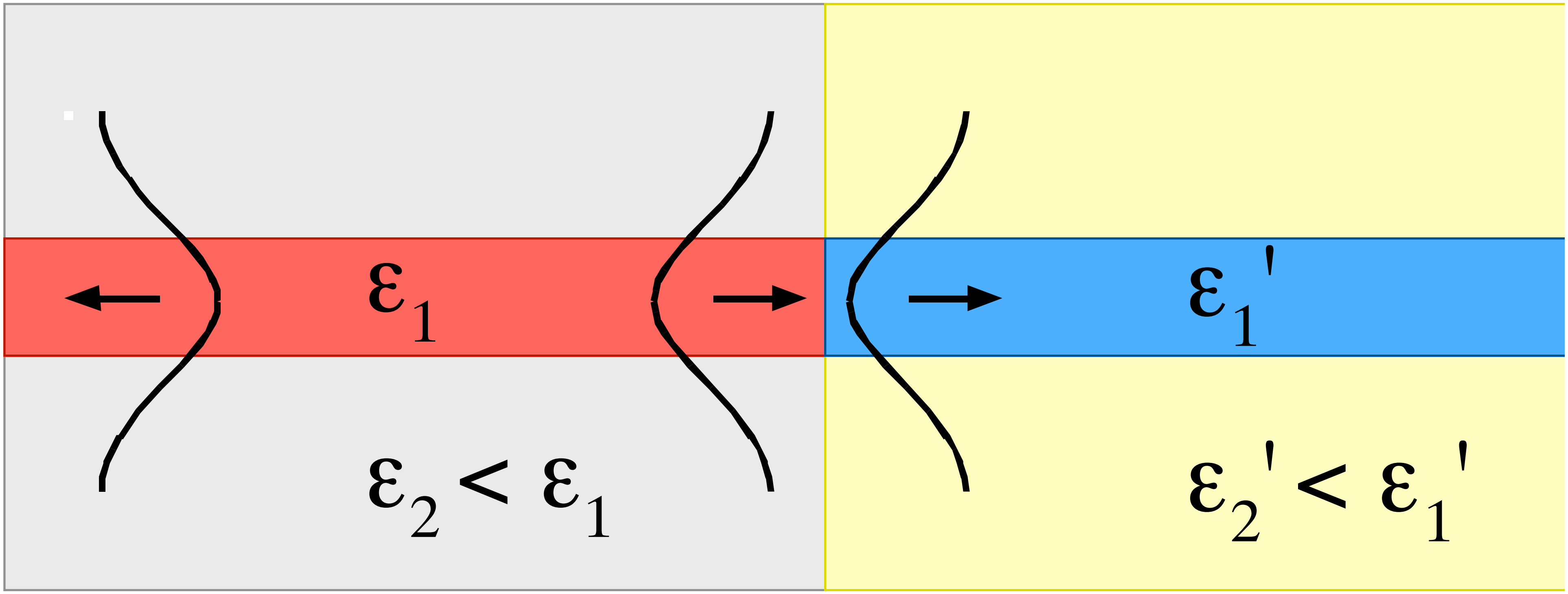
There is one other alternative…

[M. R. Watts *et al*., *Opt*. *Lett*. **27**, 1785 (2002)]

The Basic Idea, in 2d

start with:

junction of two waveguides

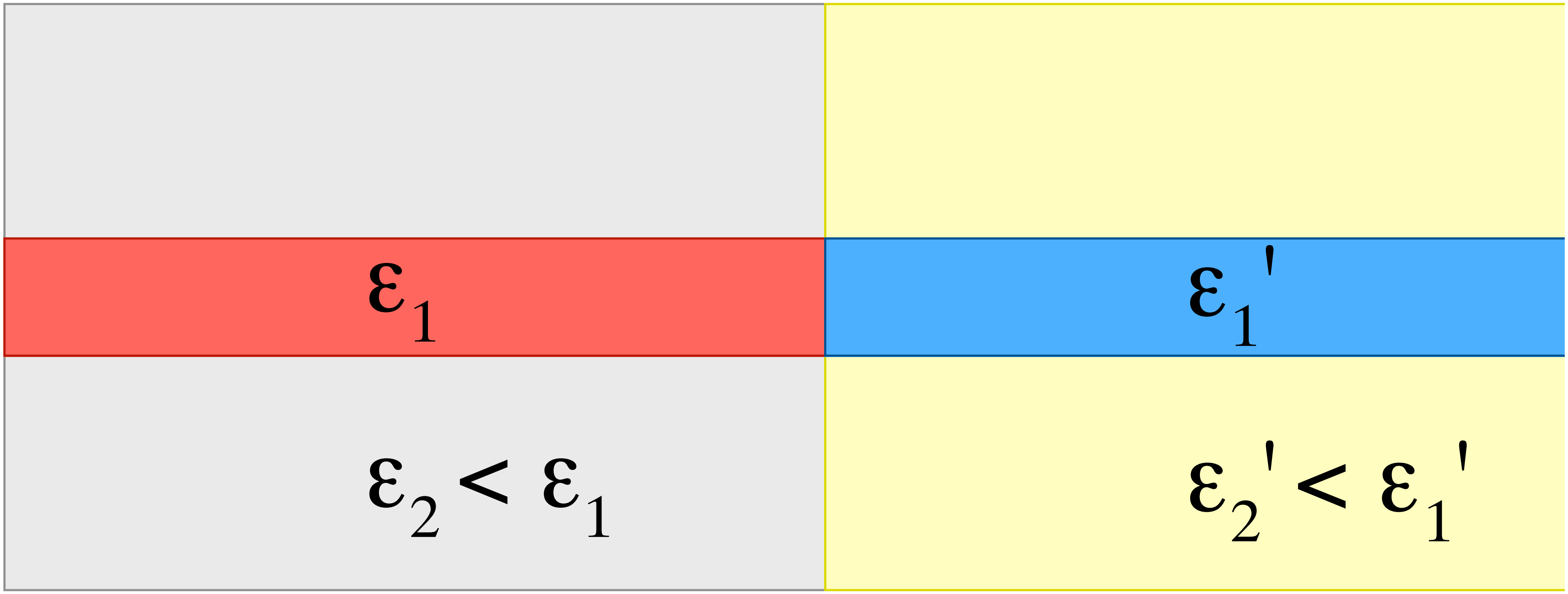


No radiation at junction

if the modes are perfectly matched

Perfect Mode Matching

requires:
same differential equations and boundary conditions



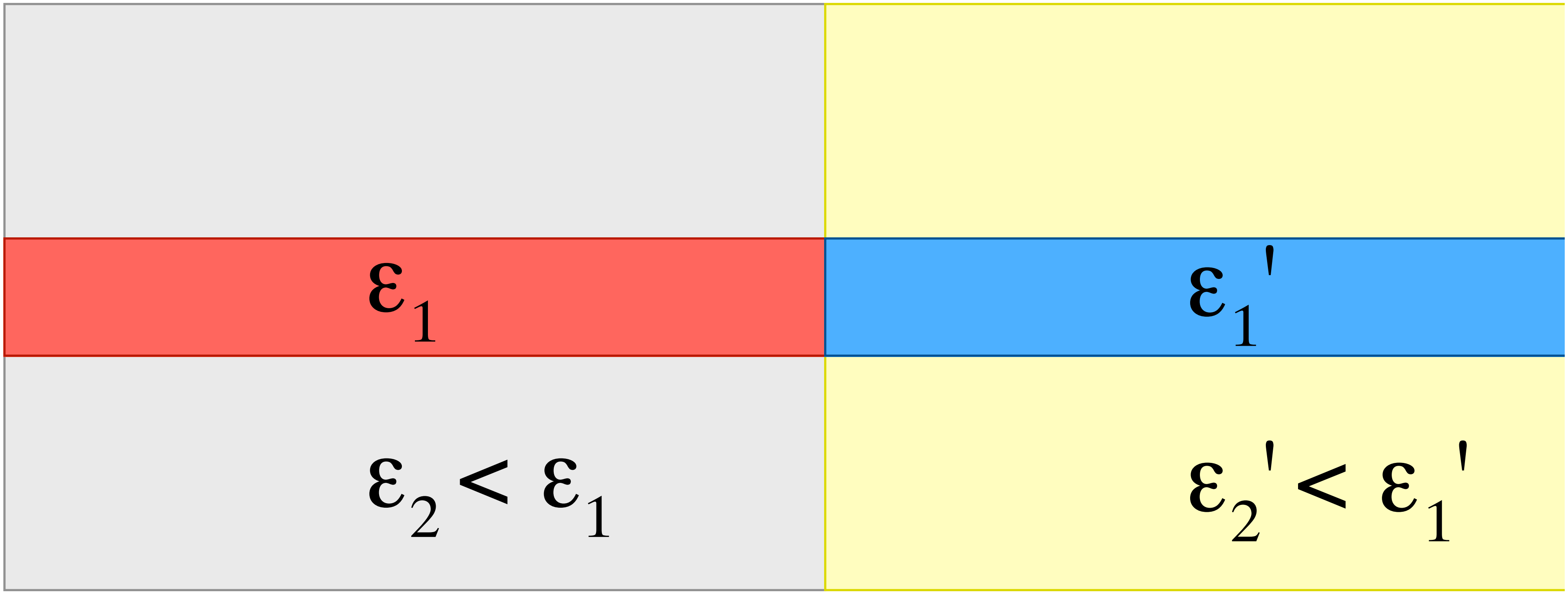
Match differential equations…

$$\varepsilon_2 - \varepsilon_1 = \varepsilon_2' - \varepsilon_1'$$

…closely related to separability
[S. Kawakami, *J. Lightwave Tech*. **20**, 1644 (2002)]

Perfect Mode Matching

requires:
same differential equations and boundary conditions



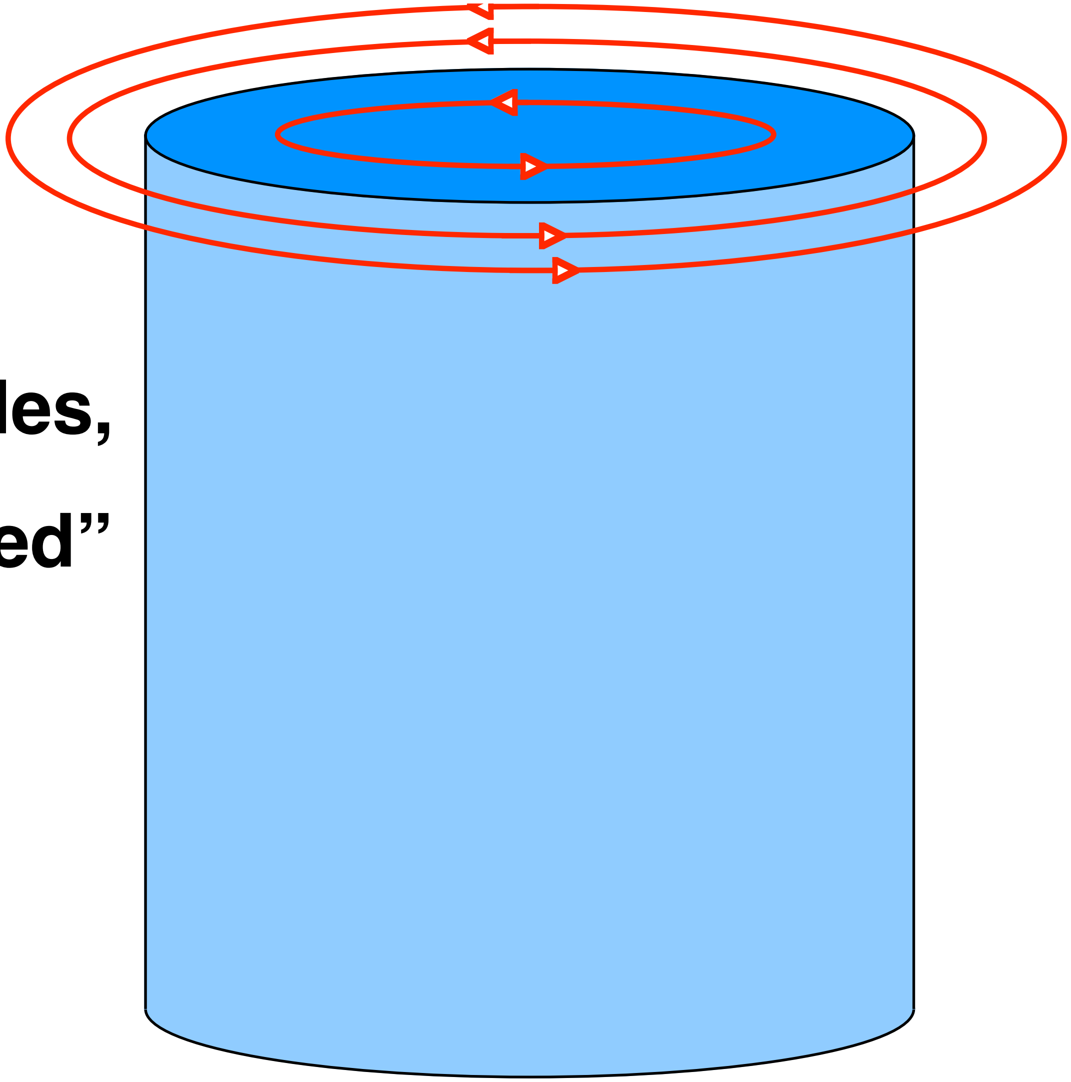
Match boundary conditions:

field must be TE

(*E* out of plane, in 2d)

(note switch in TE/TM convention)

TE modes in 3d



for

cylindrical waveguides,

“azimuthally polarized”

TE$_{0n}$ modes

A Perfect Cavity in 3d

(~ VCSEL + perfect lateral confinement)

Perfect index confinement

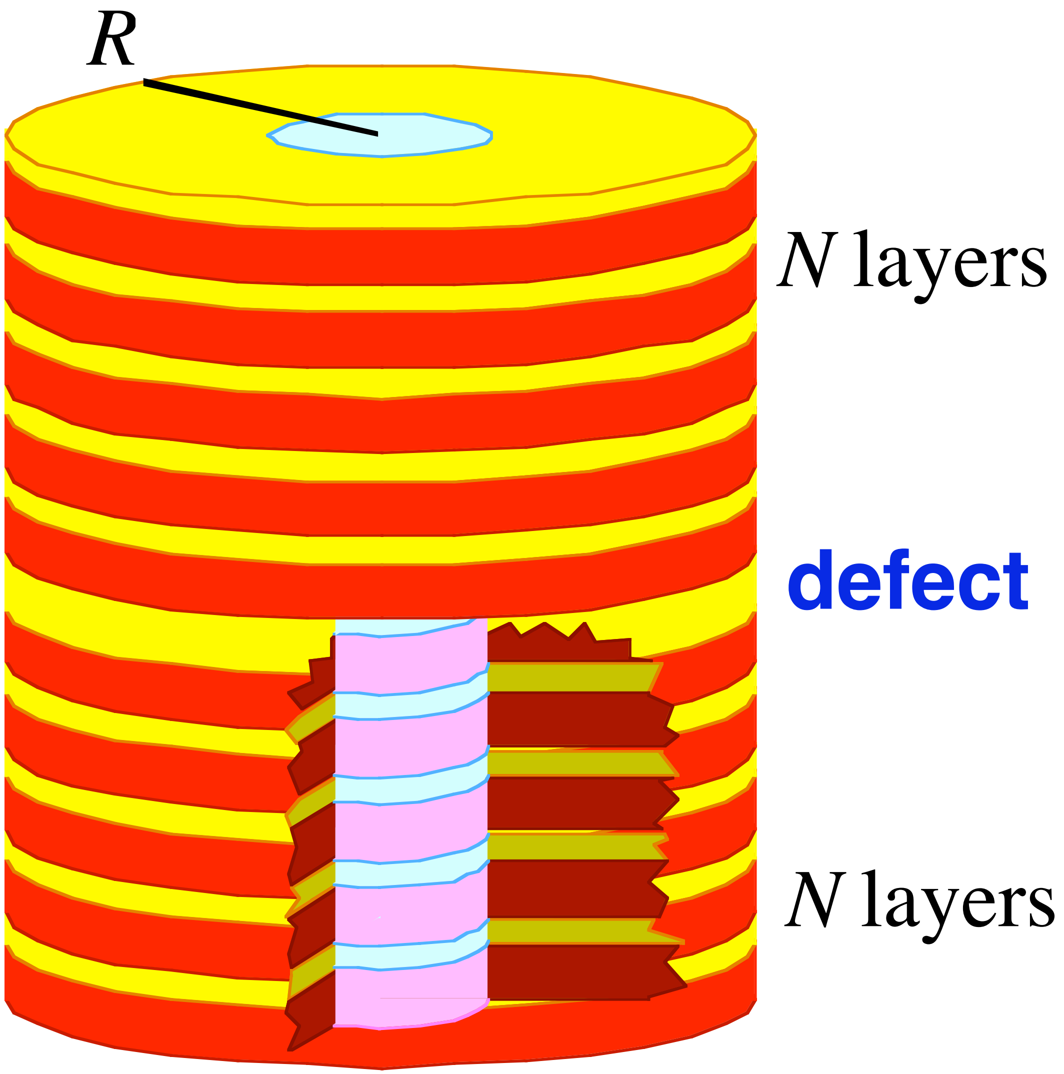
(no scattering)

+

1d band gap

=

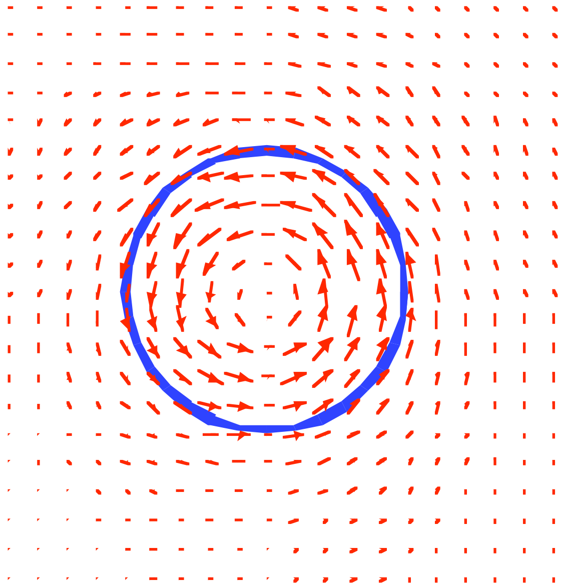
3d confinement

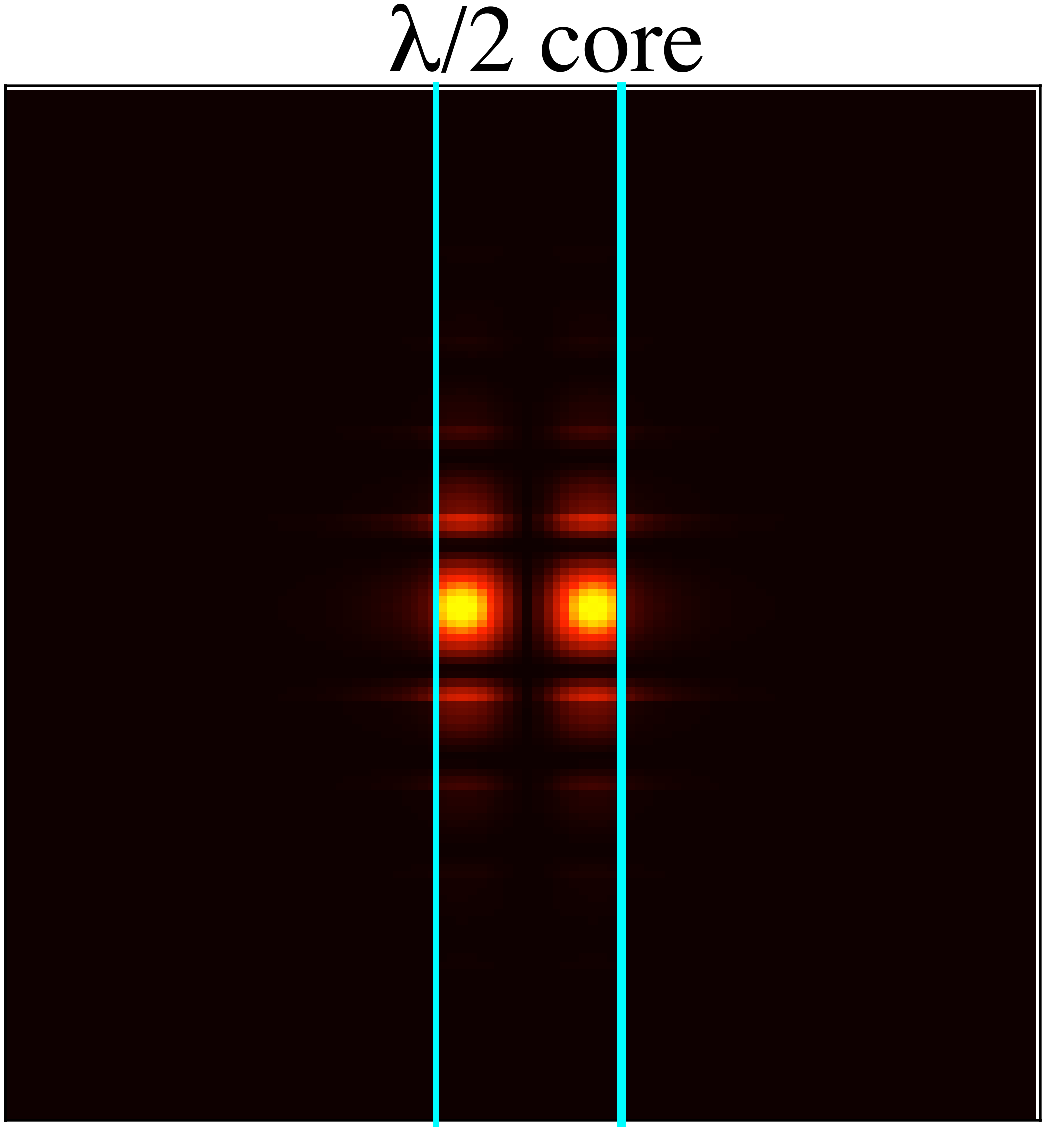


A Perfectly Confined Mode

$\varepsilon_1, \varepsilon_2 = 9, 6$

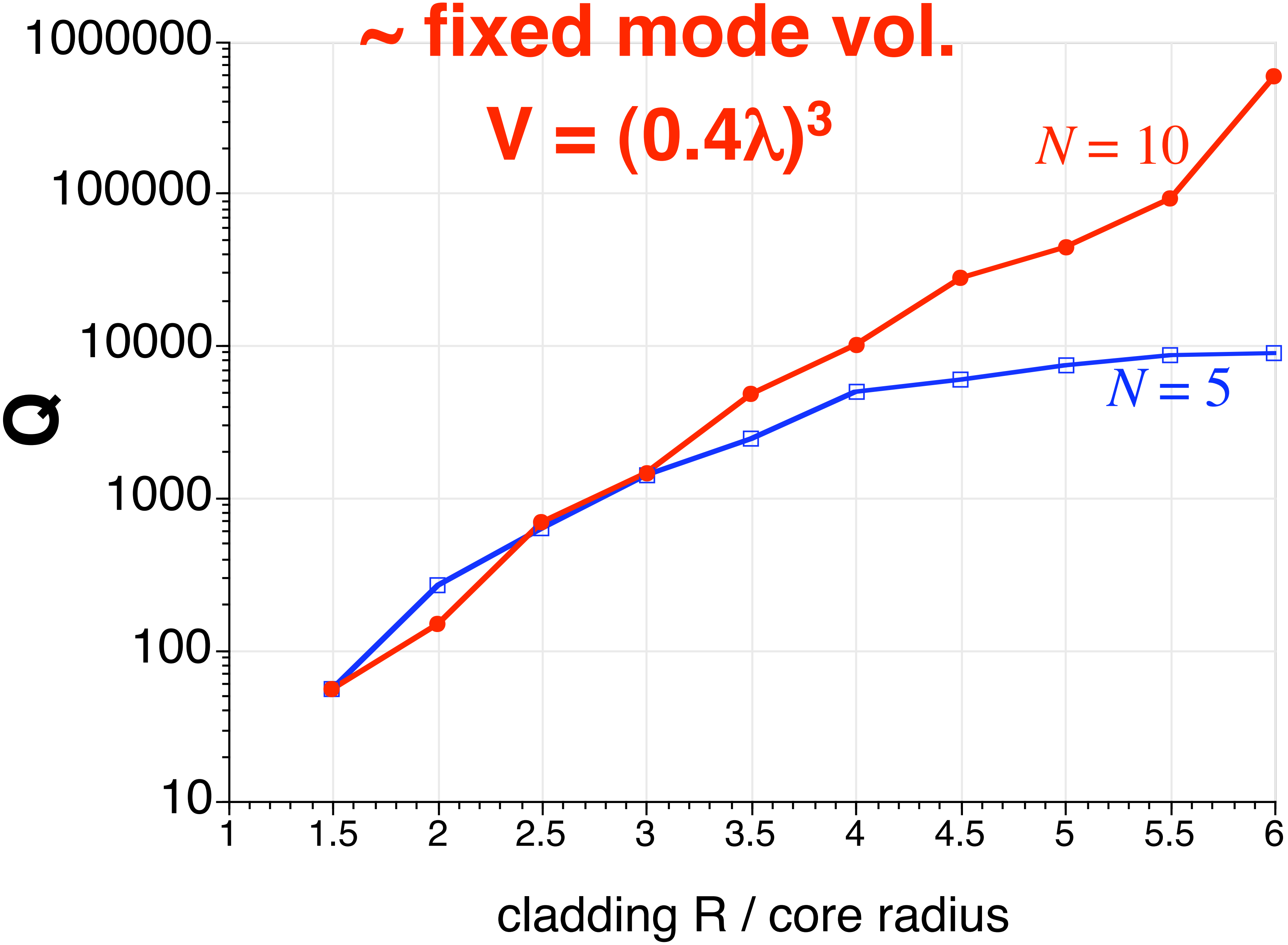
$\varepsilon_1', \varepsilon_2' = 4, 1$





E energy density, vertical slice

Q limited only by finite size



Q-tips

Three independent mechanisms for high Q:

