Appendix E

Answers to Odd-Numbered Problems

Chapter 1

1.1 Proof.

1.3 Proof.

1.5 They satisfy Maxwell's equations.

1.7 $\mathbf{E}_s = \dfrac{1}{j\omega\epsilon_o}\dfrac{H_o}{\sqrt{\rho}}\left(\dfrac{1}{\rho^2} - j\beta\right)e^{-j\beta\rho}\mathbf{a}_z.$

1.9 $\mathbf{H}_s = -j\dfrac{20}{\omega\mu_o}\left[k_y \sin(k_x x)\sin(k_y y)\mathbf{a}_x + k_x \cos(k_x x)\cos(k_y y)\mathbf{a}_y\right].$

1.11 (a) $\cos(\omega t - 2z)\mathbf{a}_x - \sin(\omega t - 2z)\mathbf{a}_y$

(b) $-10\sin x \sin \omega t\,\mathbf{a}_x - 5\cos(\omega t - 2z + 45°)\mathbf{a}_z$

(c) $2\cos 2x \cos(\omega t - 3x - 90°) + e^{3x}\cos(\omega t - 4x).$

1.13 Proof.

1.15 (a) elliptic, (b) elliptic, (c) hyperbolic, (d) parabolic.

Chapter 2

2.1 If a and d are functions of x only; c and e are functions of y only; $b = 0$; and f is the sum of a function x only and a function of y only.

2.3 (a) $V = \displaystyle\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi a}{a}\right)\sinh\left[\frac{n\pi}{a}(y-a)\right]$, where

$$A_n = \frac{2}{n \sinh\left(-\frac{n\pi b}{a}\right)} \int_0^a \frac{V_o x}{a} \sin \frac{n\pi x}{a}\, dx$$

(b) $V = V_o \dfrac{\cos \frac{\pi x}{a} \cosh \frac{\pi y}{a}}{\cosh \frac{\pi b}{a}}.$

2.5 (a) $\Phi(\rho, \phi) = \dfrac{\sin\phi}{\rho}$

(b) $\Phi(\rho, z) = \dfrac{4}{\pi} \displaystyle\sum_{n=1,3,5}^{\infty} \frac{I_0(n\pi\rho/L)}{I_0(n\pi a/L)} \frac{\sin(n\pi z/L)}{n}$

(c) $\Phi(\rho, \phi, t) = 2 \displaystyle\sum_{n=1}^{\infty} \frac{J_2(\rho x_n/a)}{x_n J_3(x_n)} \cos 2\phi \exp\left[-x_n^2 kt/a^2\right],$

where $J_2(x_n) = 0$.

2.7 $V(\rho, z) = \displaystyle\sum_{n=1}^{\infty} A_n \sin\left(\frac{n\pi z}{L}\right) I_0\left(\frac{n\pi\rho}{L}\right)$, $0.2639 V_o$.

2.9 $V(\rho, \phi) = \dfrac{4V_o}{\pi} \displaystyle\sum_{n=1,3,5}^{\infty} \frac{1}{n} \frac{\left[\left(\frac{\rho}{a}\right)^{3n} - \left(\frac{\rho}{a}\right)^{-3n}\right]}{\left[\left(\frac{b}{a}\right)^{3n} - \left(\frac{b}{a}\right)^{-3n}\right]} \sin 3n\phi.$

2.11 Proof.

2.13 Proof.

2.15 $\dfrac{a}{(\rho^2 + a^2)^{3/2}}, \dfrac{a^2 - \rho^2}{(\rho^2 + a^2)^{5/2}}.$

2.17 (a) 0, (b) $\dfrac{2}{9}$, $(c)\dfrac{1}{24}$.