Delocalization: trade off modal size for Q

Q_r *grows monotonically towards band edge*

Multipole Cancellation: force higher-order far-field pattern

Q_r *peaks* ***inside*** *gap*

New nodal planes appear in far-field pattern at peak

Mode Matching: allows arbitrary Q, finite V

Requires special symmetry & materials

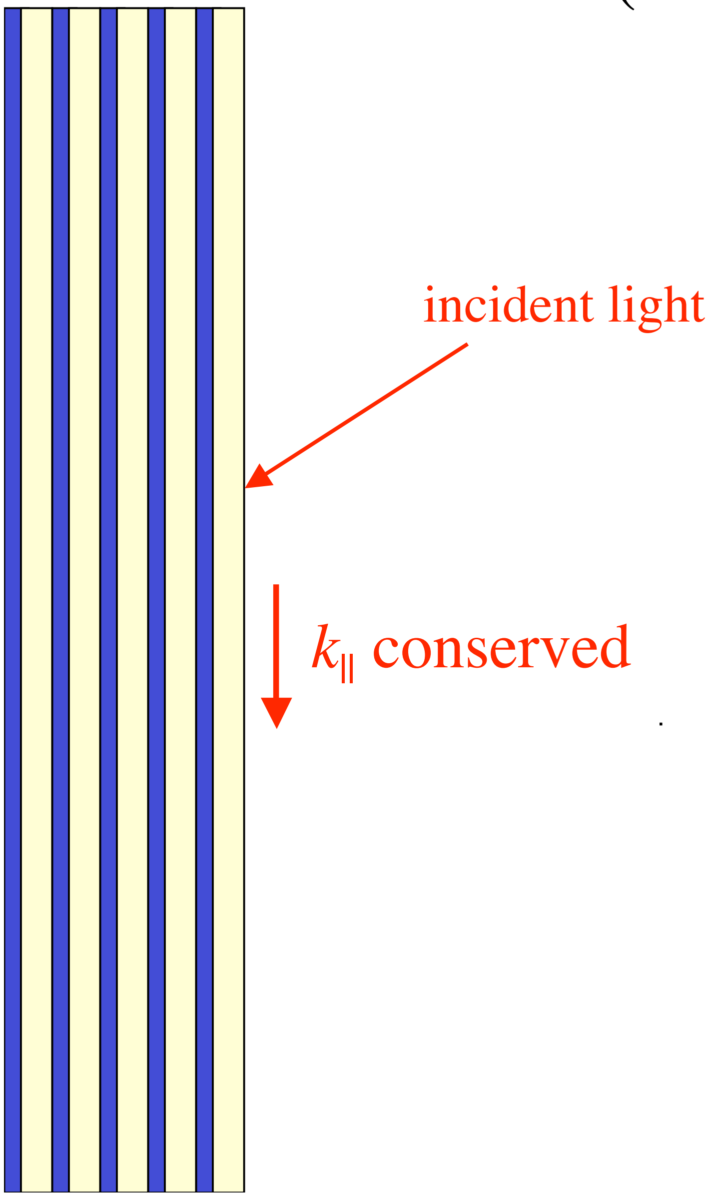
Forget these devices…

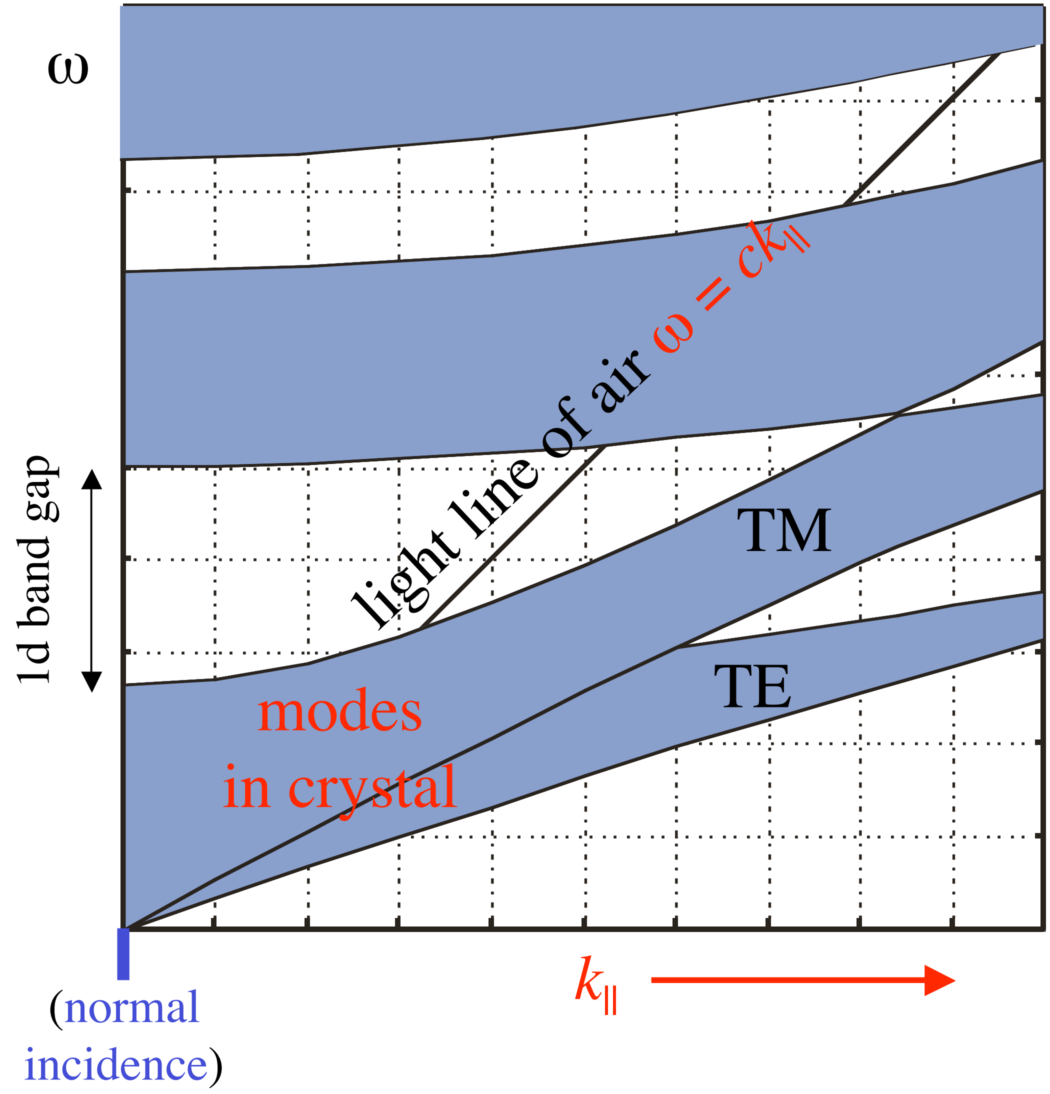
I just want a mirror.

ok

Projected Bands of a 1d Crystal

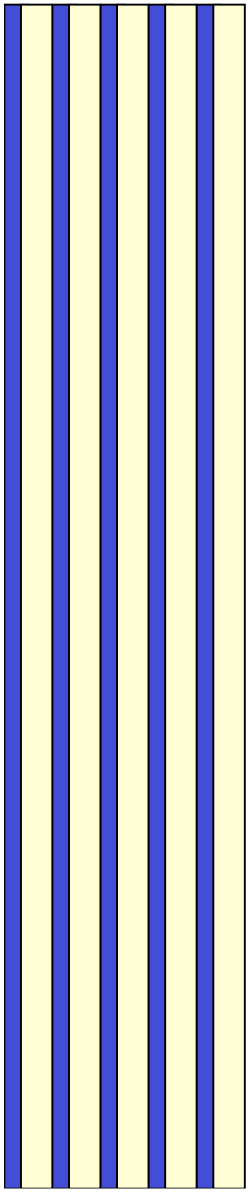
(a.k.a. a Bragg mirror)

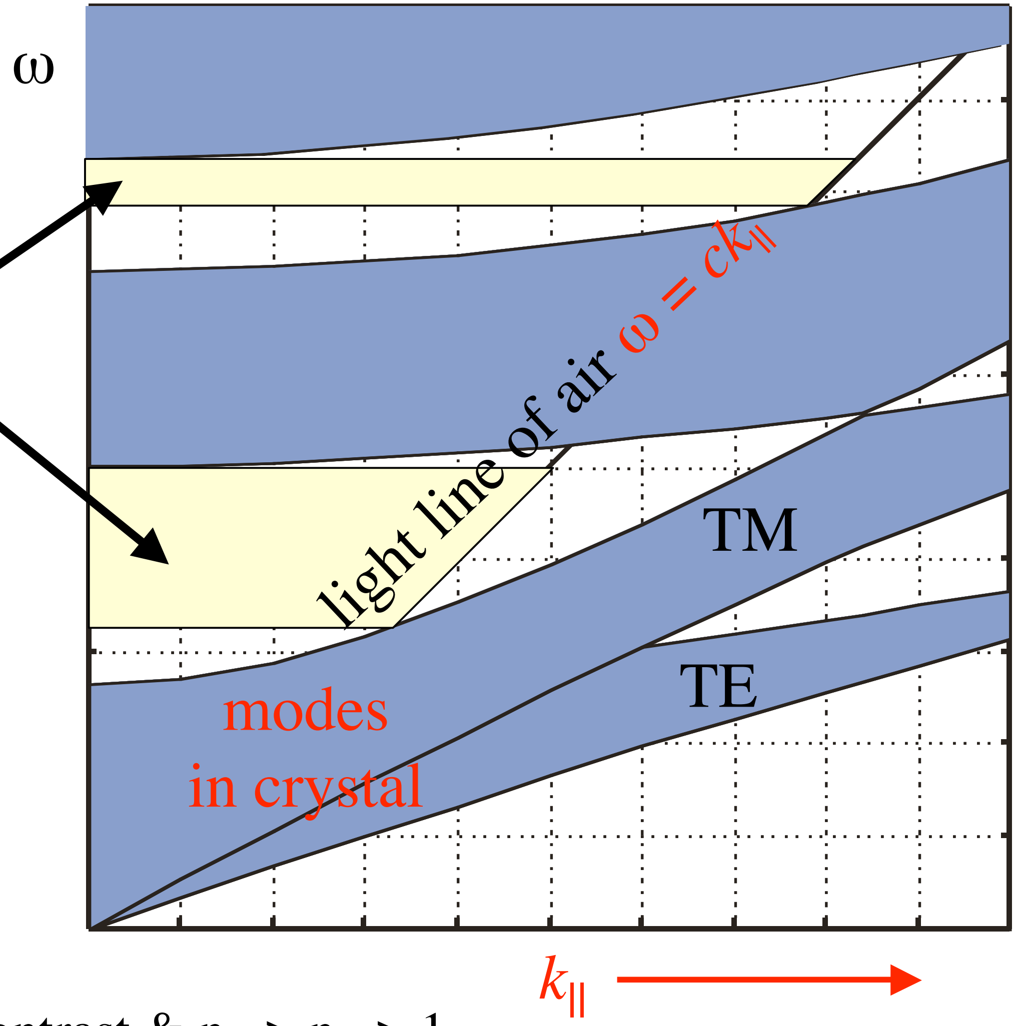




Omnidirectional Reflection

[J. N. Winn *et al*, *Opt. Lett*. **23**, 1573 (1998)]





in these ω ranges,
there is
no overlap
between modes
of air & crystal

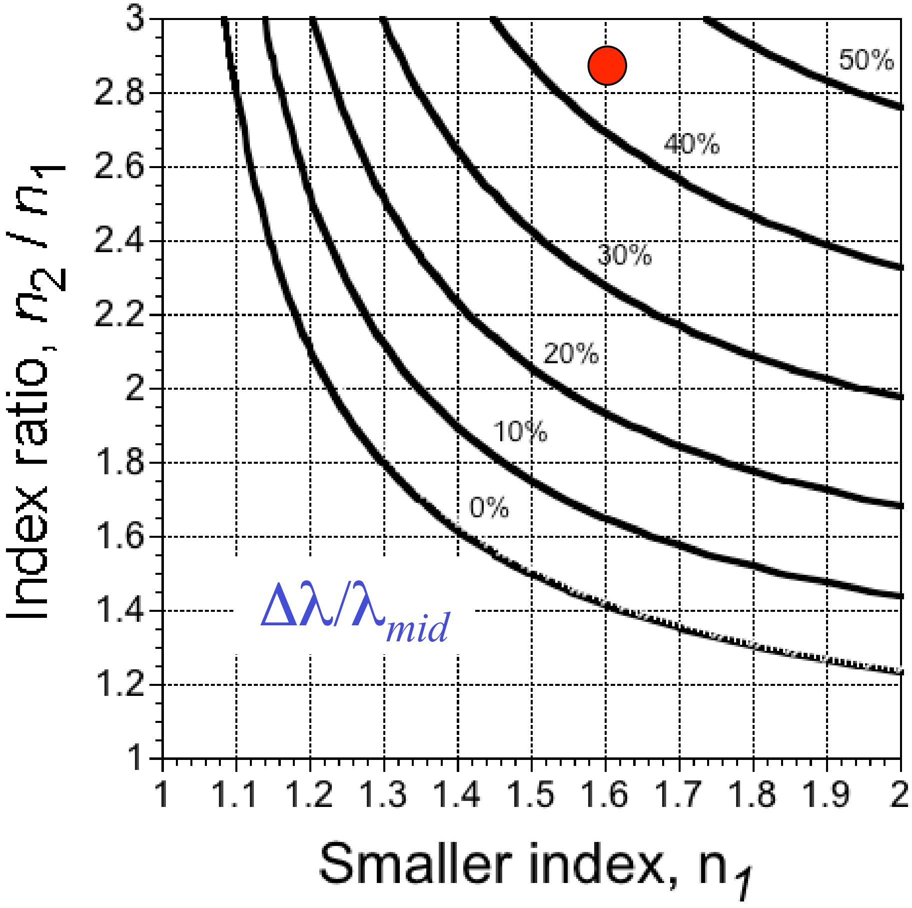
all incident light
(any angle, polarization)
is reflected
from flat surface

needs: sufficient index contrast & $n_{hi} > n_{lo} > 1$

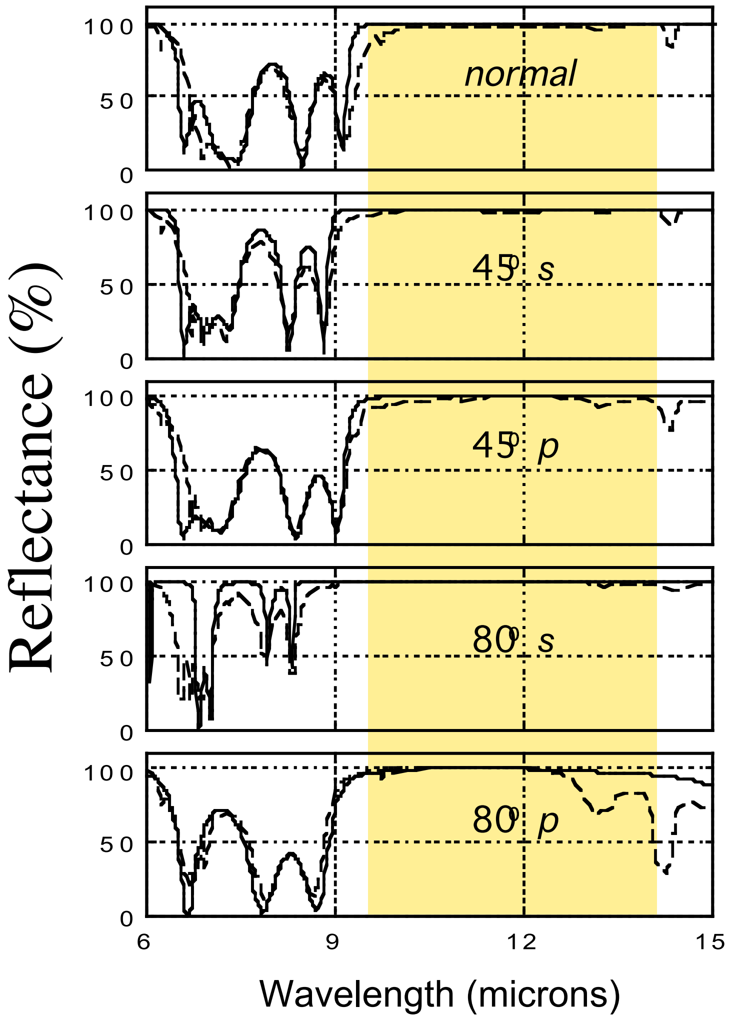
Omnidirectional Mirrors in Practice

[Y. Fink *et al*, *Science* **282**, 1679 (1998)]

contours of omnidirectional gap size



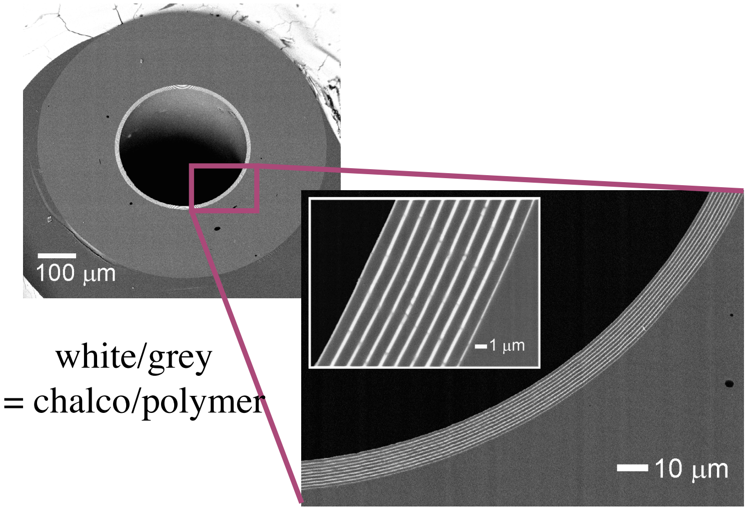
● Te / polystyrene



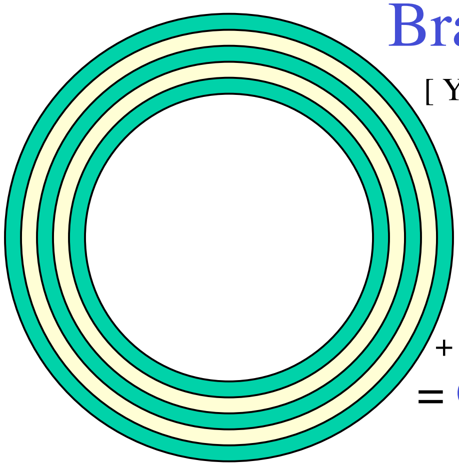
Another route to three dimensions:

Hollow-core Bandgap Fibers

[figs courtesy
Y. Fink *et al.*, MIT]