2.19 (a) $\dfrac{1}{2} + \displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)}{2n(n+1)} P_n^1(0)(r/a)^n P_n(\cos\theta)$

(b) $-\dfrac{7}{5}\dfrac{a^3}{r^2} P_1(\cos\theta) - \dfrac{3}{10}\dfrac{a^5}{r^4} P_3(\cos\theta).$

2.21 For $r < a$, $0 \leq \theta \leq \pi/2$,

$$V = \frac{a\rho}{2\epsilon}\left[-z + \sum_{n=0}^{\infty} \frac{(-1)^n (2n)!}{(n!2^n)^2} (r/a)^{2n} P_{2n}(\cos\theta)\right]$$

For $r > a$, $V = \dfrac{a\rho}{2\epsilon}\left[\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(2n+2)!}{[(n+1)!2^{n+1}]^2} (r/a)^{2n+1} P_{2n}(\cos\theta)\right].$

2.23 Proof.

2.25 $V(r,\theta) = 2(r/a)^2 P_2(\cos\theta) + \dfrac{3r}{a} P_1(\cos\theta) + 2P_0(\cos\theta).$

2.27 For $r < a$, $V(r,\theta) = -\dfrac{3E_o}{\epsilon_r + 2}\cos\theta$

For $r > a$, $V(r,\theta) = -E_o r\cos\theta + E_o \dfrac{a^3(\epsilon_r - 1)}{r^2(\epsilon_r + 2)}\cos\theta.$

2.29 $\dfrac{1}{3}\cos 2\phi P_2^2(\cos\theta).$

2.31 (a) $$\frac{32}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{m[1-(-1)^m e^{\pi}]}{(m^2+1)(2n-1)(2p-1)} \cdot \frac{\sin mx \sin(2n-l)y \sin(2p-l)z}{[m^2+(2n-1)^2+(2p-1)^2]}$$

(b) $$-\frac{128}{\pi^3}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}\frac{1}{(2m-3)(3m^2-1)(2n-1)(2p-1)} \cdot \frac{\sin(2m-1)x \sin(2n-l)y \sin(2p-l)z}{[(2m-1)^2+(2n-1)^2+(2p-1)^2]}.$$

2.33 $$V(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{2\rho_o \cos m\pi(\cos n\pi - 1)}{\pi^2 mn\epsilon[(m\pi/a)^2+(n\pi/b)^2]}\sin(m\pi x/a)\sin(n\pi y/b).$$

2.35
$$V = \begin{cases} \displaystyle\sum_{k=1}^{\infty}\sin\beta x[a_n \sinh\beta y + b_n\cosh\beta y], & 0 \le y \le c \\ \displaystyle\sum_{k=1}^{\infty} c_n \sin\beta x \sinh\beta y, & c \le y \le b \end{cases}$$

where $\beta = \dfrac{n\pi}{a}$, $n = 2k-1$.

2.37 $P_1^0 = 0.5$, $P_2^0 = -1.250$, $P_2^1 = 1.29904$, $P_3^2 = 2.25$, $P_3^0 = -0.4375$, $P_3^1 = 0.32476$, $P_3^2 = 5.625$, $P_3^3 = 9.74279$, etc.

2.39 Derivation/Proof.

2.41 Proof.

2.43 Proof.

2.47 See Fig. E.1.

2.49 See Fig. E.2.

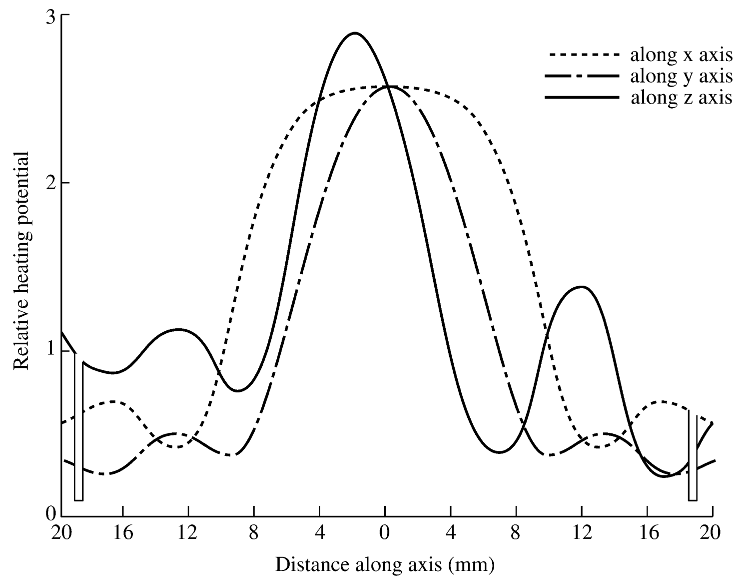

Figure E.1
For Problem 2.47.

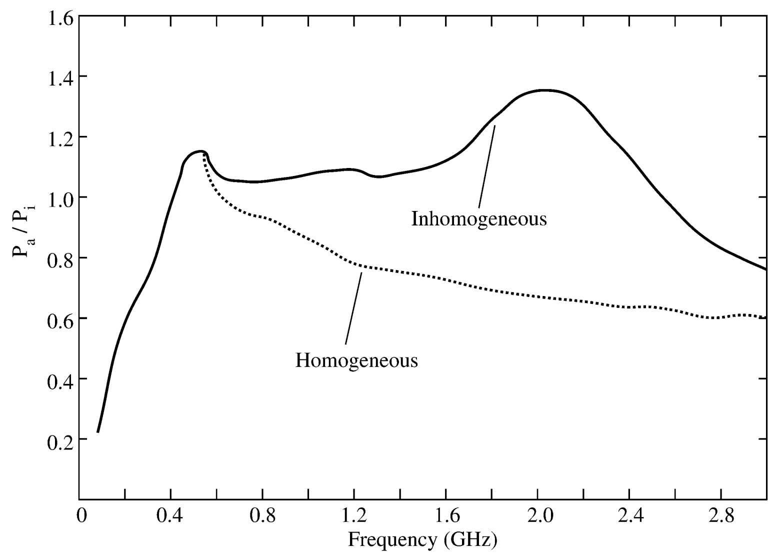

Figure E.2
For Problem 2.49.

Chapter 3

3.1 Proof.

3.3 $(1-a^2)[\Phi(i+1,j+1)+\Phi(i-1,j-1)]-2[\Phi(i,j+1)+\Phi(i,j-1) -a^2\Phi(i+1,j)-a^2\Phi(i-1,j)]+\Phi(i-1,j+1)+\Phi(i-1,j-1)-\Phi(i+1,j-1)-\Phi(i-1,j-1)=0.$

3.5 Proof.

3.7 $V_A = 61.46 = V_E$, $V_B = 21.95 = V_D$, $V_C = 45.99V$.

3.9 $16.67V$.

3.11 $r \leq 1/2$.

3.13 (a) Proof, (b) Proof, (c) Euler: $r \leq 1/4$ for stability, Leapfrog: unstable, Dufort-Frankel: unconditionally stable.

3.15 (a) After 5 iterations, $V_1 = 73.79$, $V_2 = 79.54$, $V_3 = 40.63$, $V_4 = 45.31$, $V_5 = 61.33$

(b) After 5 iterations, $V_1 = 19.93$, $V_2 = 19.96$, $V_3 = 6.634$, $V_4 = 6.649$.

3.19 (a) 60.51 Ω, (b) 50.44 Ω.

3.21 $k_c = 4.443$ (exact), $k_c = 3.5124$ (exact).

3.23 Proof.

3.25 See Fig. E.3.

3.27 68.85, 23.32, 6.4, 10.23, 10.34.