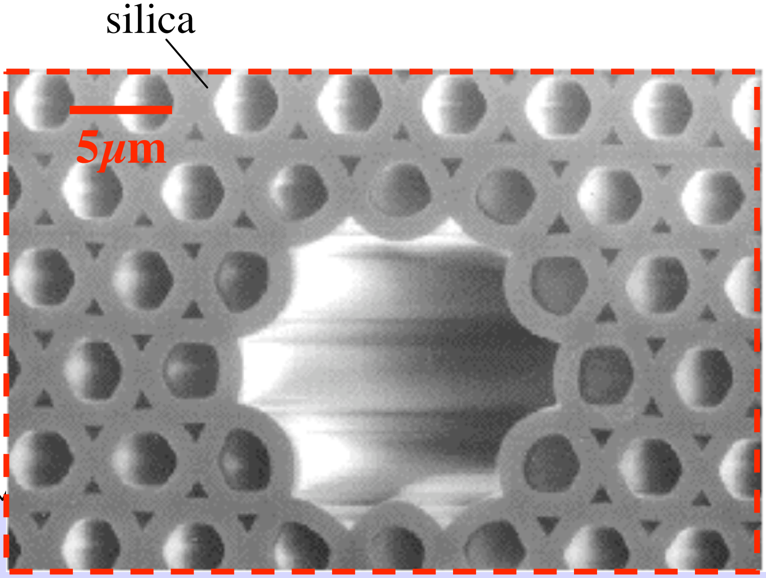


white/grey
= chalco/polymer

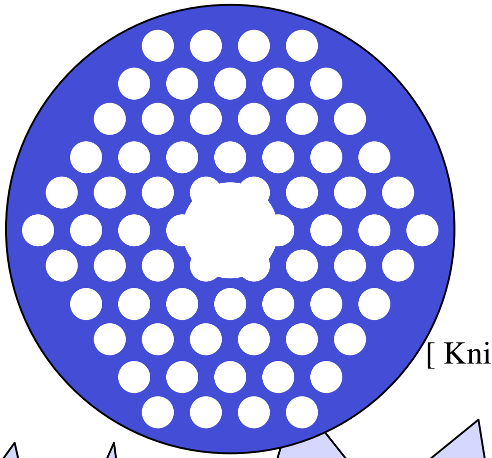


Bragg fiber
[Yeh *et al.*, 1978]

+ omnidirectional
= OmniGuide
fibers

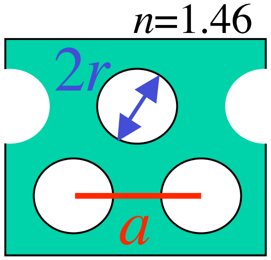


[R. F. Cregan
et al.,
Science **285**,
1537 (1999)]

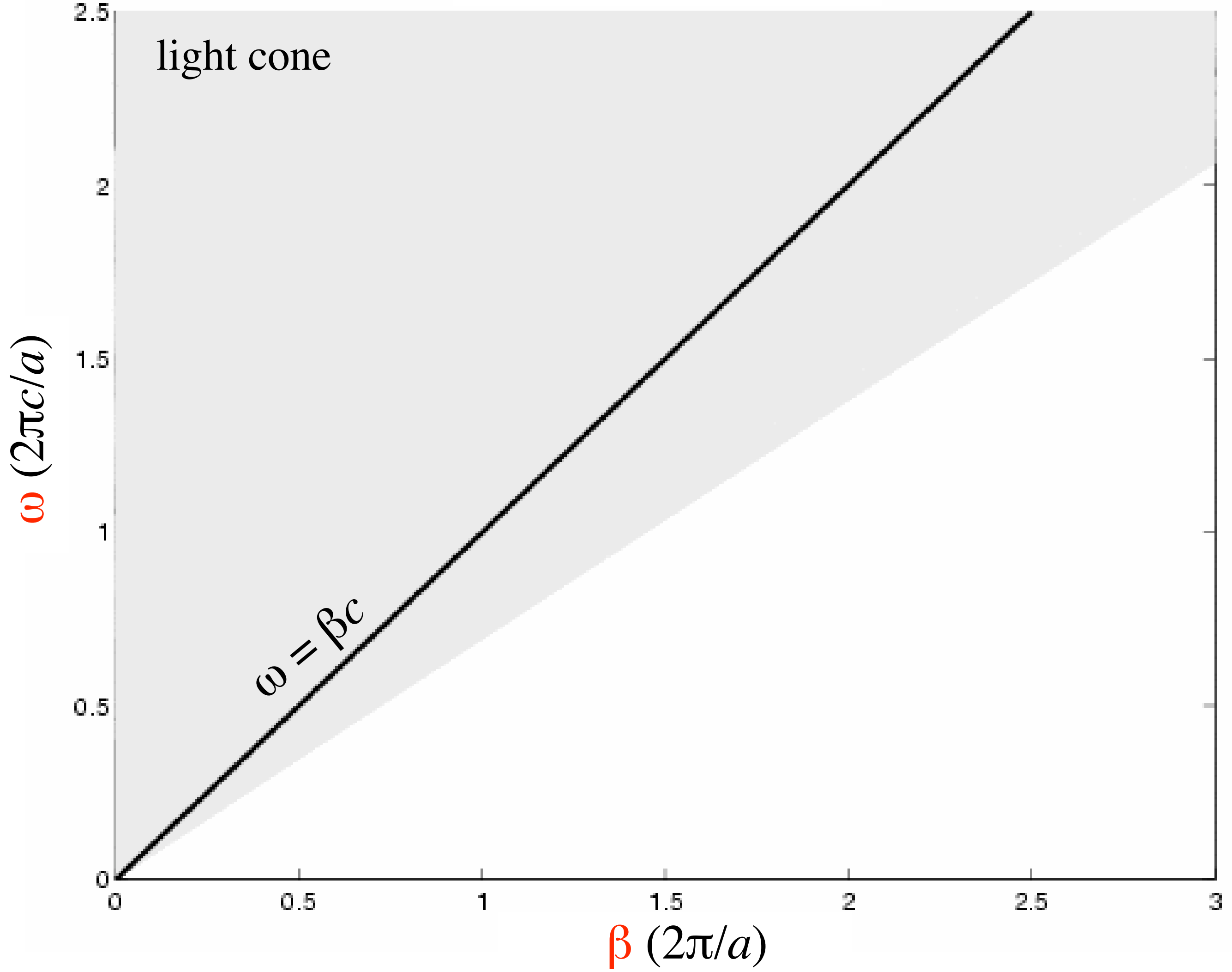


PCF
[Knight *et al.*, 1998]