3.29 Proof.

3.31

$$
\begin{aligned}
\mu_o \frac{\partial H_{xy}}{\partial t} + \sigma_y^* H_{xy} &= -\frac{\partial}{\partial y}\left(E_{zx} + E_{zy}\right) \\
\mu_o \frac{\partial H_{xz}}{\partial t} + \sigma_y^* H_{xz} &= \frac{\partial}{\partial z}\left(E_{yx} + E_{yz}\right) \\
&\vdots \\
\epsilon_o \frac{\partial}{\partial t} E_{zx} + \sigma_x E_{zx} &= \frac{\partial}{\partial x}\left(H_{yx} + E_{yz}\right) \\
\epsilon_o \frac{\partial}{\partial t} E_{zy} + \sigma_y E_{zy} &= -\frac{\partial}{\partial y}\left(H_{xy} + E_{xz}\right).
\end{aligned}
$$

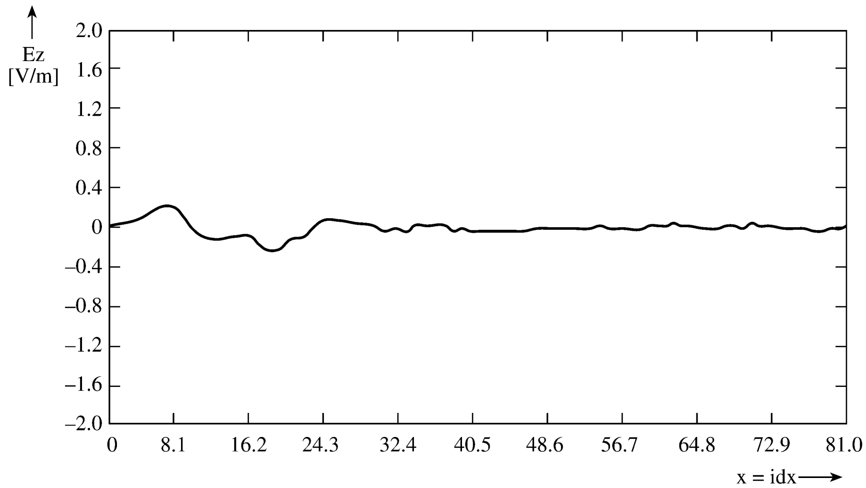

Figure E.3
For Problem 3.25.

3.33 Proof.

3.35 (a) 0.9047, (b) 0.05324.

3.37 1.218.

3.43 (a) 1.724, (b) 3.963, (c) 15.02.

Chapter 4

4.1 (a) 1.3333, (b) −4.667, (c) 157.08.

4.3 (a) $y'' = 0$, (b) $1 + y'^2 - yy'' = 0$,
(c) $xy' \cos(xy') + \sin(xy') = 0$, (d) $y'' + y = 0$,
(e) $2y^{iv} - 10y = 0$, (f) $3u + 2v'' = 0$.

4.5 Proof.

4.7 Proof.

4.9 $\rho_v = \nabla \cdot \mathbf{D}$.

4.11 $I = \dfrac{1}{2}\int_v \left[\epsilon_x \left(\dfrac{\partial V}{\partial x}\right)^2 + \epsilon_y \left(\dfrac{\partial V}{\partial y}\right)^2 + \epsilon_z \left(\dfrac{\partial V}{\partial z}\right)^2 - 2\rho_v V\right] dv.$

4.13 $\frac{1}{2}\int \left[(y')^2 + y^2 - 2y\sin\pi x\right] dx$

4.15 For exact, $\Phi = 2.1667x - 0.1667x^3$,

for $N = 1$, $\widetilde{\Phi} = 2.25x - 0.25x^2$

for $N = 2, 3$, $\widetilde{\Phi}$ is the same as the exact solution.

4.17 (a) $a_1 = 10.33, a_2 = -1.46, a_3 = 0.48$

(b) $a_1 = 10.44, a_2 = -1.61, a_3 = 0.67$

(c) $a_1 = 10.21, a_2 = -1.32, a_3 = 0.35$

(d) $a_1 = 10.21, a_2 = -1.32, a_3 = 0.35$

4.19 See table below.

Method	a_1	a_2
Collocation	0.9292	−0.05115
Subdomain	0.9237	−0.05991
Galerkin	0.9334	−0.05433
Least squares	0.9327	−0.06813
Rayleigh–Ritz	0.09334	−0.05433

4.21 $\widetilde{\Phi} = (1 - x)(1 - 0.2090x - 0.789x^2 + 0.2090x^3)$.