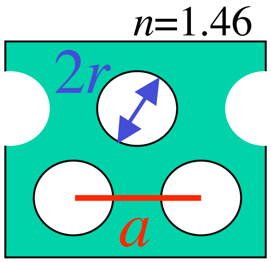
PCF: Holey Silica Cladding



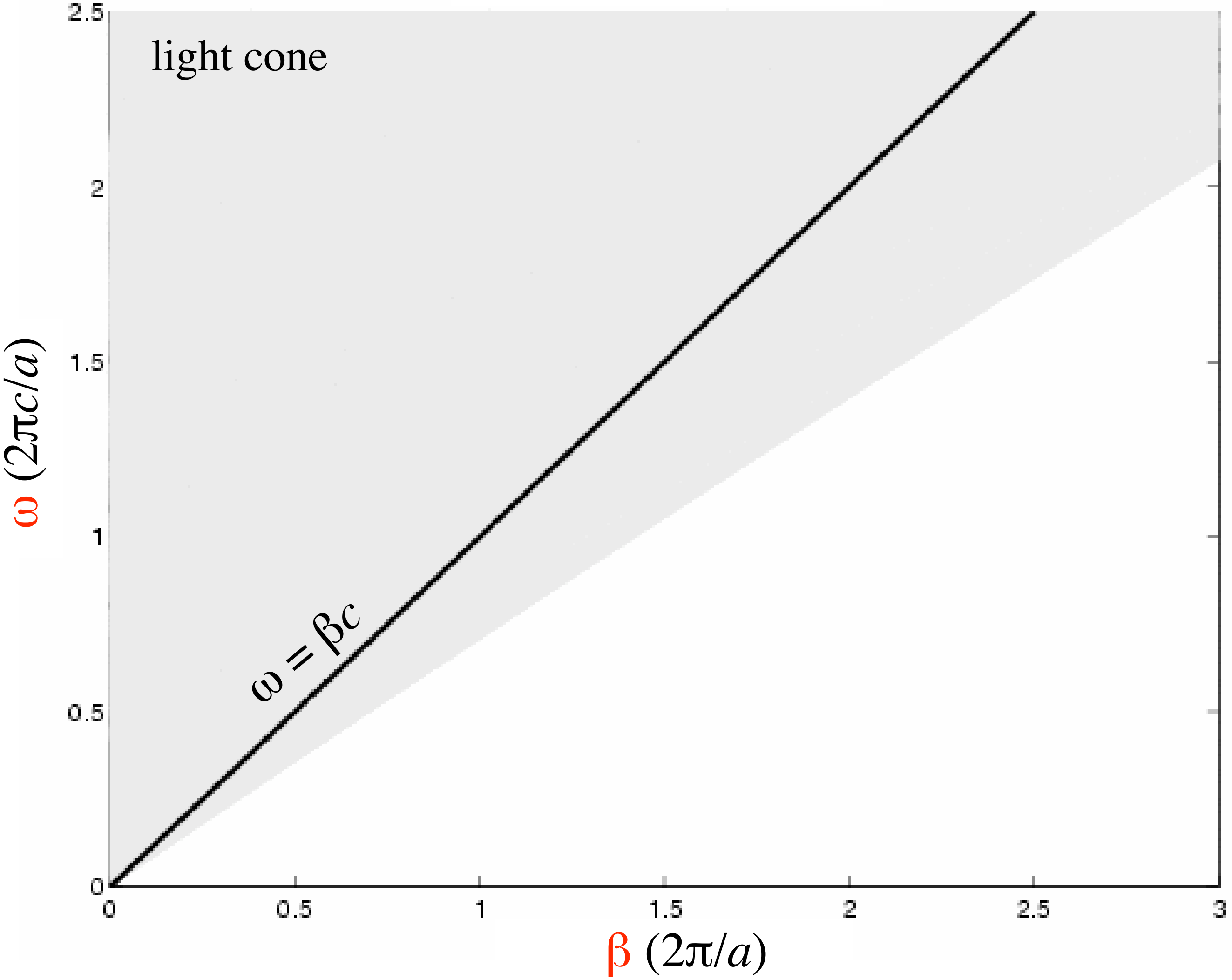
$r = 0.1a$



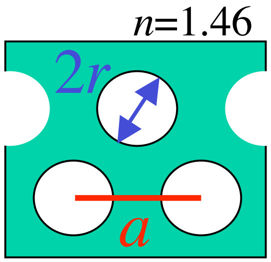
PCF: Holey Silica Cladding



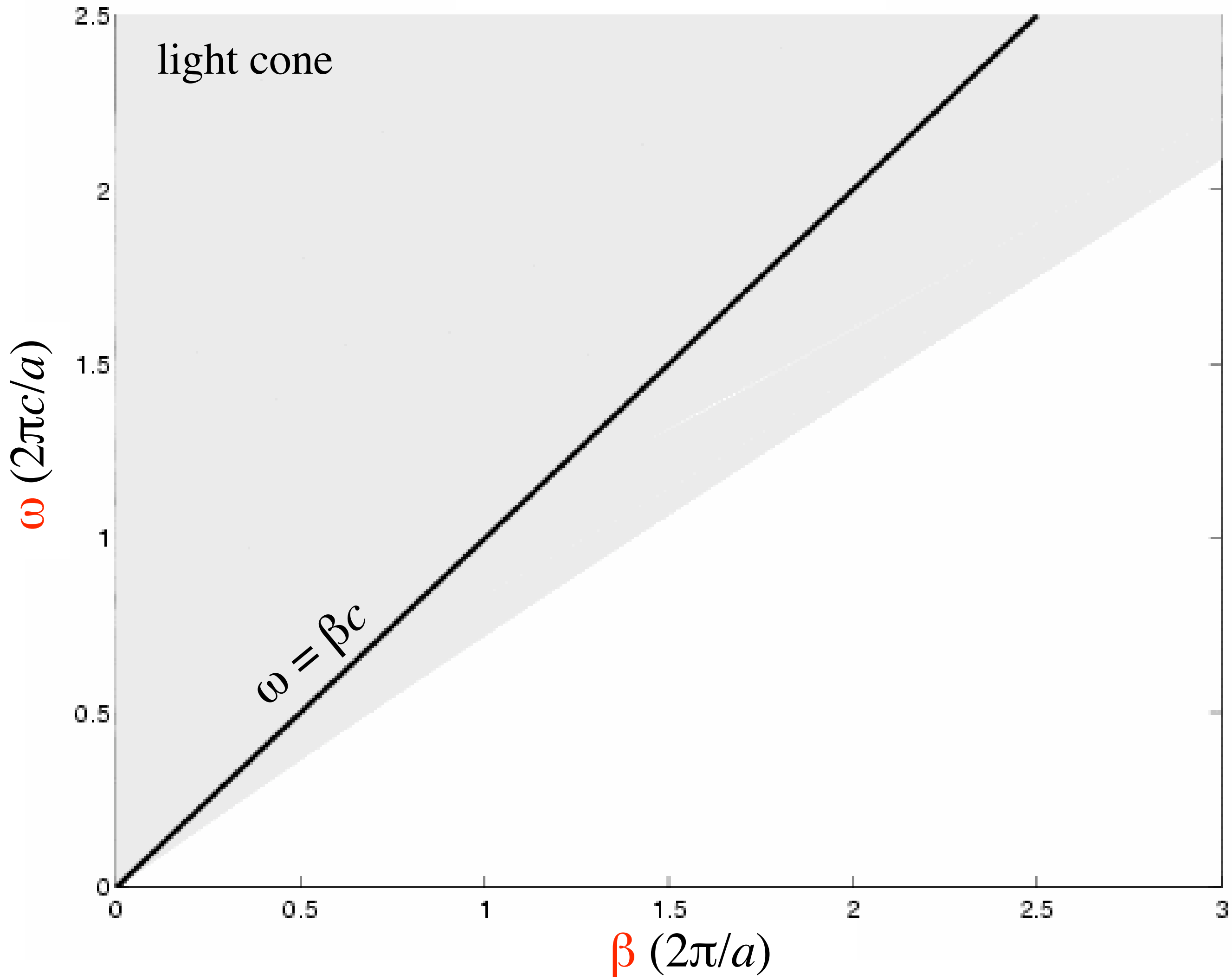
$r = 0.17717a$



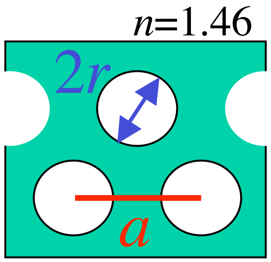
PCF: Holey Silica Cladding



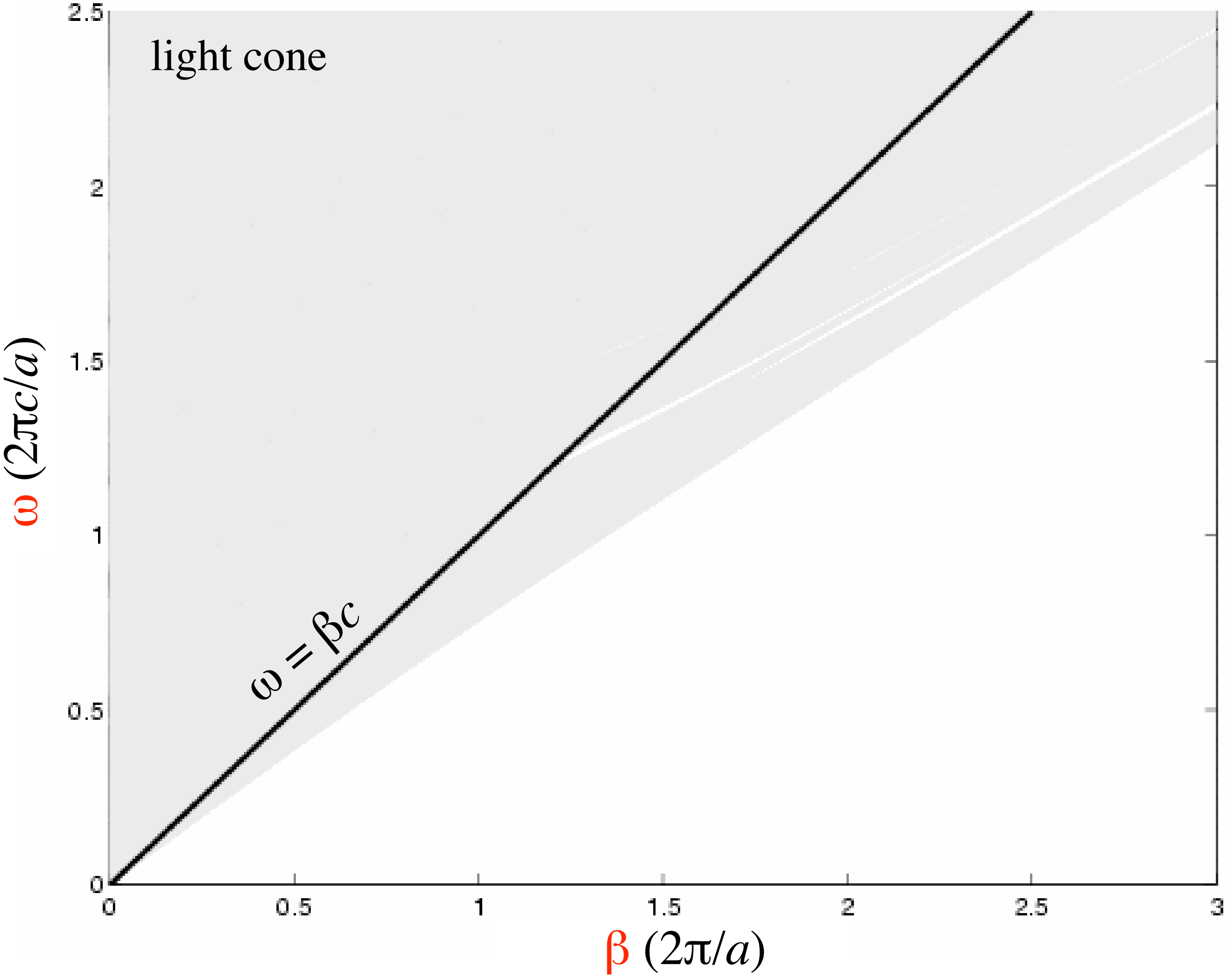
$r = 0.22973a$



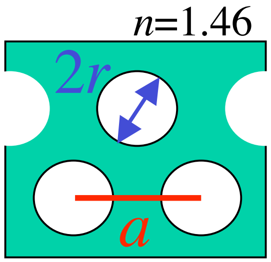
PCF: Holey Silica Cladding



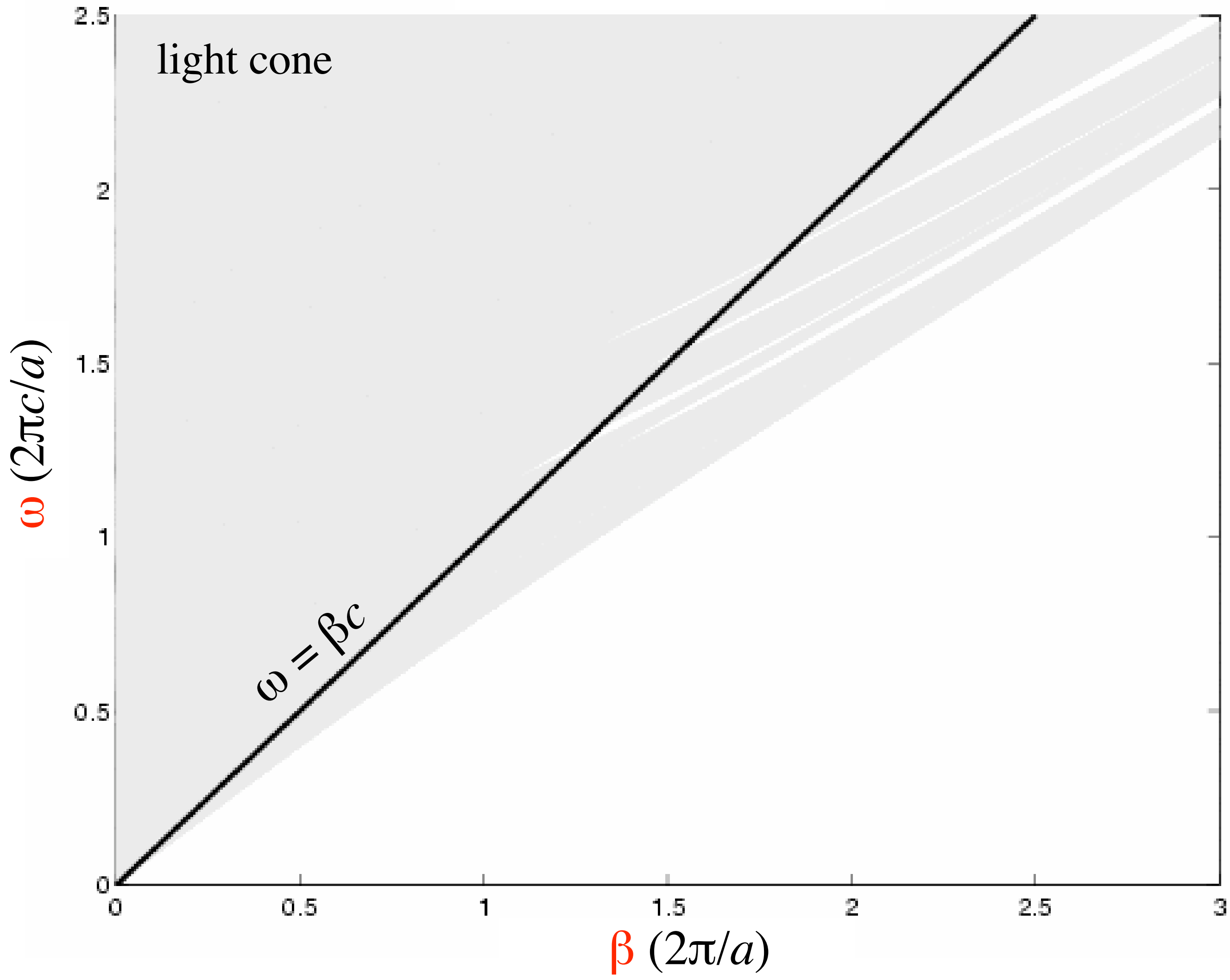
r = 0.30912a



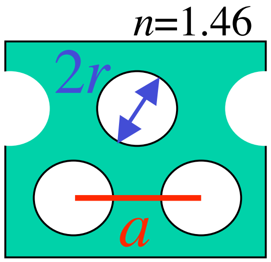
PCF: Holey Silica Cladding



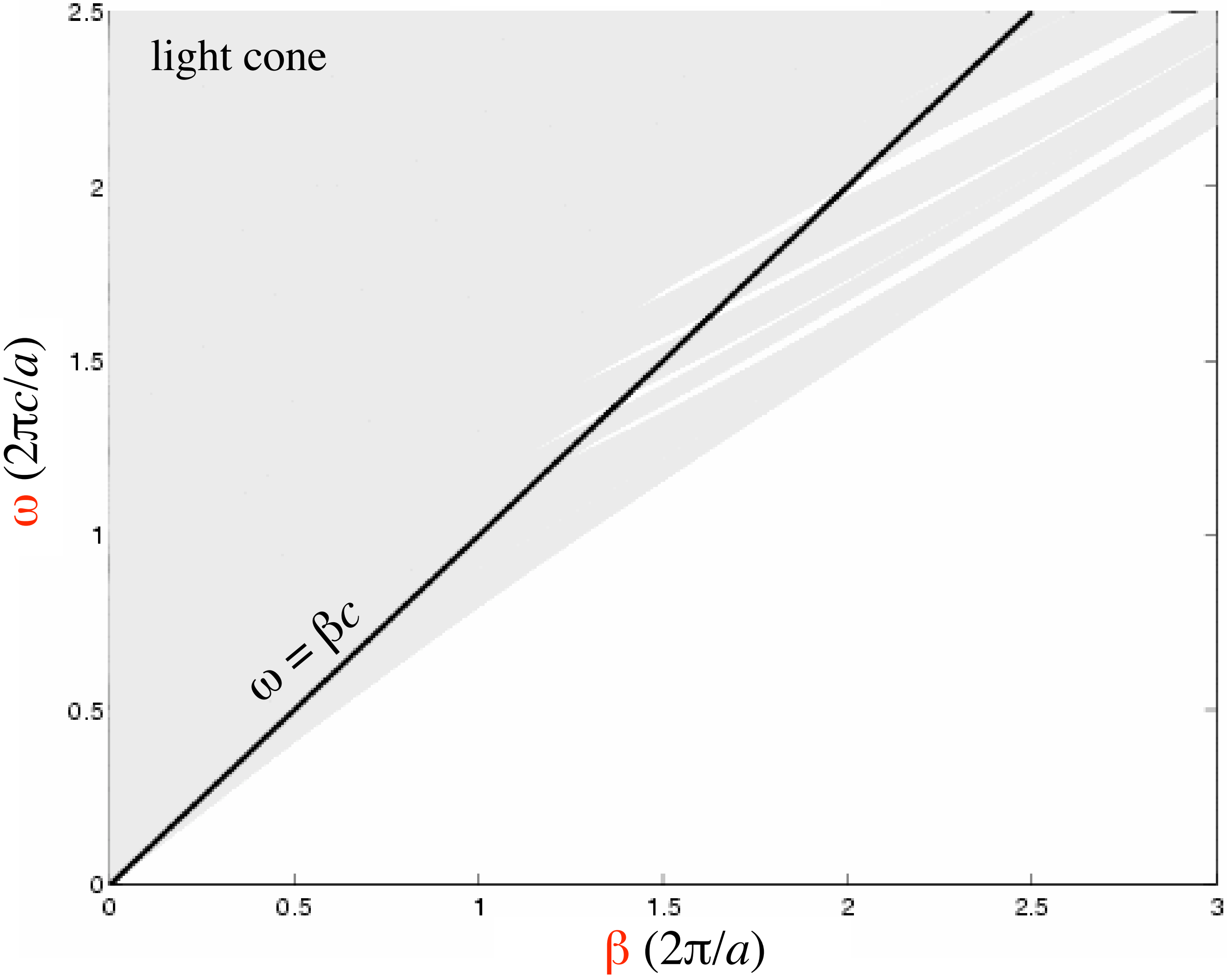
$r = 0.34197a$



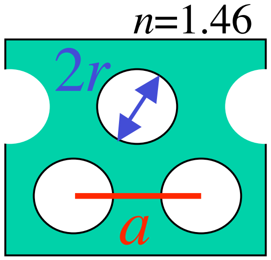
PCF: Holey Silica Cladding



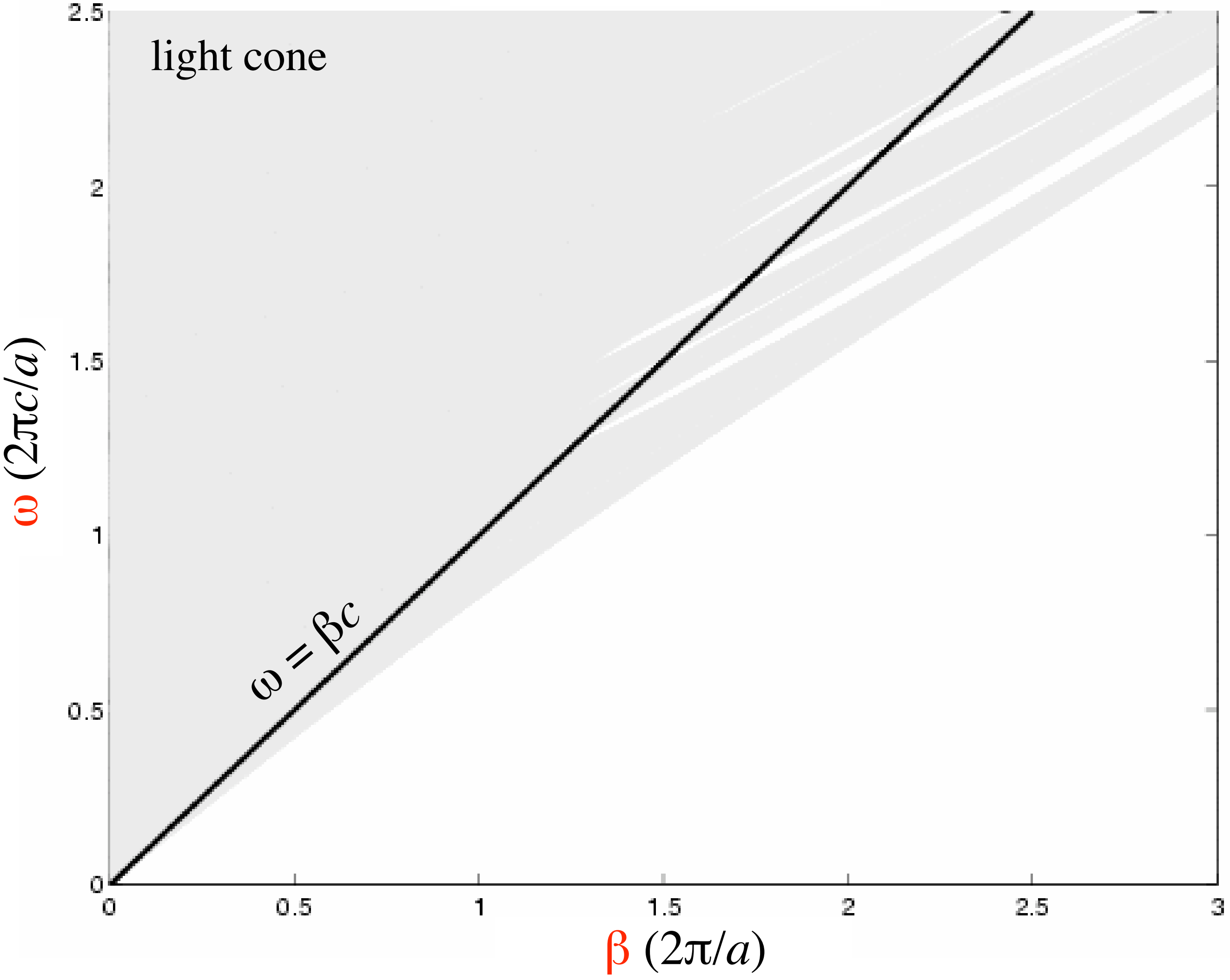
$r = 0.37193a$



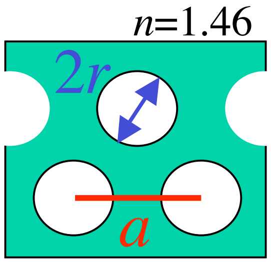
PCF: Holey Silica Cladding



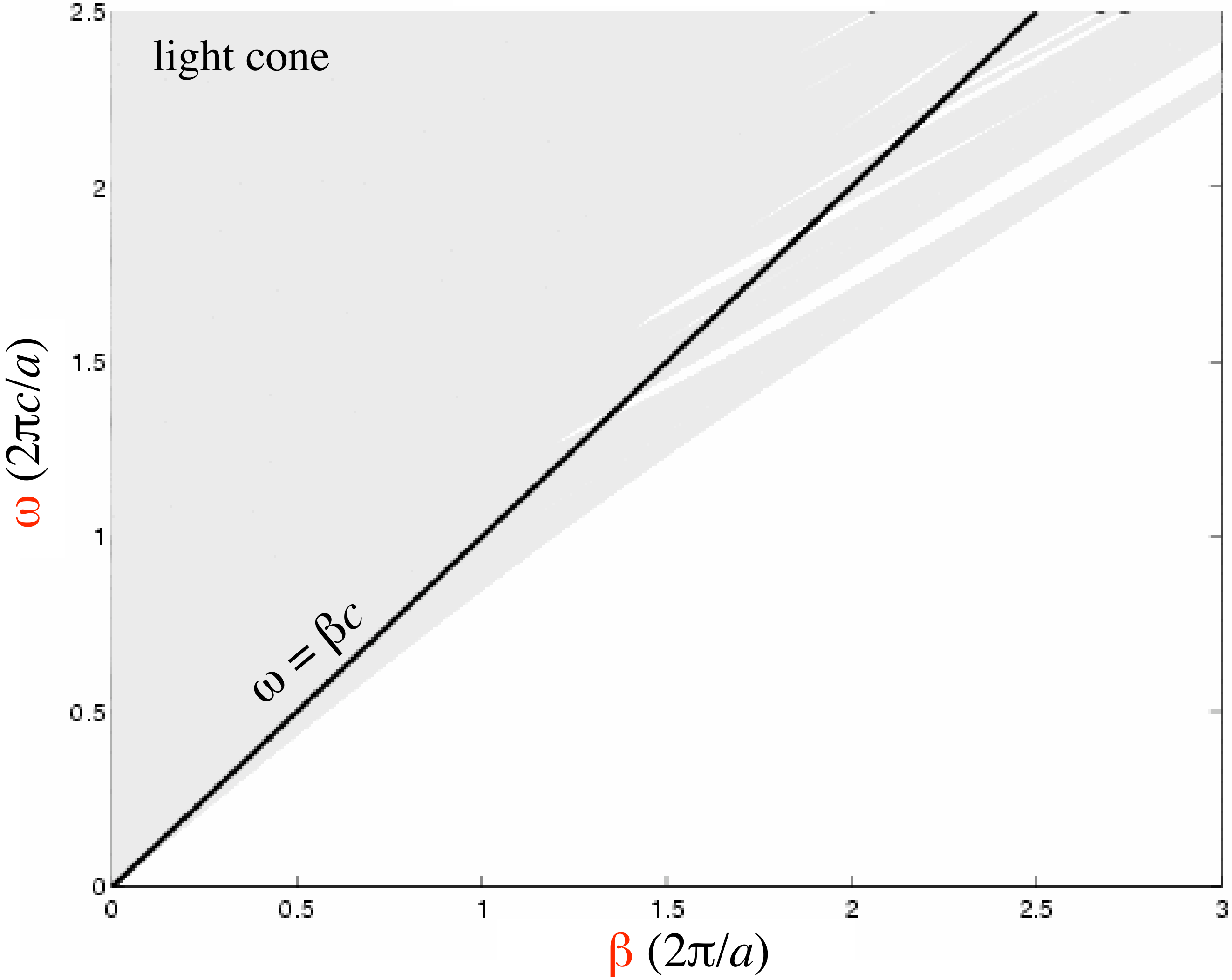
$r = 0.4a$



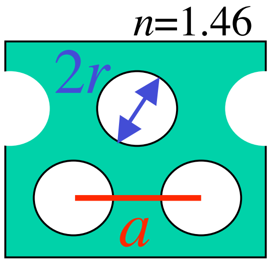
PCF: Holey Silica Cladding



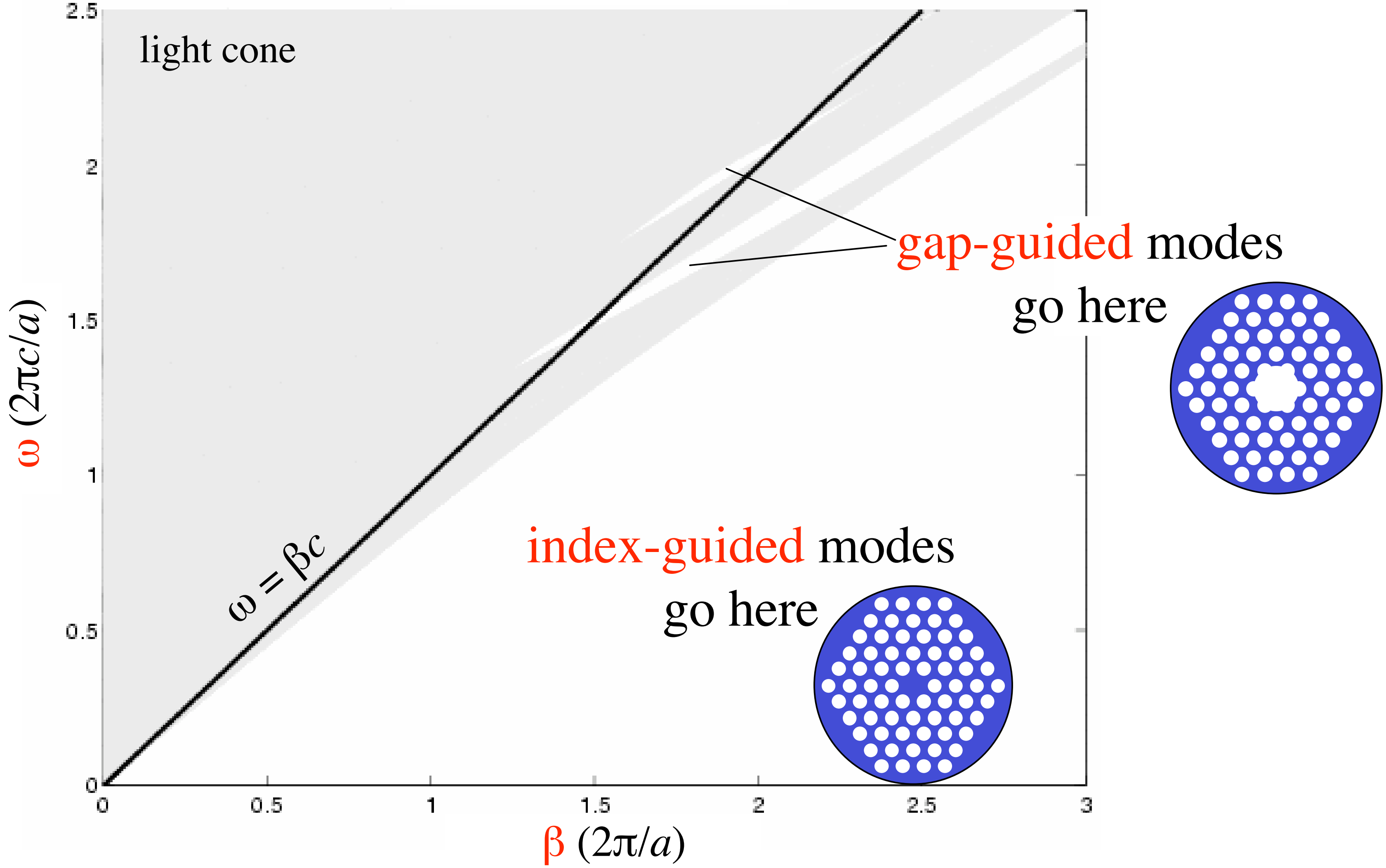
$r = 0.42557a$



PCF: Holey Silica Cladding



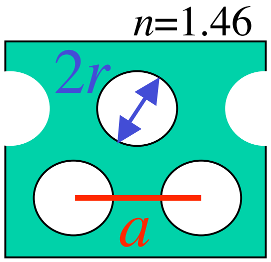
$r = 0.45a$



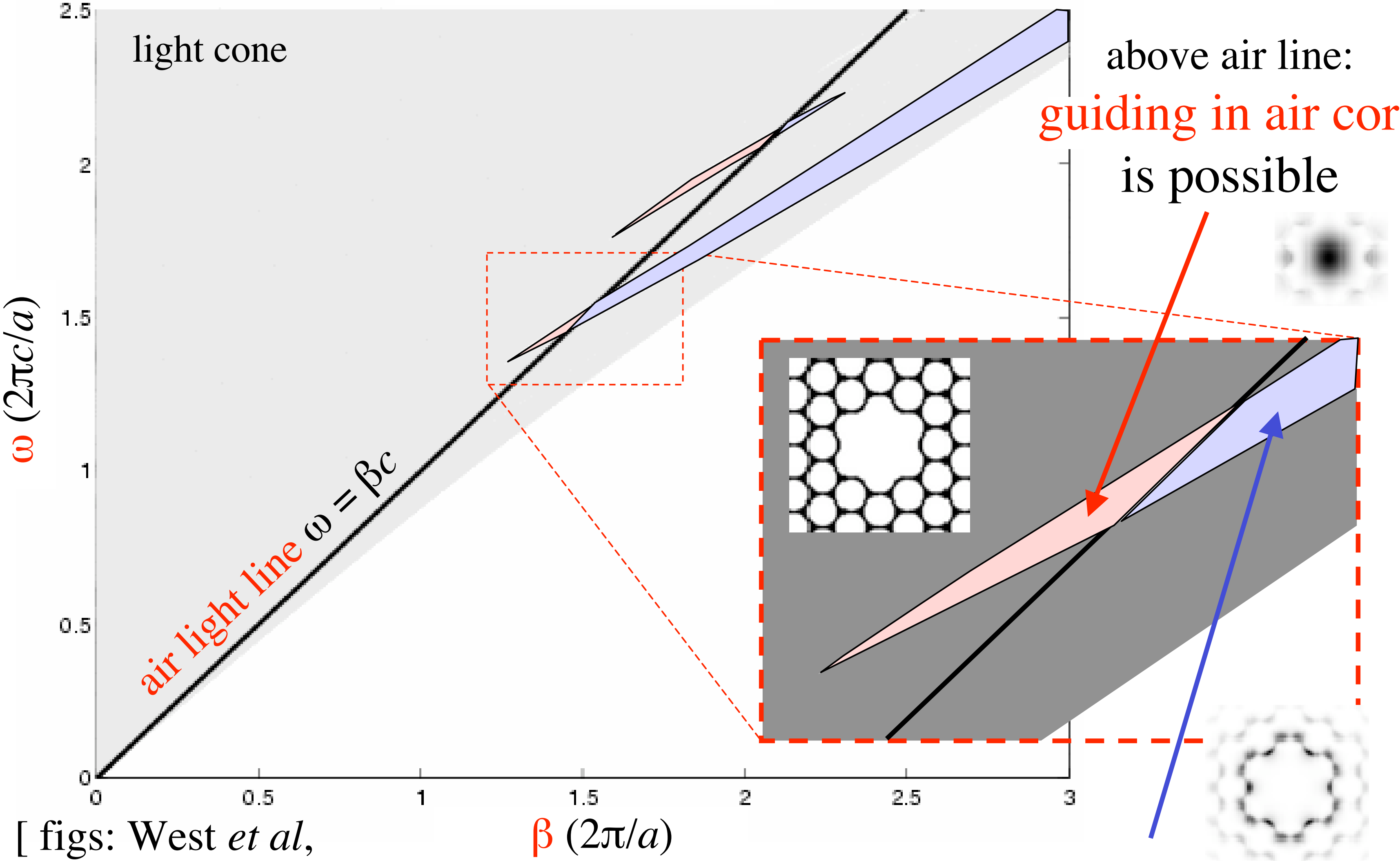
gap-guided modes go here

index-guided modes go here

PCF: Holey Silica Cladding



$r = 0.45a$



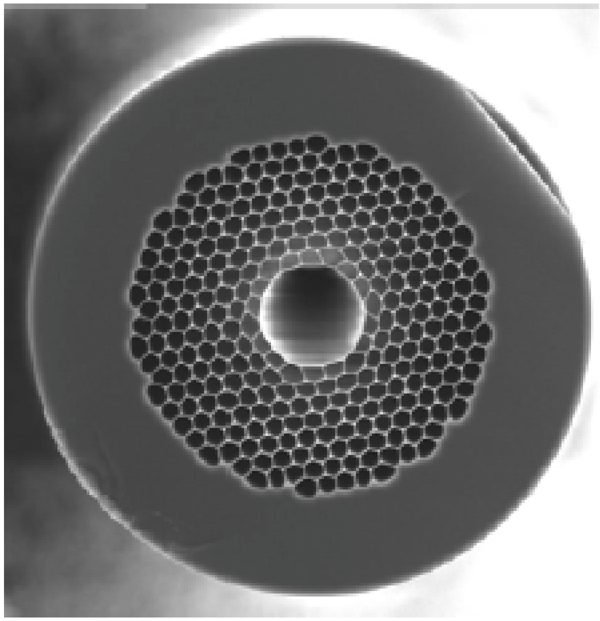
above air line:
guiding in air core
is possible

[figs: West *et al*,
Opt. Express **12** (8), 1485 (2004)]

below air line: surface states of air core

Bandgap fibers: Air-guiding records

1.7dB/km @1.57μm



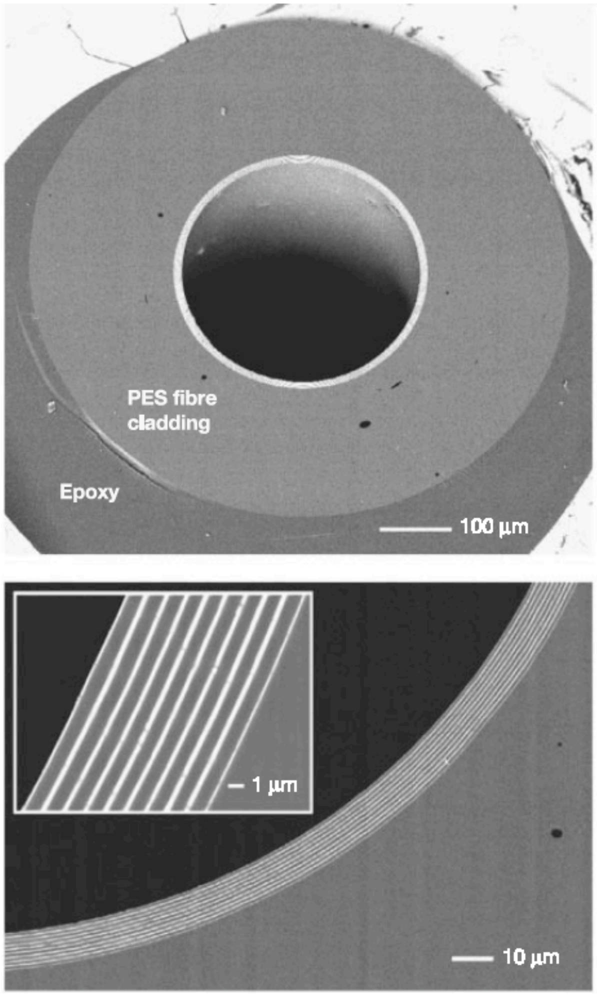
[Mangan, *et al.*, OFC 2004 PDP24]

Not limited by material absorption!

(at 10.6μm, absorption > 10^4 dB/m!)

(still lots of work until
0.2dB/km of conventional fiber)

0.5 dB/m @10.6μm



[B. Temelkuran *et al.*, *Nature* **420**, 650 (2002)]
[C. Anastassiou *et al.*, *Phot. Spectra* (Mar. 2004)]

Outline

- Preliminaries: waves in periodic media
- Photonic crystals in theory and practice
- Bulk crystal properties
- Intentional defects and devices
- Index-guiding and incomplete gaps
- Perturbations, tuning, and disorder

All Imperfections are Small

(or the device wouldn't work)

- Material absorption: small imaginary $\Delta\varepsilon$
- Nonlinearity: small $\Delta\varepsilon \sim |\mathbf{E}|^2$ (Kerr)
- Stress (MEMS): small $\Delta\varepsilon$ or small ε boundary shift
- Tuning by thermal, electro-optic, etc.: small $\Delta\varepsilon$
- Roughness: small $\Delta\varepsilon$ or boundary shift

Weak effects, long distance/time: hard to compute directly

— use semi-analytical methods

Semi-analytical methods for small perturbations

- Brute force methods (FDTD, *etc*.):
  expensive and give limited insight

- Semi-analytical methods
  — numerical solutions for perfect system
  + analytically bootstrap to imperfections

… coupling-of-modes, perturbation theory,
Green's functions, coupled-wave theory, …

Perturbation Theory

for Hermitian eigenproblems

given eigenvectors/values: $\hat{O}|u\rangle = u|u\rangle$

…find change Δu & $\Delta|u\rangle$ for small $\Delta\hat{O}$

Solution:

expand as power series in $\Delta\hat{O}$

$$\Delta u = 0 + \Delta u^{(1)} + \Delta u^{(2)} + \ldots$$

& $\Delta|u\rangle = 0 + \Delta|u\rangle^{(1)} + \ldots$

$$\boxed{\Delta u^{(1)} = \frac{\langle u|\Delta\hat{O}|u\rangle}{\langle u|u\rangle}}$$

(first order is usually enough)

Perturbation Theory

for electromagnetism

$$\Delta\omega^{(1)} = \frac{c^2}{2\omega} \frac{\langle \mathbf{H} | \Delta\hat{A} | \mathbf{H} \rangle}{\langle \mathbf{H} | \mathbf{H} \rangle}$$

$$= -\frac{\omega}{2} \frac{\int \Delta\varepsilon |\mathbf{E}|^2}{\int \varepsilon |\mathbf{E}|^2}$$

…e.g. absorption
gives imaginary Δω
= decay!

$$or: \quad \Delta k^{(1)} = \Delta\omega^{(1)} / v_g$$

$$v_g = \frac{d\omega}{dk}$$

$$\Rightarrow \frac{\Delta\omega^{(1)}}{\omega} = -\frac{\Delta n}{n} \cdot \left(\text{fraction of } \varepsilon \left|\mathbf{E}\right|^2 \text{ in } \Delta n\right)$$

A Quantitative Example

Gas can have
low loss
& nonlinearity

…but what about
the cladding?

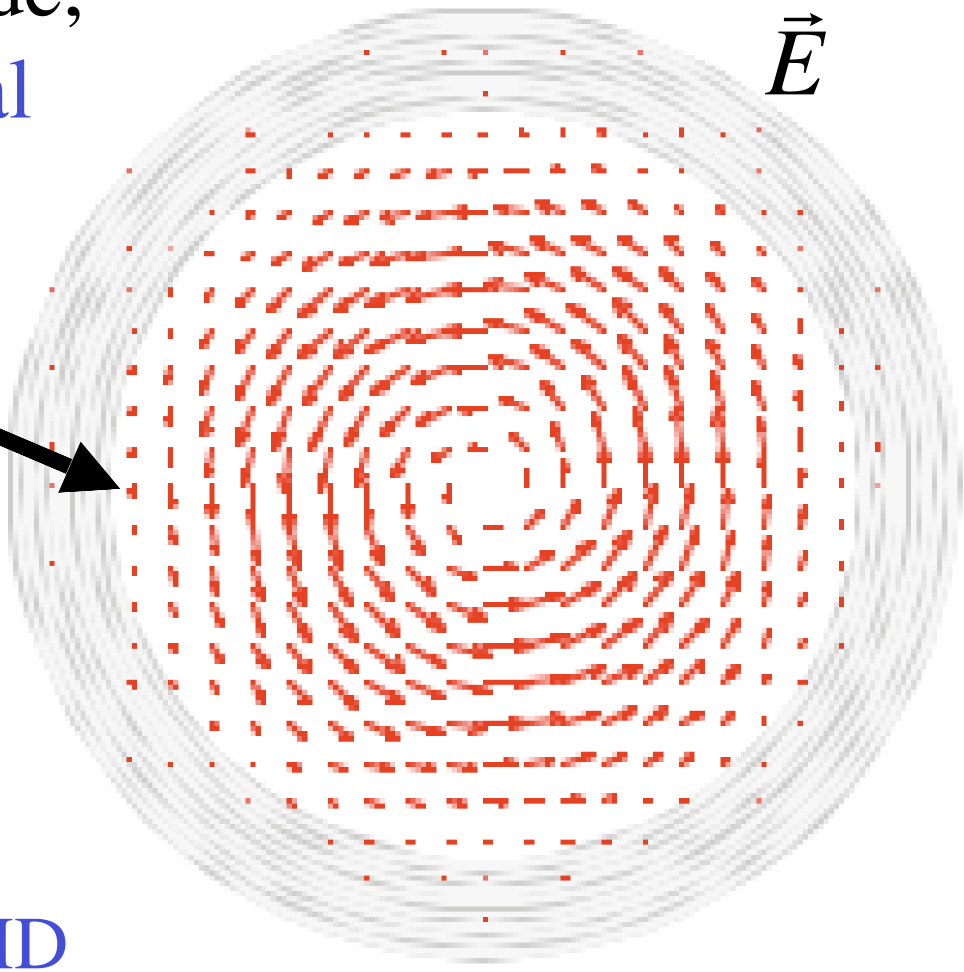
…*some* field
penetrates!

& may need to use
very "bad" material
to get high index contrast

An Old Friend: the TE_{01} mode

lowest-loss mode,
just as in metal

(near) node at interface
= strong confinement
= low losses



non-degenerate mode
— cannot be split
= no birefringence or PMD

[Johnson, *Opt. Express* **9**, 748 (2001)]

Suppressing Cladding Losses

Mode Losses

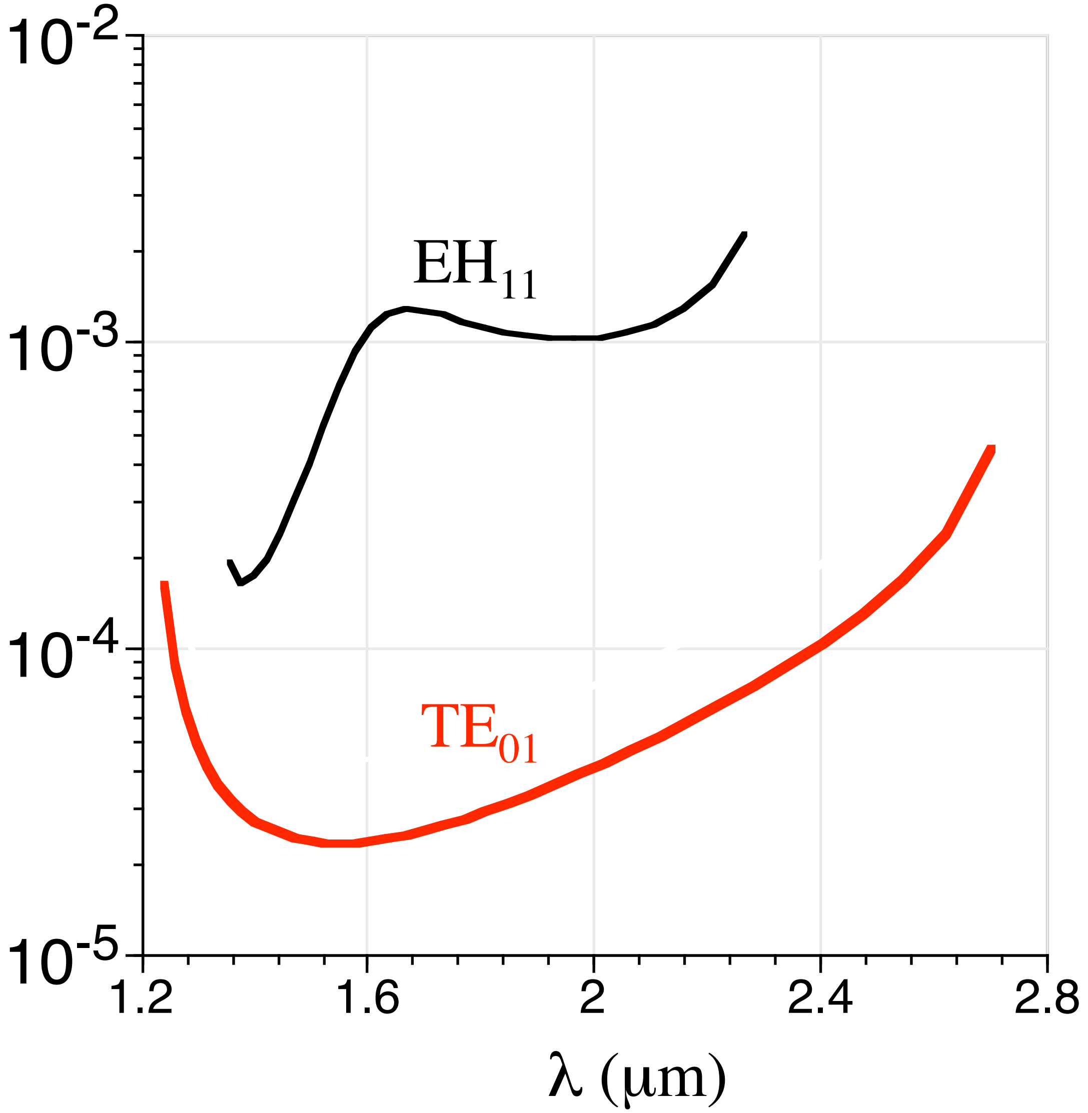
÷

Bulk Cladding Losses

Large differential loss

TE_{01} strongly suppresses cladding absorption

(like ohmic loss, for metal)



[Johnson, *Opt. Express* **9**, 748 (2001)]

Quantifying Nonlinearity

$\Delta\beta$ ~ power P ~ 1 / lengthscale for nonlinear effects

$$\gamma = \Delta\beta / P$$

= nonlinear-strength parameter determining
self-phase modulation (SPM), four-wave mixing (FWM), …

(unlike “effective area,”
tells *where* the field is,
not just how big)

[Johnson, *Opt. Express* **9**, 748 (2001)]   [R. Ramaswami & K. N. Sivarajan, *Optical Networks: A Practical Perspective*]

Suppressing Cladding Nonlinearity

[Johnson, *Opt. Express* **9**, 748 (2001)]

Mode Nonlinearity*

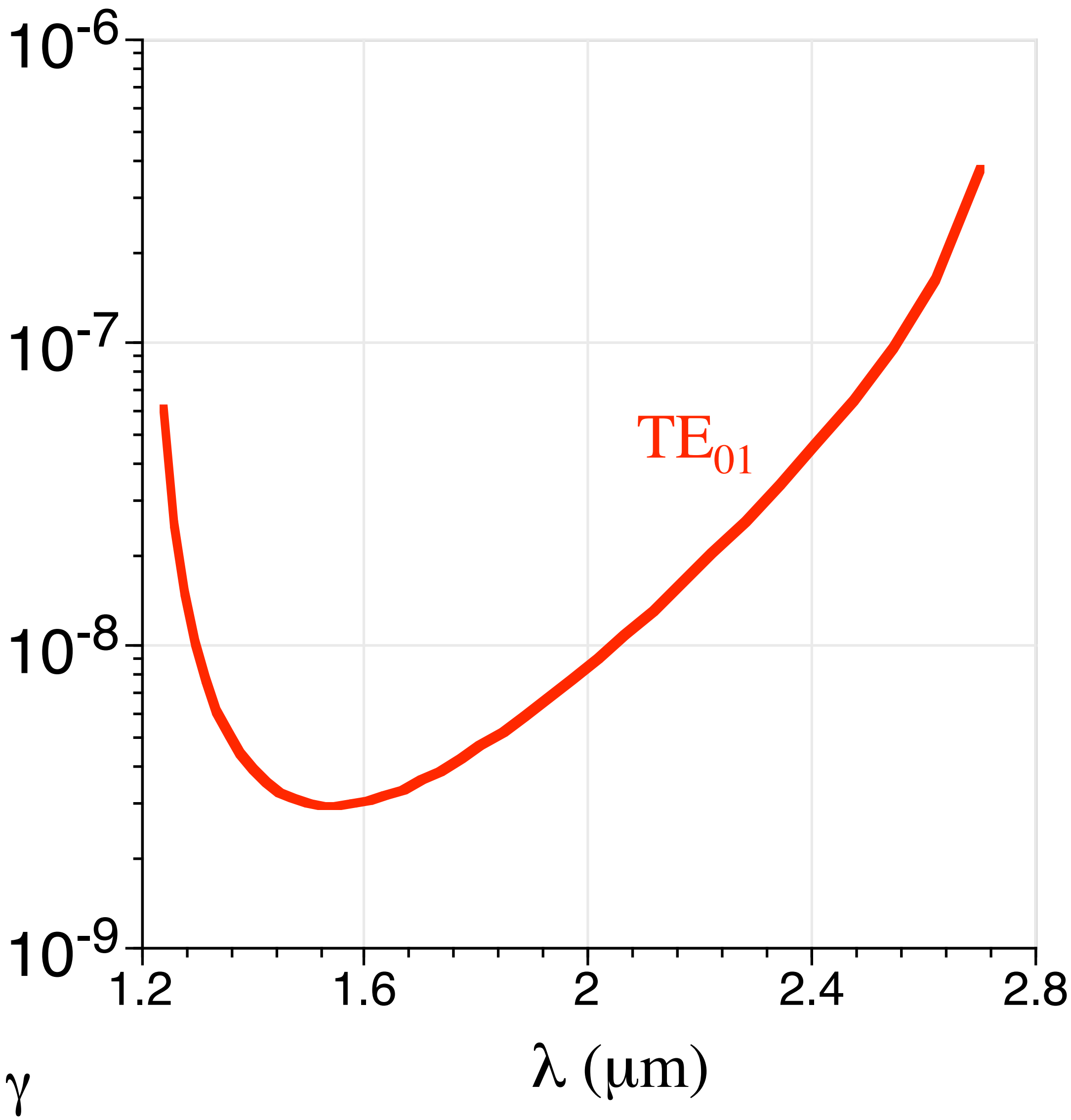
÷

Cladding Nonlinearity

Will be dominated by nonlinearity of air

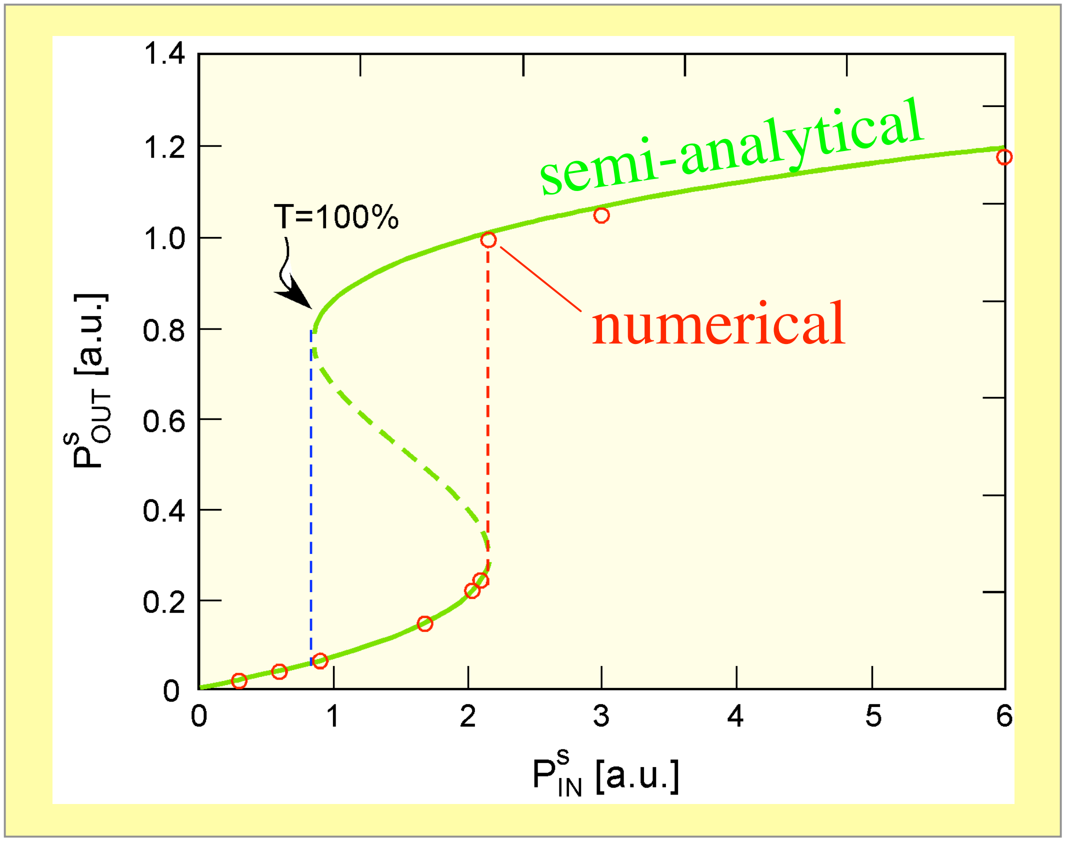
~10,000 times weaker than in silica fiber

(including factor of 10 in area)



* "nonlinearity" = $\Delta\beta^{(1)} / P = \gamma$

A ~~Linear~~ *Nonlinear* “Transistor”



Bistable (hysteresis) response

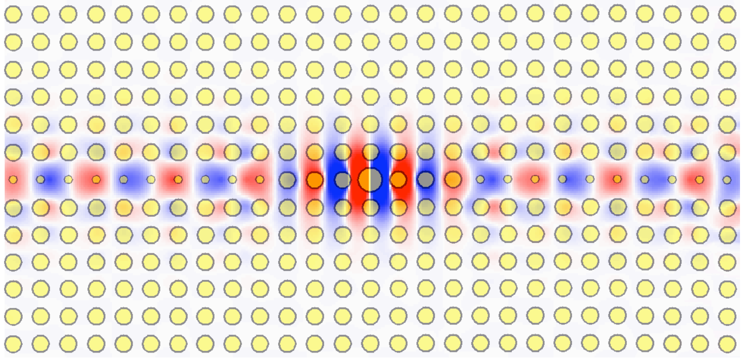
Entire nonlinear response from *one* linear calculation:

Lorentzian mode ω, *Q*

+

Kerr $\Delta\omega \sim |\mathbf{E}|^2$

(to first order)



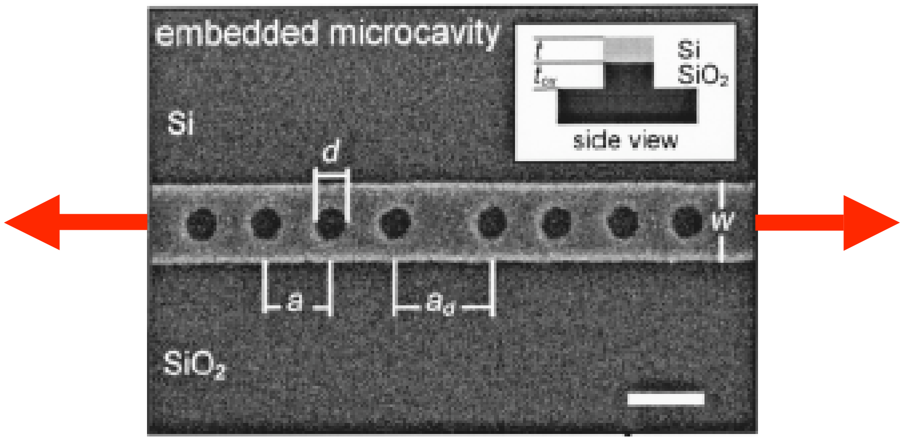
[Soljacic *et al.*, *PRE Rapid. Comm.* **66**, 055601 (2002).]