4.23 $\widetilde{\lambda} = 0.2$, $\lambda_o = 0.1969$ (exact).

4.25 $a/\lambda_c = 0.2948$.

Chapter 5

5.1 (a) Nonsingular, Fredholm IE of the 2nd kind,

(b) nonsingular, Volterra IE of the 2nd kind,

(c) nonsingular, Fredholm IE of the 2nd kind.

5.3 (a) $y = x - \int_0^x (x - t)y\,dt$

(b) $y = 1 + x - \cos x - \int_0^x (x - t)y(t)\,dt$

5.5 Proof.

5.7

$$G(x, y; x', y') = -\frac{4}{ab} e^{x'-x} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[\frac{\sin \frac{m\pi x}{a} \sin \frac{m\pi y}{b} \sin \frac{m\pi x'}{a} \sin \frac{m\pi y'}{b}}{1 + (m\pi/a)^2 + (n\pi/b)^2} \right]$$

5.11 Proof.

5.13 Proof.

5.15 (a) 62.71 Ω, (b) 26.75 Ω.

5.21 (a) Proof, (b) See Fig. E.4(a), (c) See Fig. E.4(b).

5.23 (a) See Fig. E.5(a), (b) See Fig. E.5(b).

5.25 The distribution of normalized field, $|\mathbf{E}_x|/|\mathbf{E}^i|$, is shown in Fig. E.6.

5.27 See table below.

Cell	E_n
64	0.1342
65	0.3966
66	0.4292
67	0.1749
74	0.0965
75	0.3925
76	0.4173
77	0.1393
84	0.1342
85	0.3966
86	0.4293
87	0.1749

Chapter 6

6.1 (a) $\begin{bmatrix} 0.5909 & -0.1364 & -0.4545 \\ -0.1364 & 0.4545 & -0.3182 \\ -0.4545 & -0.3182 & 0.7727 \end{bmatrix}$

(b) $\begin{bmatrix} 0.6667 & -0.6667 & 0 \\ -0.6667 & 1.042 & -0.375 \\ 0 & -0.375 & 0.375 \end{bmatrix}$

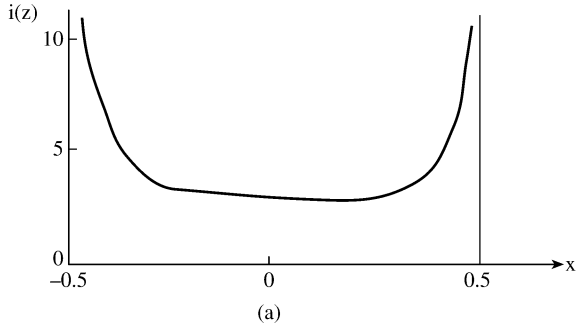

Figure E.4
(a) For Problem 5.21.

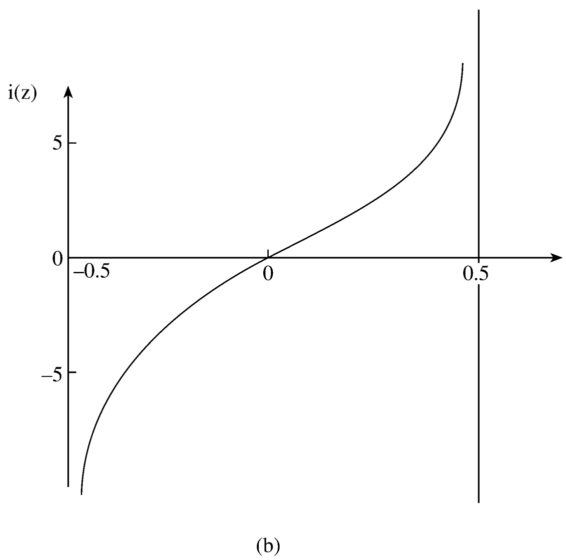

Figure E.4
(b) For Problem 5.21.