Tuning Microcavities

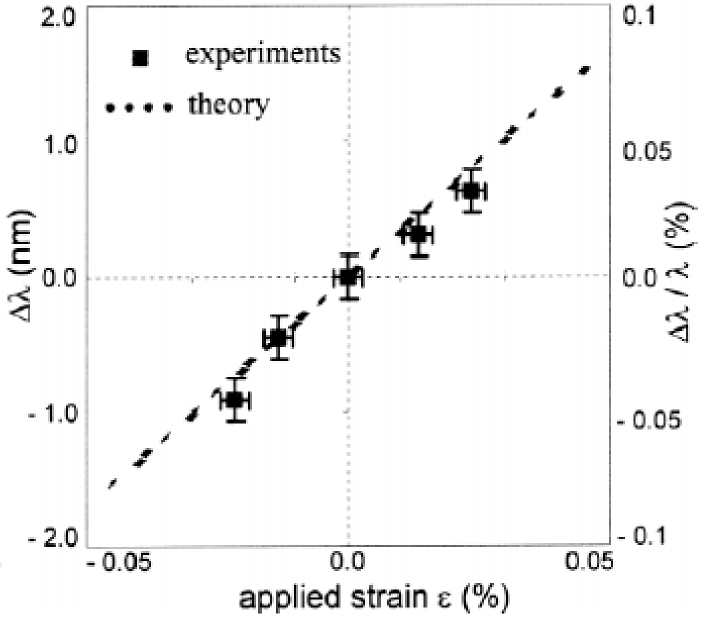
Fabrication accurate to 10^{-3} or 10^{-6} (bandwidth) is challenging
…need post-fabrication tuning

Tuning mechanisms: electro-optic, thermal, conductivity, liquid crystal…
alter cavity index *or shape*

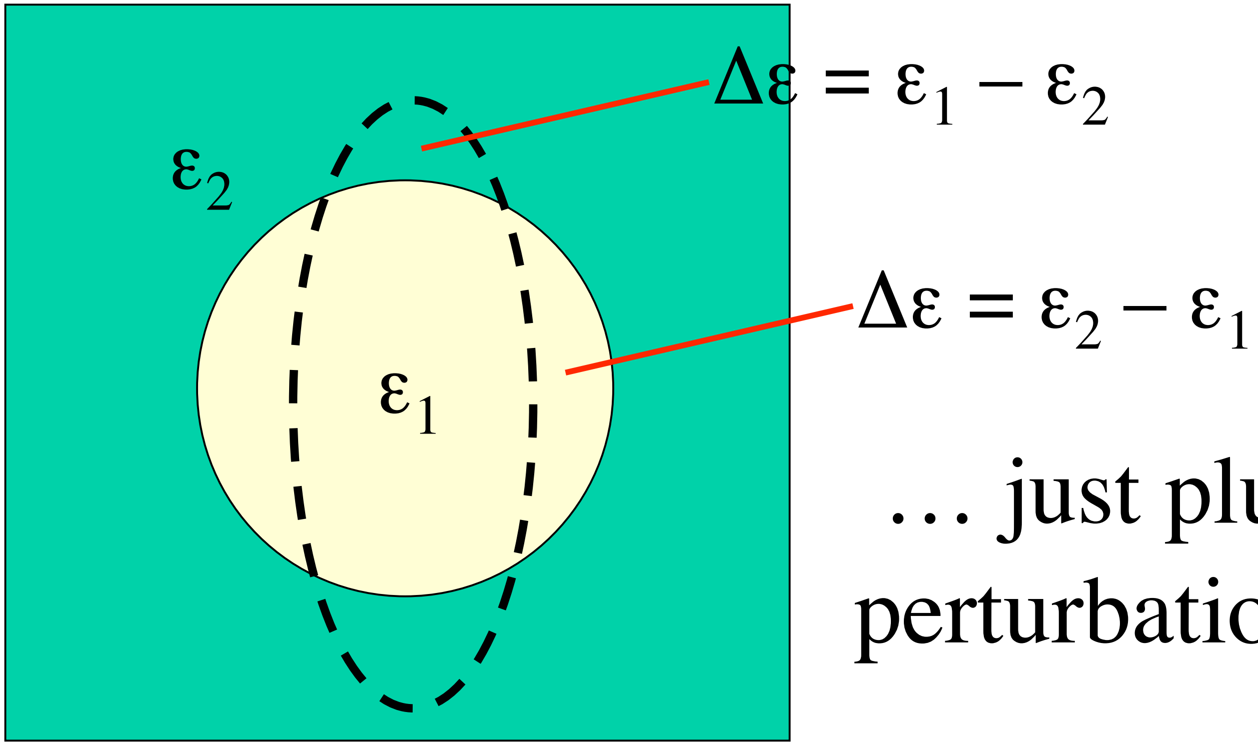
[C.-W. Wong, *Appl*. *Phys*. *Lett*. **84**, 1242 (2004).]



stretch piezo-electrically
(MEMS)



Boundary-perturbation theory



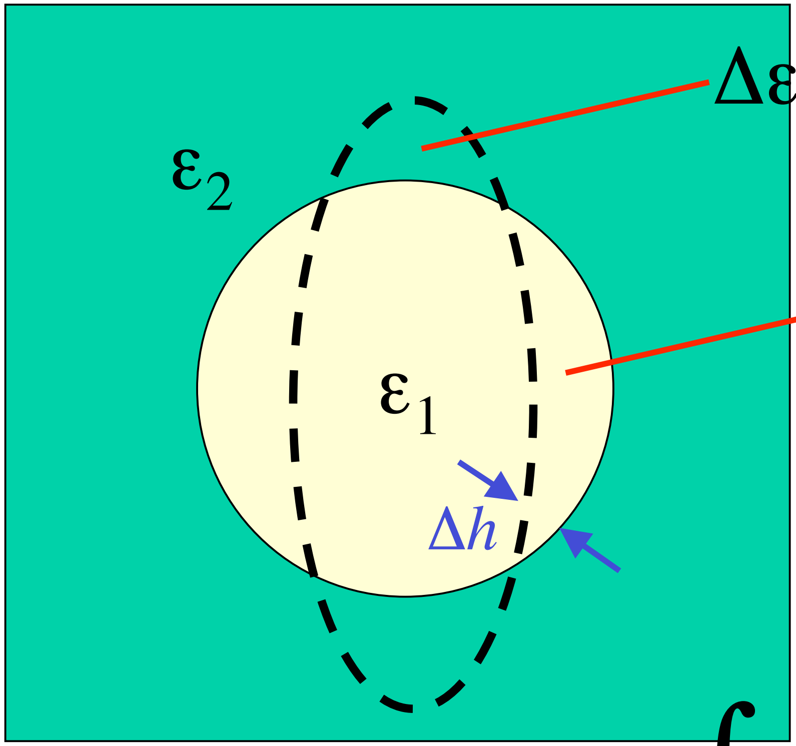
… just plug $\Delta\varepsilon$'s into perturbation formulas?

FAILS for high index contrast!

beware field discontinuity…
fortunately, a simple correction exists

[S. G. Johnson *et al.*, *PRE* **65**, 066611 (2002)]

Boundary-perturbation theory



$\Delta\varepsilon = \varepsilon_1 - \varepsilon_2$

$\Delta\varepsilon = \varepsilon_2 - \varepsilon_1$

(continuous field components)

$$\Delta\omega^{(1)} = -\frac{\omega}{2}\frac{\int_{\text{surf.}} \Delta h\left[\Delta\varepsilon\left|\mathbf{E}_{\parallel}\right|^2 - \Delta\frac{1}{\varepsilon}\left|D_{\perp}\right|^2\right]}{\int \varepsilon\left|\mathbf{E}\right|^2}$$

[S. G. Johnson *et al.*,
PRE **65**, 066611 (2002)]

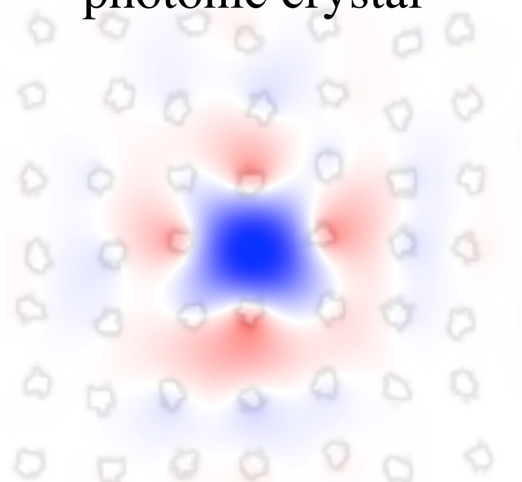
Surface roughness disorder?

[http://www.physik.uni-wuerzburg.de/TEP/Website/groups/opto/etching.htm]

conventional
ring resonator

loss limited by disorder

disordered
photonic crystal



[A. Rodriguez, MIT]

theorem: [S. Fan *et. al.*, *J. Appl. Phys.* **78**, 1415 (1995).]

small (bounded) disorder does not destroy the bandgap

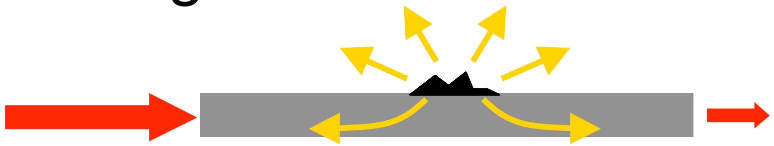
Q limited only by crystal size (for a 3d complete gap) …

… but waveguides have more trouble …

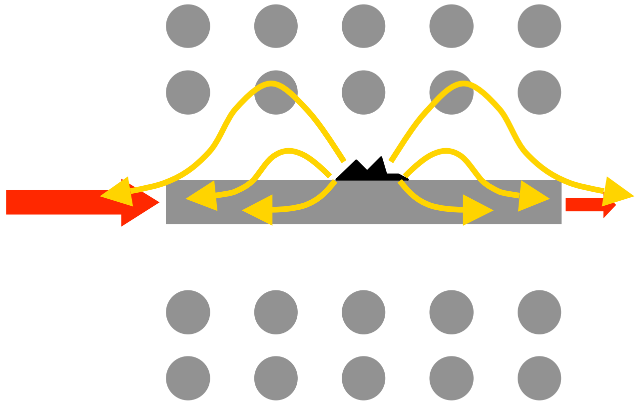
Effect of Gap on Disorder (e.g. Roughness) Loss?

[with M. Povinelli]

index-guided waveguide

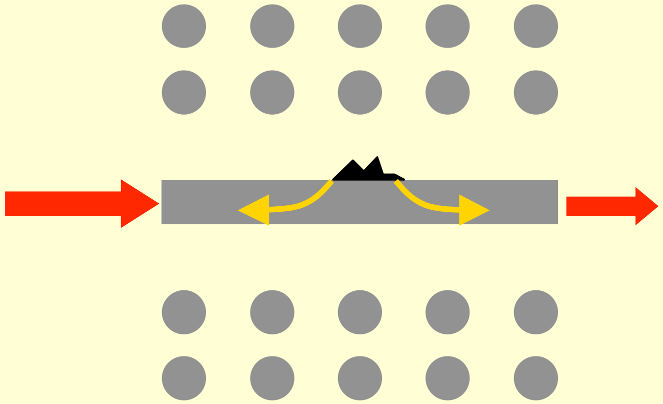


photonic-crystal waveguide: which picture is correct?



radiation blocked
increased reflection

OR



radiation blocked
no increase in reflection

Coupled-mode theory

Expand state in ideal eigenmodes, for constant ω:

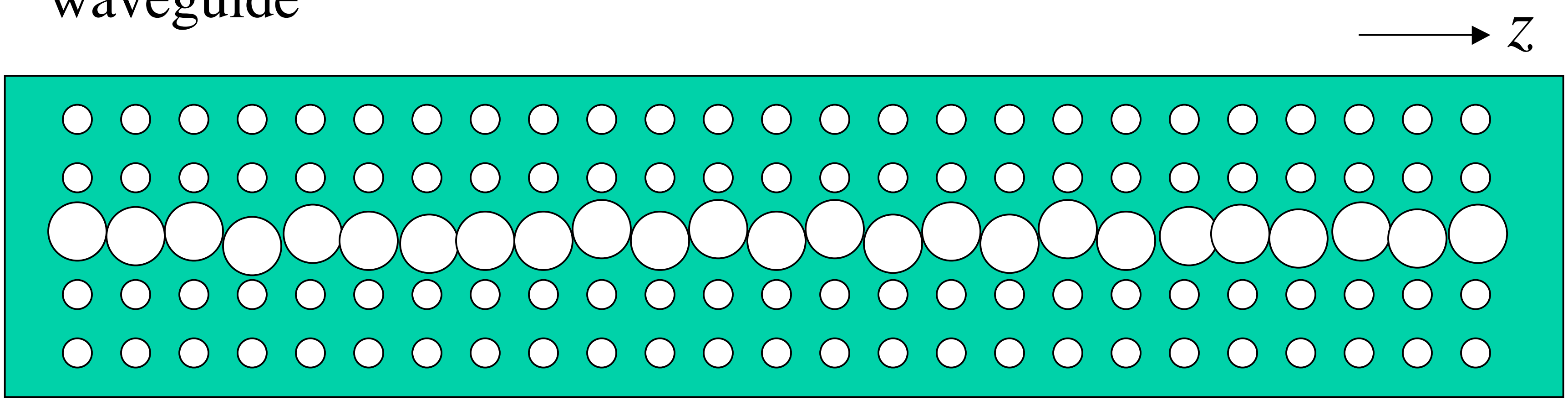
$$|\psi\rangle = \sum_n c_n(z)\boxed{|n\rangle}e^{i\beta_n z}$$

wavenumber

state (field)
of disordered
waveguide

expansion
coefficient

eigenstate of perfect waveguide



What’s New in Coupled-Mode Theory?

- Traditional methods (Marcuse, 1970): weak periodicity only
- Strong perodicity (Bloch modes expansion):
  - — de Sterke *et al*. (1996): coupling in *time* (nonlinearities)
  - — Russell (1986): weak perturbations, slowly varying only

scalar

NEW: exact extension, for *z*-dependent (constant ω), and:

arbitrary periodicity,
arbitrary index contrast (full vector),
arbitrary disorder [and/or tapers]

full-vector

[S. G. Johnson *et al*., *PRE* **66**, 066608 (2002).]
[M. L. Povinelli *et al*., *APL* **84**, 3639 (2004).]

[M. Skorobogatiy *et al*.,
Opt. Express **10**, 1227 (2002).]

Coupled-wave Theory

(skipping all the math…)

$$\frac{dc_n}{dz} = \sum_{m \neq n} [\text{coupling}]_{m,n} \, e^{i\Delta\beta z} \, c_m$$

mode expansion coefficients

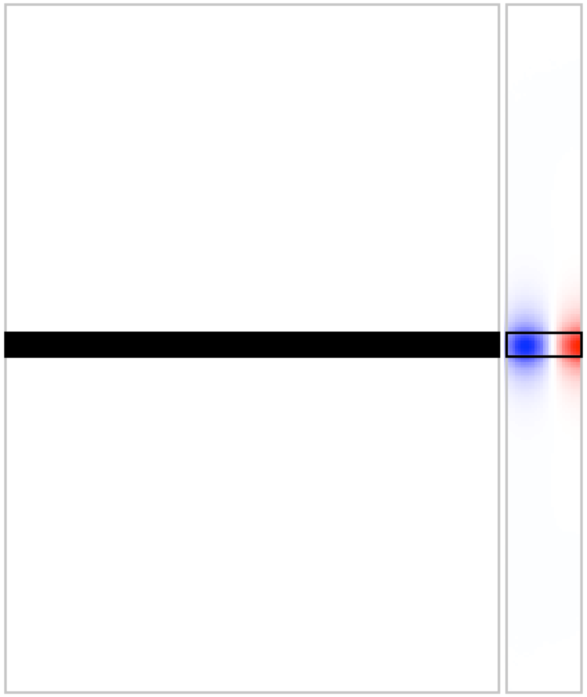
Depends only on: [M. L. Povinelli *et al.*, *APL* **84**, 3639 (2004).]

- strength of disorder
- mode field at disorder
- group velocities

Weak disorder, short correlations: refl. ~ |coupling|^2
if disorder and modes are "same,"
then reflection is the same

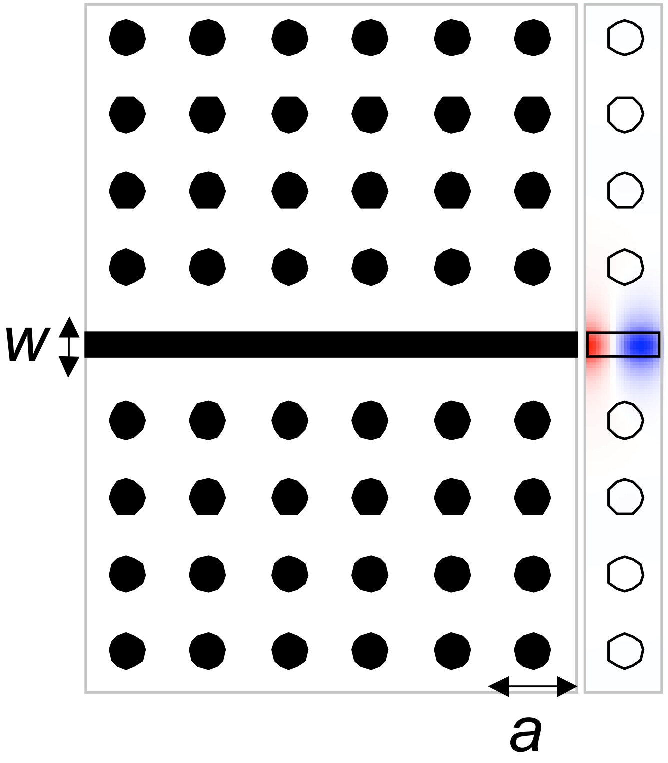
A Test Case

[M. L. Povinelli *et al.*, *APL* **84**, 3639 (2004).]



strip waveguide

index-guided



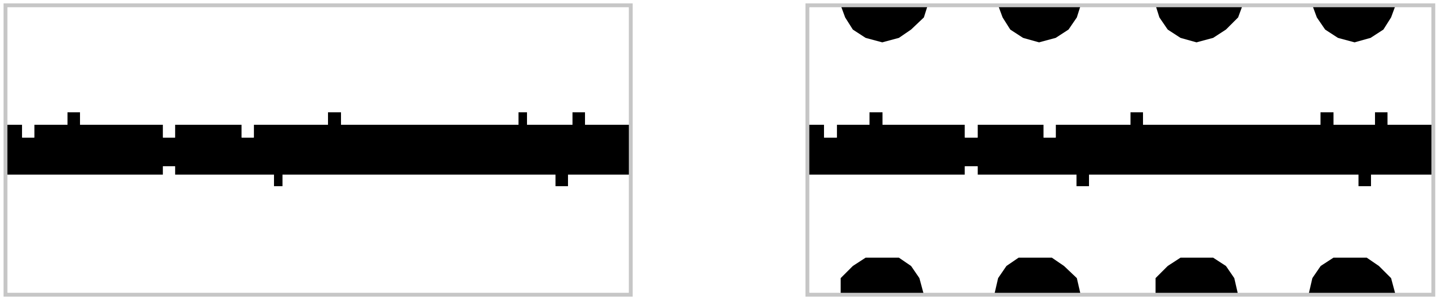
PC waveguide

gap-guided, same ω(β)

Apples to Apples

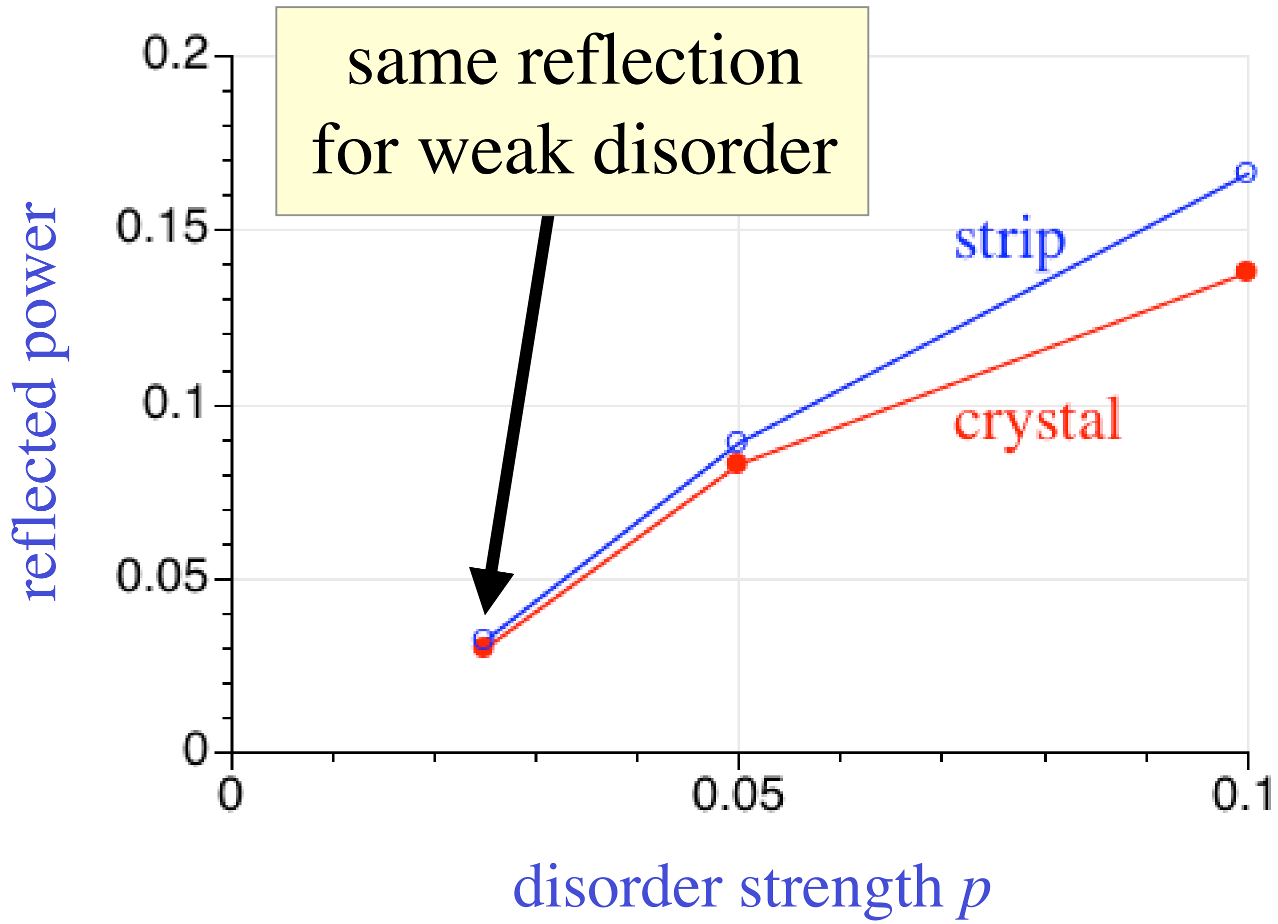
A Test Case

pixels added/removed with probability *p*

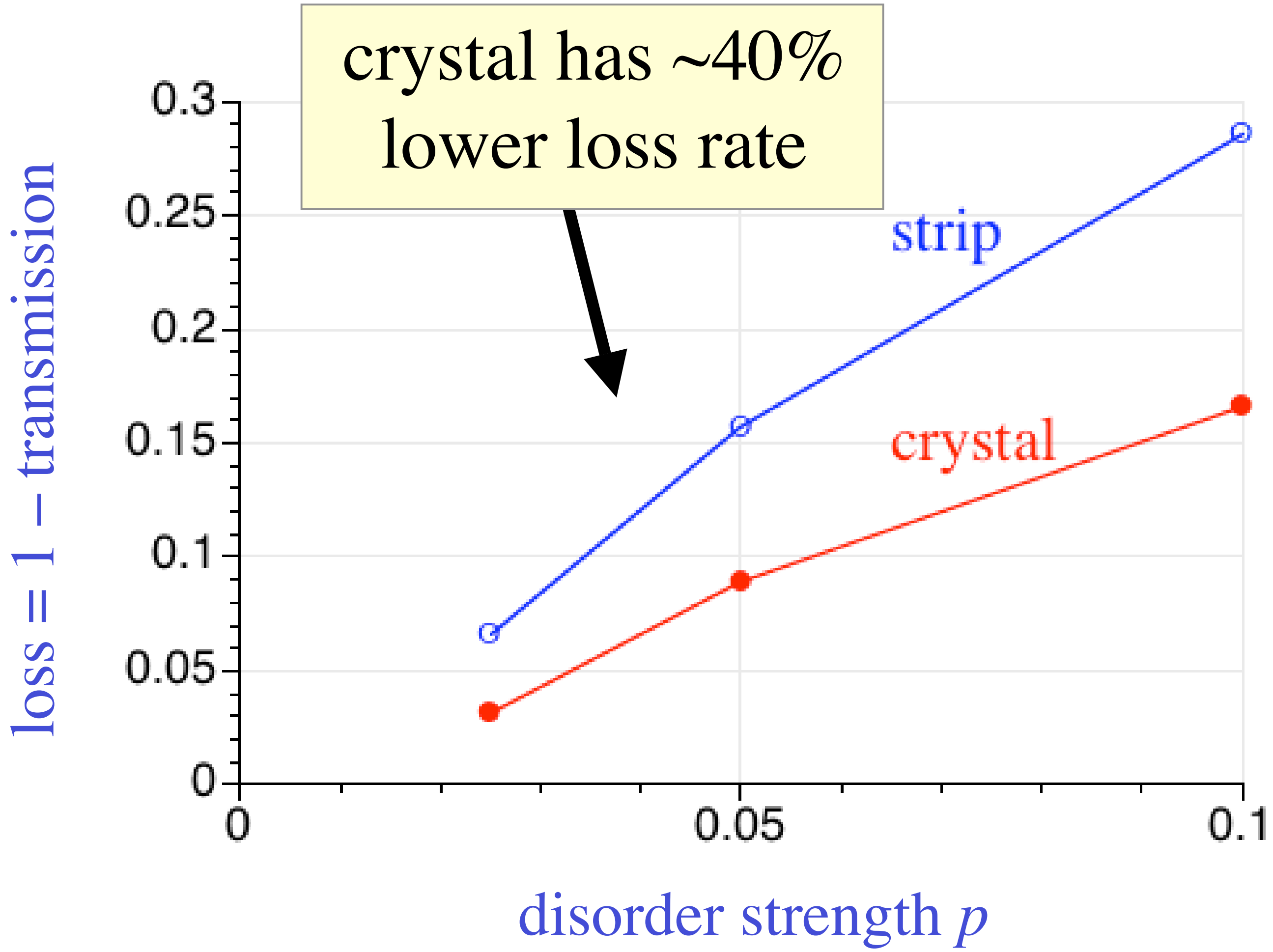


same disorder in both cases, averaged over many FDTD runs

Test Case Results: Reflection


same reflection for weak disorder

Test Case Results: Total Loss



crystal has ~40% lower loss rate

photonic bandgap
(all other things equal)
= unambiguous improvement

But, the news isn't all good…

Group-velocity (v) dependence

other things being equal

[S. G. Johnson *et al*., *Proc*. *2003 Europ*. *Symp*. *Phot*. *Cryst*. **1**, 103.]
[S. Hughes *et al*., *Phys*. *Rev*. *Lett*. **94**, 033903 (2005).]

absorption/radiation-scattering loss

(per distance) $\sim 1/v$

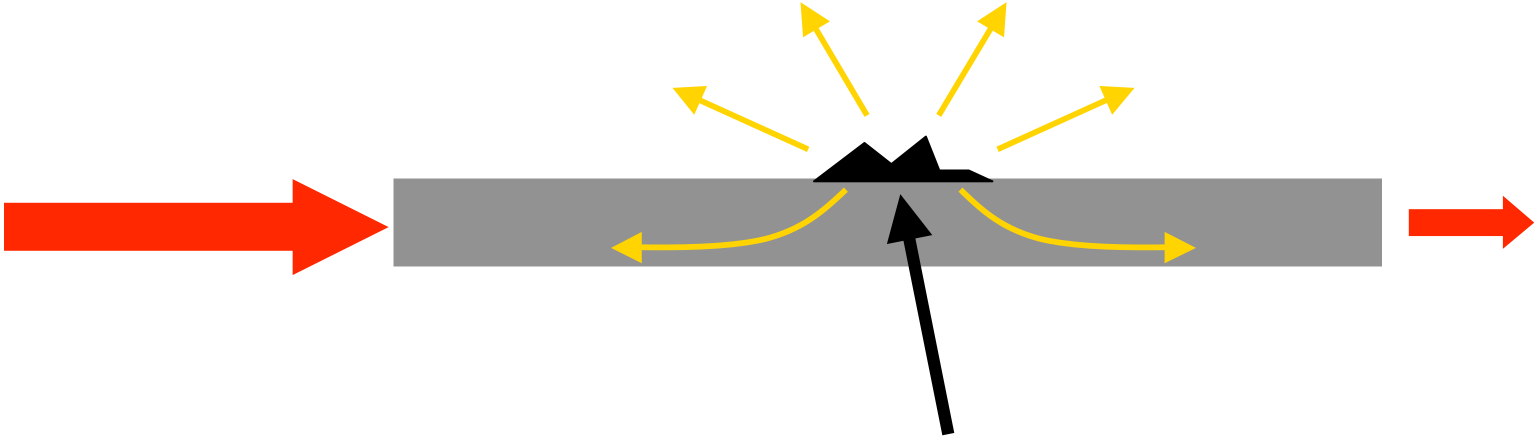
reflection loss

(per distance) $\sim 1/v^2$

(per time) $\sim 1/v$

Losses a challenge for slow light…

An Easier Way to Compute Loss



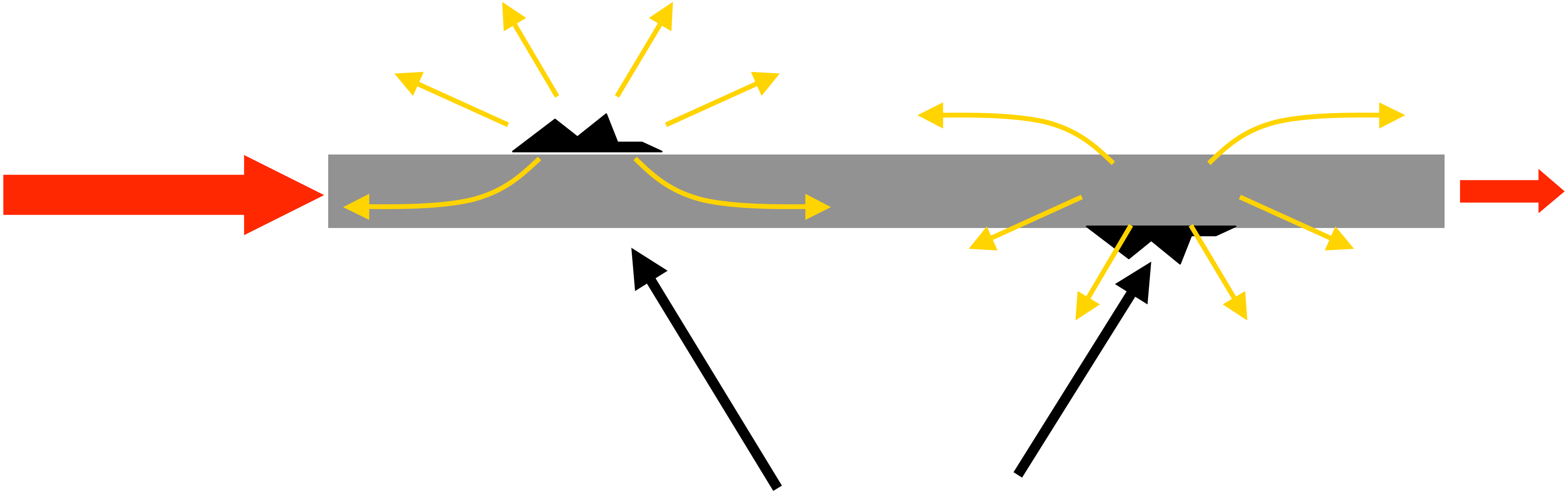
imperfection acts like a volume current

$$\vec{J} \sim \Delta\varepsilon\,\vec{E}_0$$

volume-current method

(i.e., first Born approx. to Green’s function)

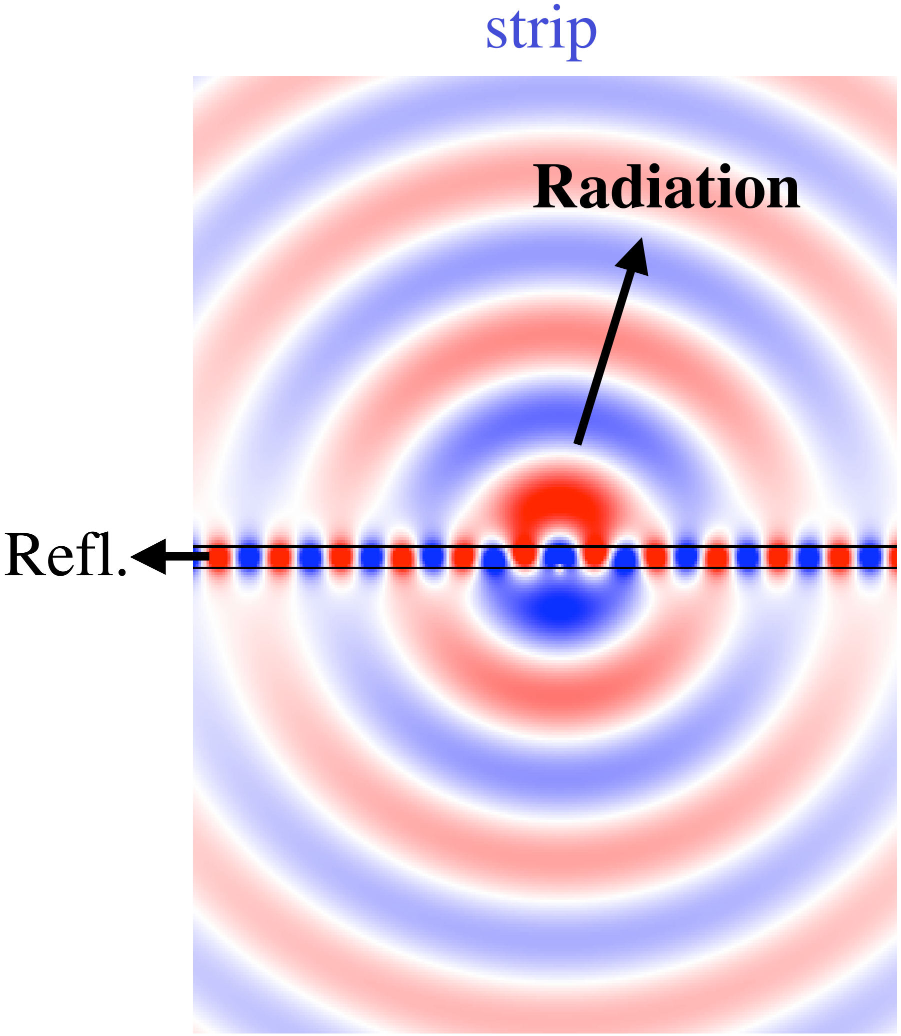
An Easier Way to Compute Loss

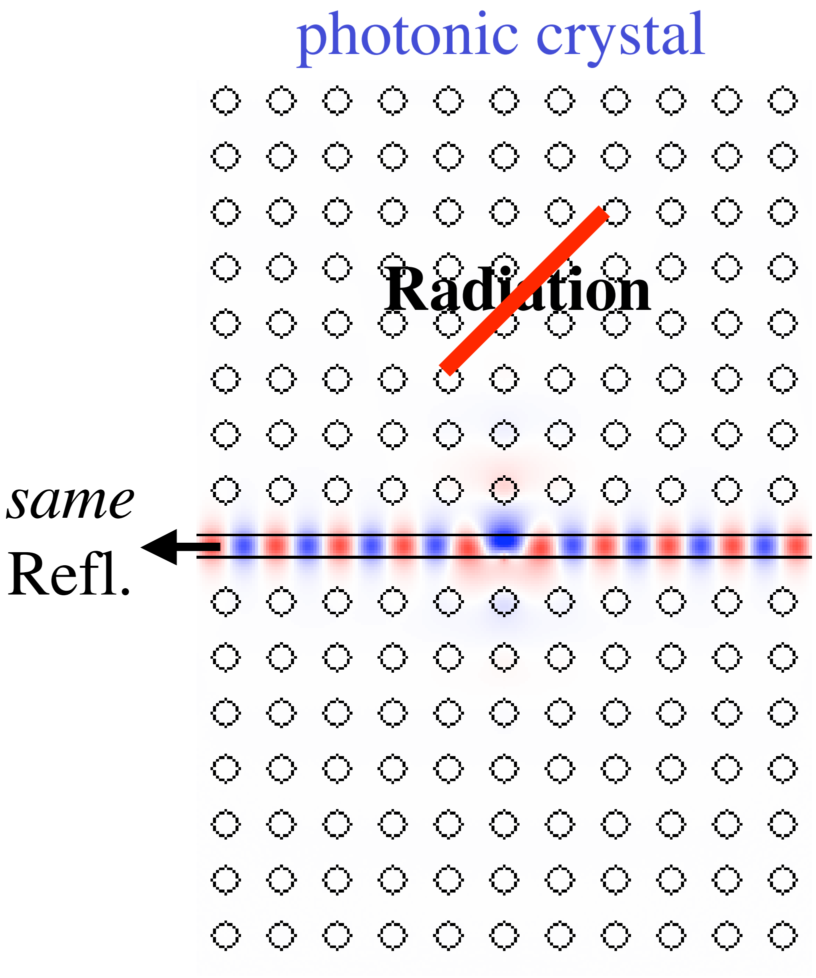


uncorrelated disorder adds *incoherently*

So, compute power P radiated by *one* localized source *J*, and loss rate ~ P * (mean disorder strength)

Losses from Point Scatterers

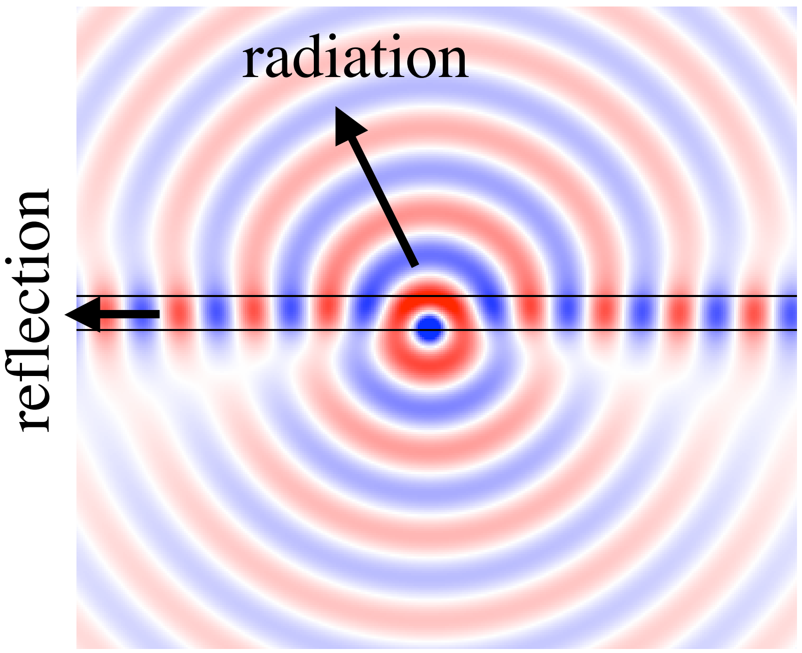




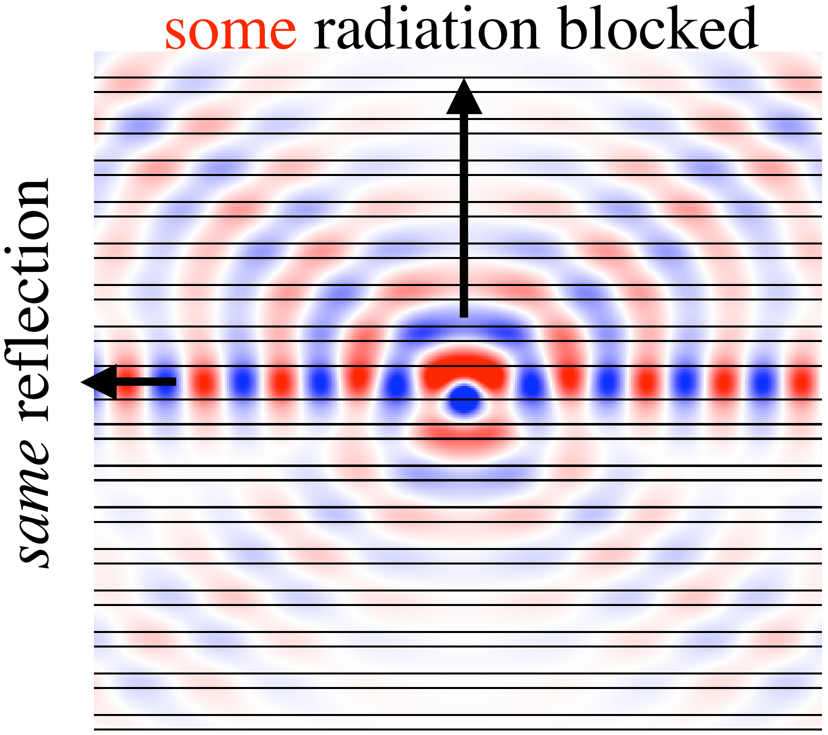
Loss rate ratio = (Refl. only) / (Refl. + Radiation) = 60% ✓

Effect of an *Incomplete* Gap

on uncorrelated surface roughness



Conventional waveguide
(matching modal area)

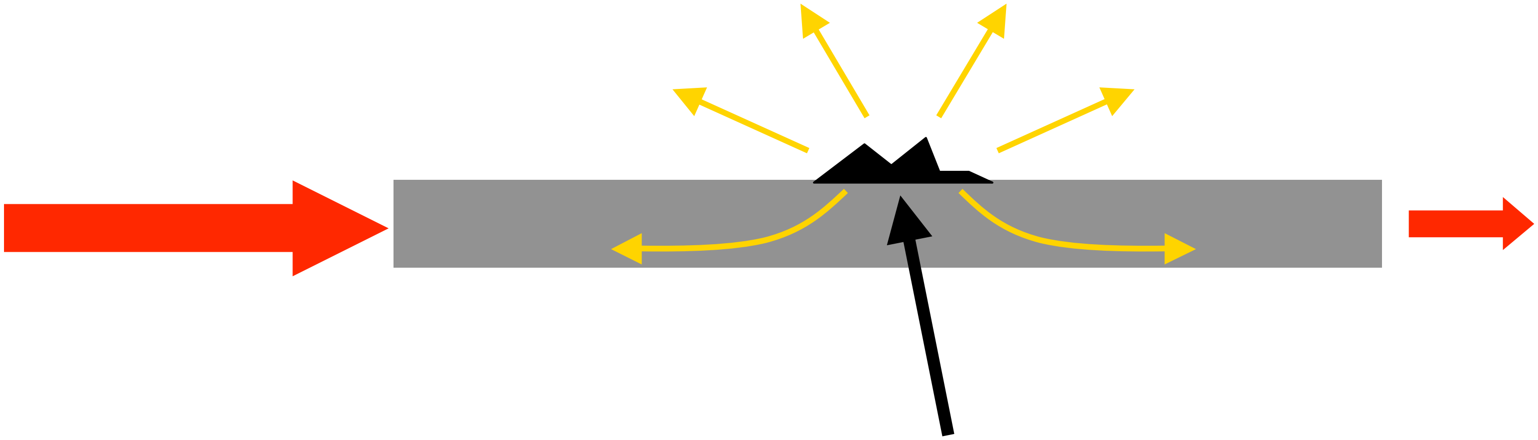


…with Si/SiO_2 Bragg mirrors (1D gap)

50% lower losses (in dB)

same reflection

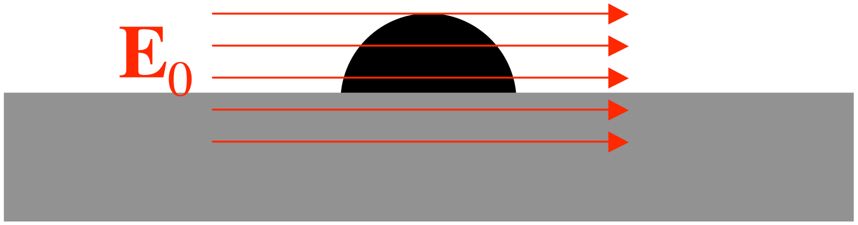
Failure of the Volume-current Method



imperfection acts like a volume current

$$\vec{J} \sim \cancel{\Delta\varepsilon\, \vec{E}_0}$$

Incorrect for large Δε (except in 2d TM polarization)

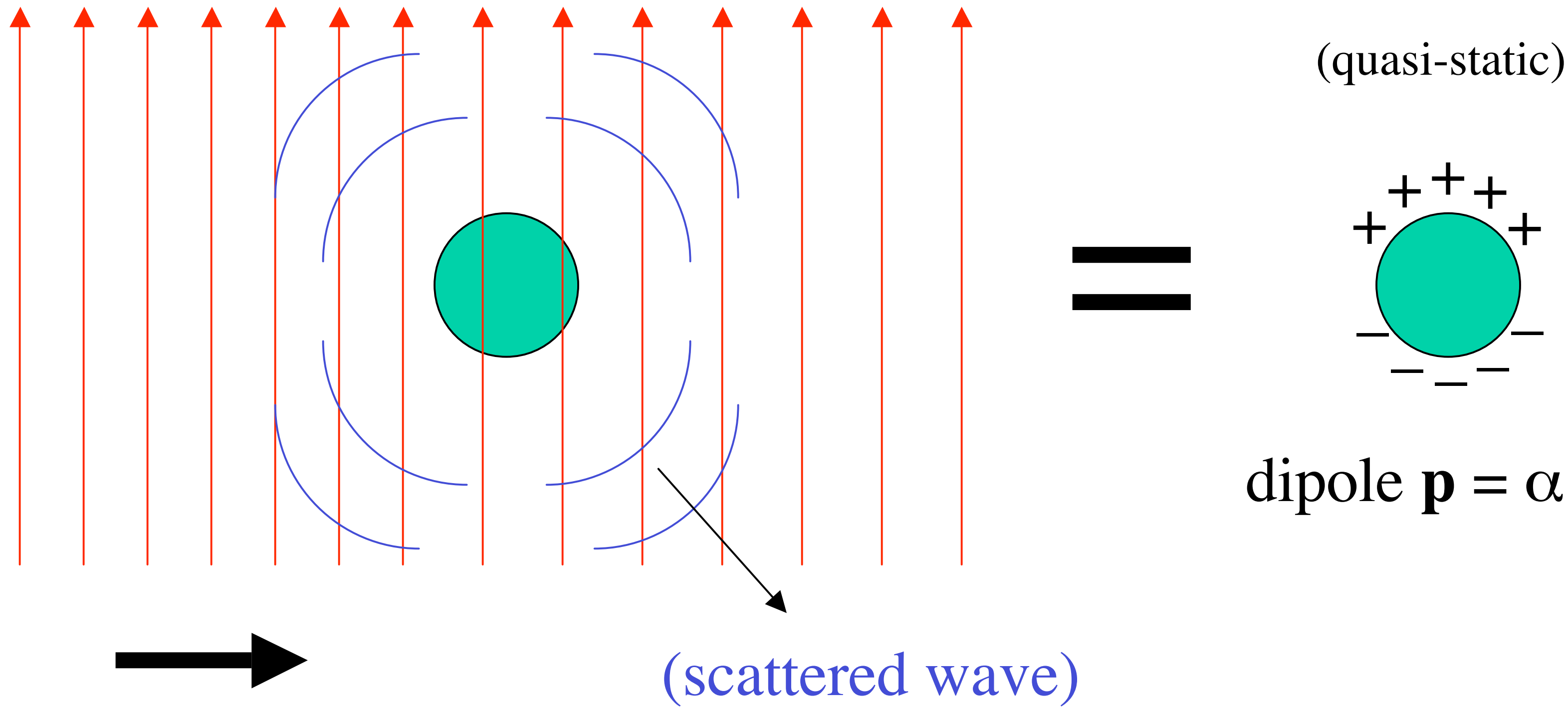


Δε "bump" *changes* **E**

($E_\perp$ is *discontinuous*)