6.3 $\alpha_1 = \dfrac{1}{23}[4x + 3y - 24], \alpha_2 = \dfrac{1}{23}[-5x + 2y + 30], \alpha_3 = \dfrac{1}{23}[x - 5y + 17].$

6.5 $$\frac{1}{2}\epsilon\,[V_1 \quad V_2 \quad V_3]\begin{bmatrix} \dfrac{h_y V_1}{2h_x} - \dfrac{h_y V_2}{2h_x} \\ -\dfrac{h_y V_1}{2h_x} + \dfrac{V_2(h_x^2 + h_y^2)}{2h_x h_y} - \dfrac{V_3 h_x}{2h_y} \\ -\dfrac{h_x V_2}{2h_y} + \dfrac{h_x V_3}{2h_y} \end{bmatrix}$$

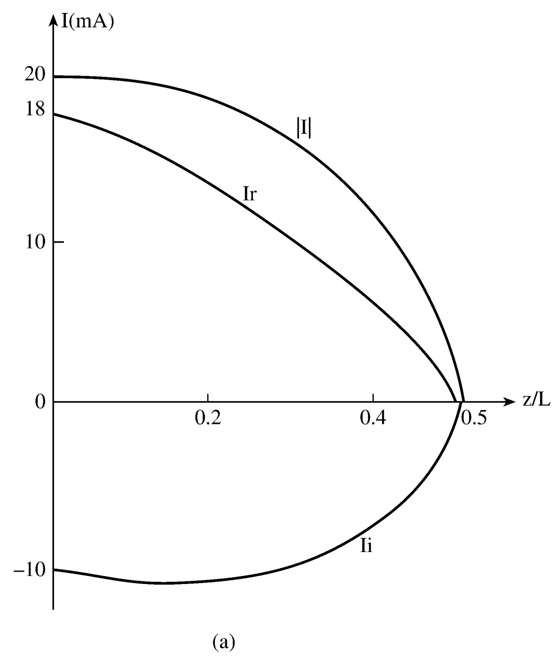

Figure E.5
(a) For Problem 5.23.

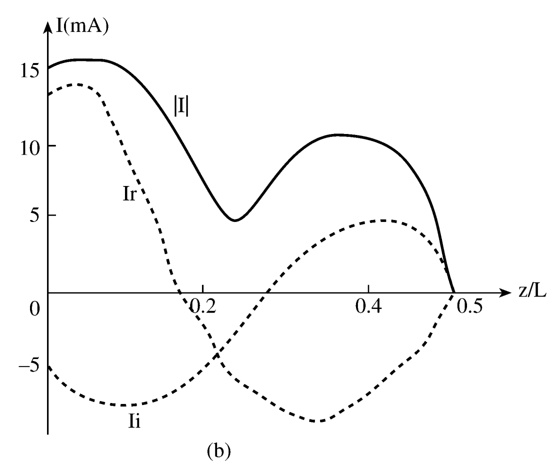

Figure E.5
(b) For Problem 5.23.

0.0159	0.0161	0.0155	0.21
0.011	0.112	0.108	0.0155
0.0115	0.0116	0.0112	0.016
0.0112	0.0114	0.011	0.0158
0.0112	0.0114	0.011	0.0158
0.0112	0.0114	0.011	0.0158
0.0112	0.0114	0.011	0.0158
0.0112	0.0114	0.011	0.0158
0.0112	0.0114	0.011	0.0158
0.0115	0.0116	0.0112	0.0155
0.0110	0.0112	0.0108	0.0155
0.0159	0.0161	0.0155	0.0211

Figure E.6
For Problem 5.25.