Scattering Theory (for small scatterers)

[e.g. Jackson, *Classical Electrodynamics*]



(quasi-static)

dipole $\mathbf{p} = \alpha\, \mathbf{E}_0$

(scattered wave)

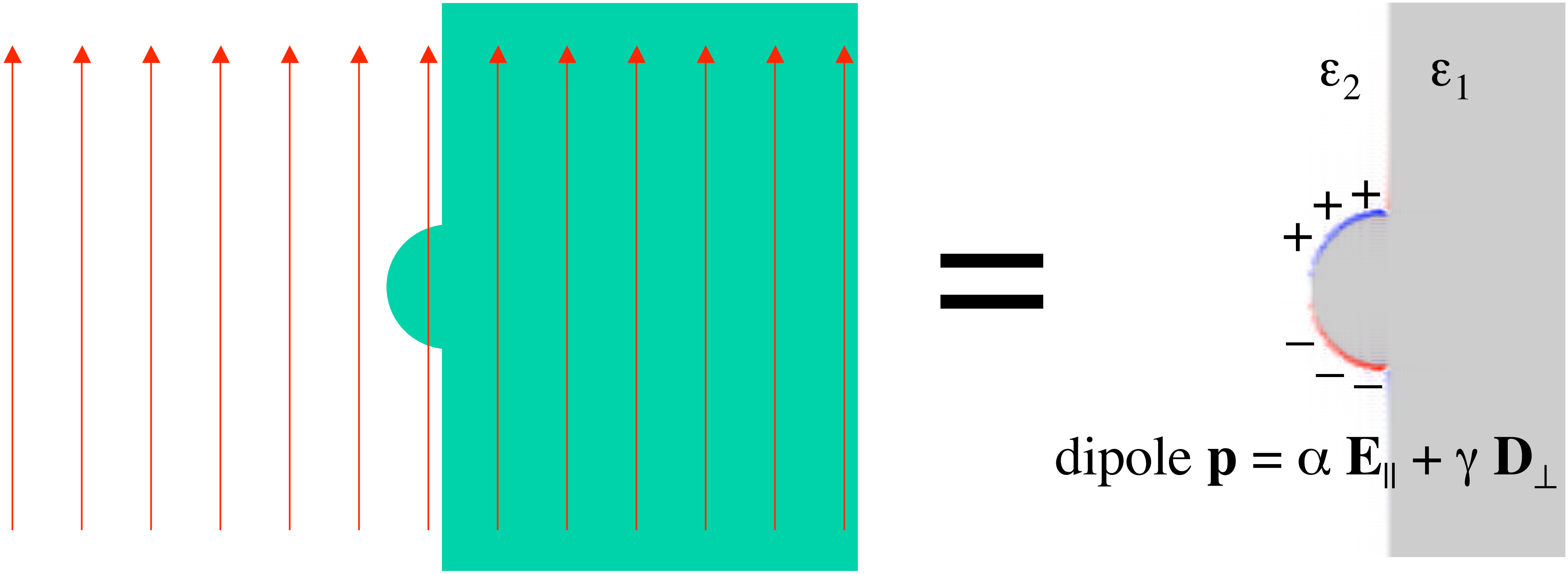
incident wave $\mathbf{E}_0$ ($\lambda >> d$)

sphere: *effective* point current $\mathbf{J} \sim \mathbf{p} / \Delta V$

$= 3\, \Delta\varepsilon\, \mathbf{E}_0 / (\Delta\varepsilon + 3)$

$= \Delta\varepsilon\, \mathbf{E}_0$ for small $\Delta\varepsilon$, but very different for large $\Delta\varepsilon$

Corrected Volume Current for Large Δε



dipole $\mathbf{p} = \alpha\, \mathbf{E}_{\|} + \gamma\, \mathbf{D}_{\perp}$

unperturbed field **E**

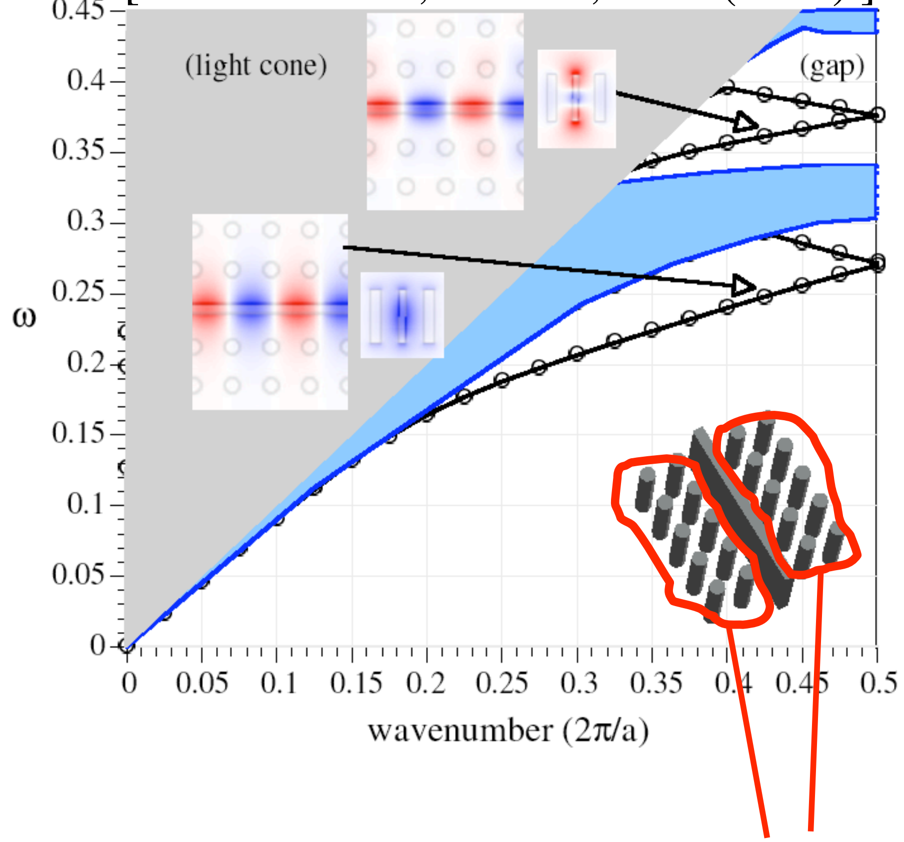
(compute polarizability *numerically*)

effective point current **J** $\sim \left(\frac{\varepsilon_1 + \varepsilon_2}{2} \mathbf{p}_{\|} + \varepsilon \mathbf{p}_{\perp} \right) / \Delta V$

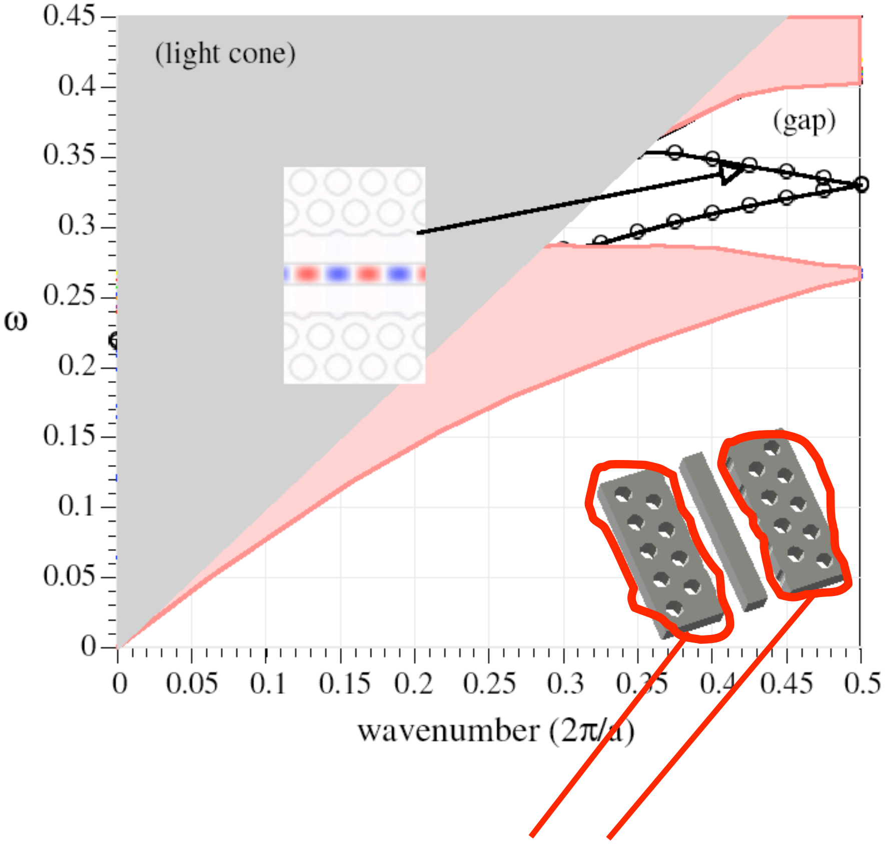
[S. G. Johnson *et al.*, *Applied Phys. B*, in press (2005).]

Strip Waveguides in Photonic-Crystal Slabs (3d)

[Johnson *et al.*, *PRB* **62**, 8212 (2000)]

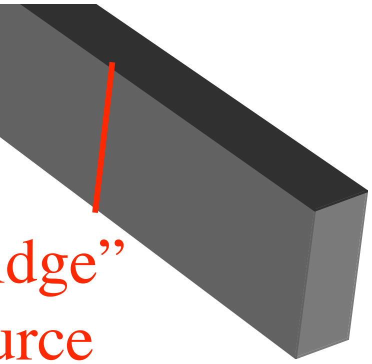


[Lau *et al.*, *APL* **81**, 3915 (2002)]

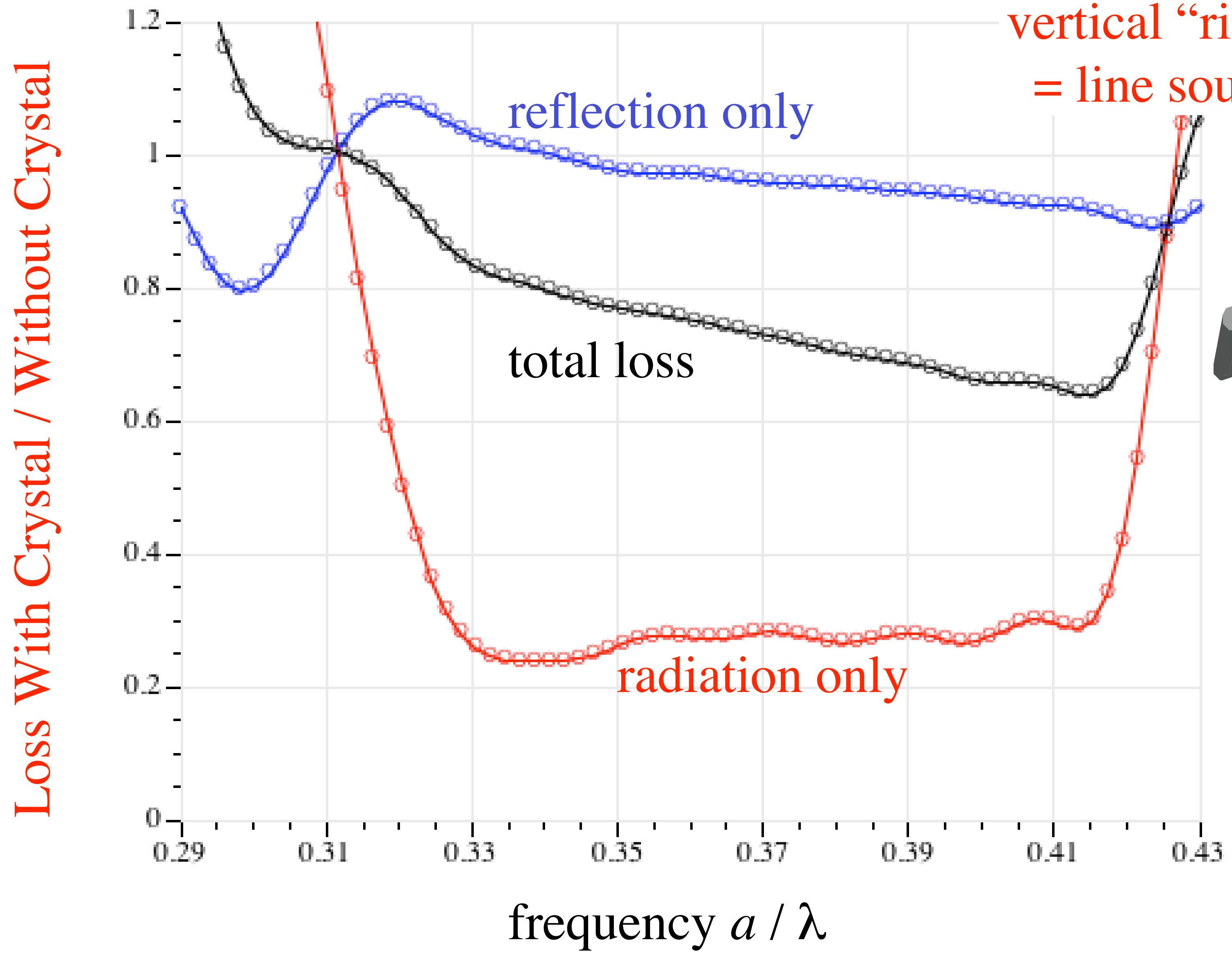


How does *incomplete 3d gap* affect roughness loss?

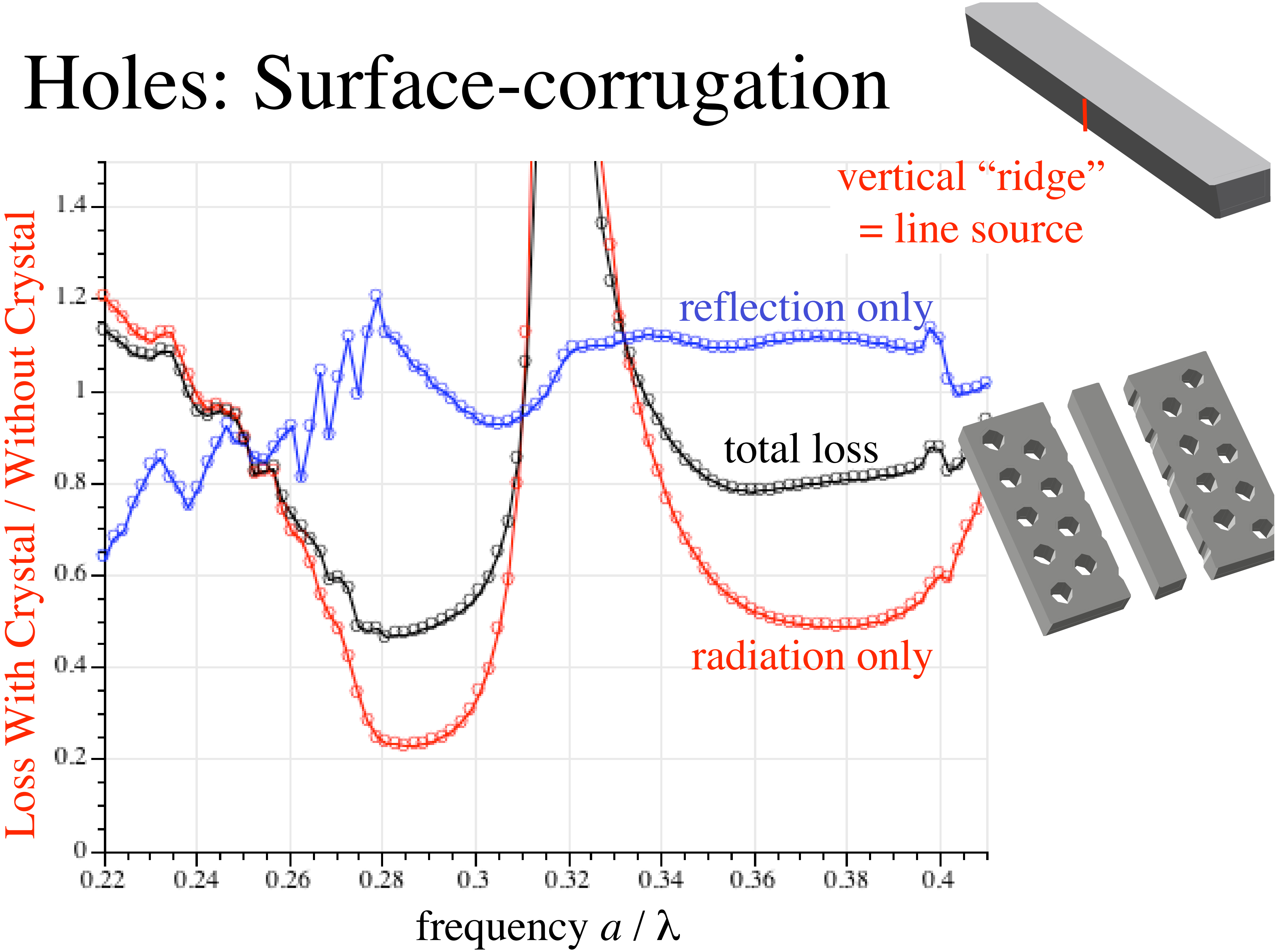
Rods: Surface-corrugation



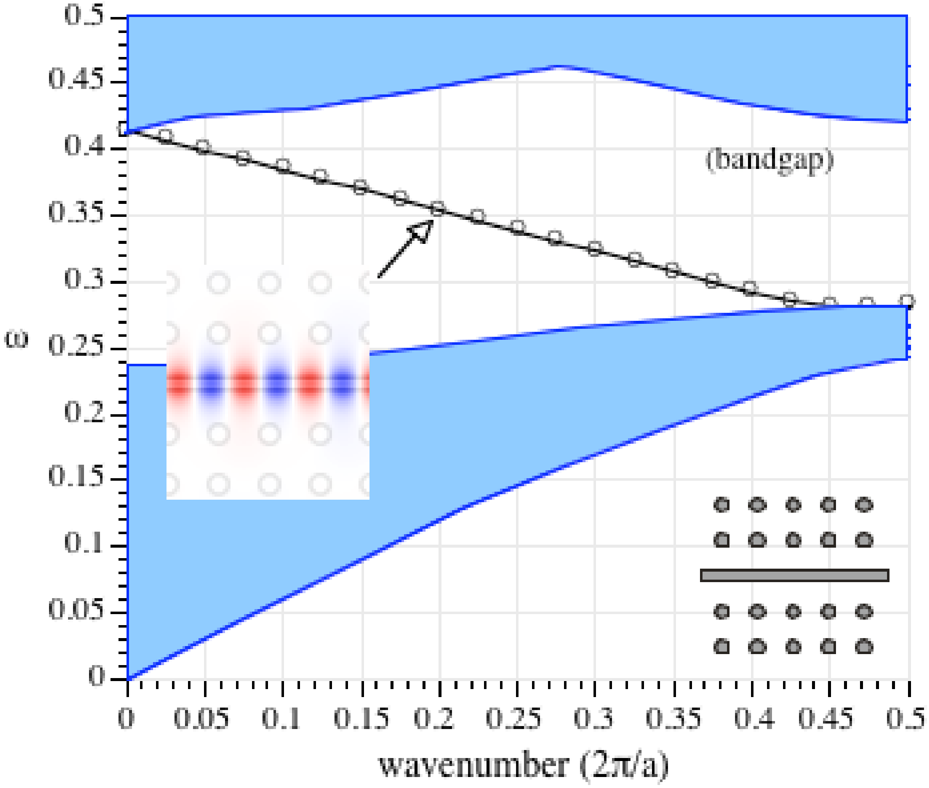
vertical “ridge”
= line source

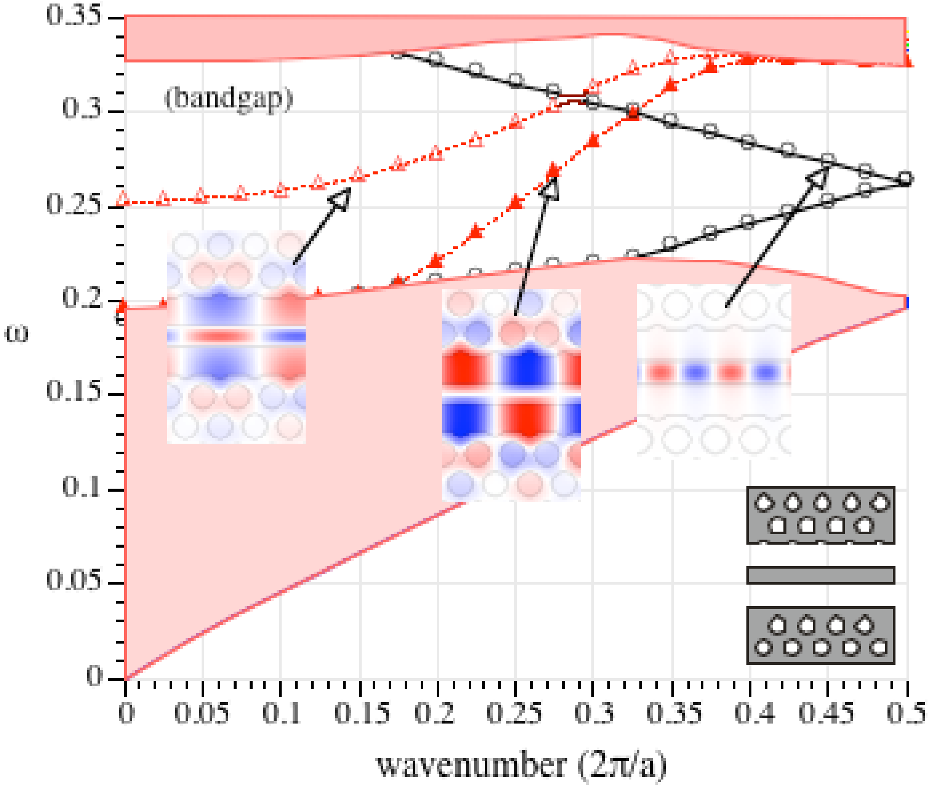


Holes: Surface-corrugation



Rods vs. Holes? *Answer is in 2d.*





The hole waveguide is not single mode

— crystal introduces new modes (in 2d)

and new leaky modes (in 3d)

The story of photonic crystals:

Finding New Materials / Processes

⇒ Designing New Structures

Further Reading

Books:

- J. D. Joannopoulos, R. D. Meade, and J. N. Winn, *Photonic Crystals: Molding the Flow of Light* (Princeton Univ. Press, 1995).

- S. G. Johnson and J. D. Joannopoulos, *Photonic Crystals: The Road from Theory to Practice* (Kluwer, 2002).

- K. Sakoda, *Optical Properties of Photonic Crystals* (Springer, 2001).

This Presentation, Free Software, …

`http://ab-initio.mit.edu/photons/tutorial`