6.9 Calculate **E** using

$$\mathbf{E} = -\frac{1}{2A}\sum_{i=1}^{3} P_i V_{ei}\,\mathbf{a}_x - \frac{1}{2A}\sum_{i=1}^{3} Q_i V_{ei}\,\mathbf{a}_y$$

6.11 See Fig. E.7.

6.13 $\lambda_{c,mn} = \frac{a}{2}\sqrt{(m+n)^2 + n^2}$, $a = 1$.

6.17 $B = 14$. See the mesh in Fig. E.8, $B = 4$.

6.19 Proof.

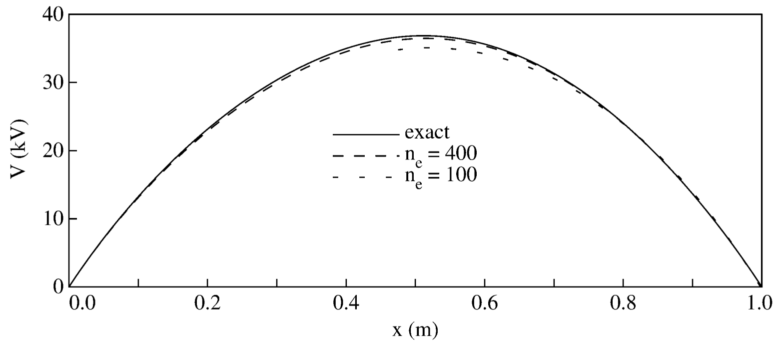

Figure E.7
For Problem 6.11.

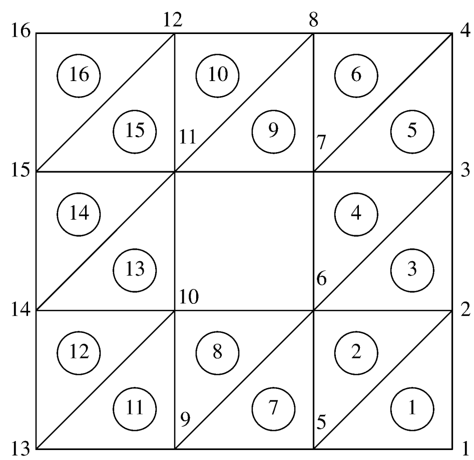

Figure E.8
For Problem 6.17.

6.21 (a) $\dfrac{32A}{180}$, (b) $-\dfrac{A}{45}$, (c) 0.

6.23 $\dfrac{A}{180}\begin{bmatrix} 6 & 0 & 0-1 & 4 & -1 & \\ 0 & 32 & 16 & 0 & 16 & -4 \\ 0 & 16 & 32 & -4 & 16 & 0 \\ -1 & 0 & -4 & 6 & 0 & -1 \\ -4 & 16 & 16 & 0 & 32 & 0 \\ -1 & -4 & 0 & -10 & 6 & \end{bmatrix}$

6.25

$$D^{(1)} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad D^{(2)} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad D^{(3)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$D^{(1)} = \begin{bmatrix} 3 & 1 & 1 & -1 & -1 & -1 \\ 0 & 2 & 0 & 4 & 2 & 0 \\ 0 & 0 & 2 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 3 & 1 & -1 \\ 0 & 0 & 2 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D^{(3)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 4 & 2 & 0 \\ -1 & -1 & 1 & -1 & 1 & 3 \end{bmatrix}$$

6.27 Proof.

6.29 $\dfrac{v}{20}\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

6.31 $B_1 = \dfrac{\partial}{\partial \rho} + jk + \dfrac{1}{2\rho}$

$$B_2 = \frac{\partial}{\partial \rho} + jk + \frac{1}{2\rho} - \frac{1}{8\rho(1 + jk\rho)} - \frac{1}{2\rho(1 + jk\rho)}\frac{\partial^2}{\partial \phi^2}.$$

Chapter 7

7.1 Proof.

7.3 Proof.

7.5 $\Delta\ell/\lambda = 0.0501$.

7.7 See Fig. E.9.

7.9 Proof.

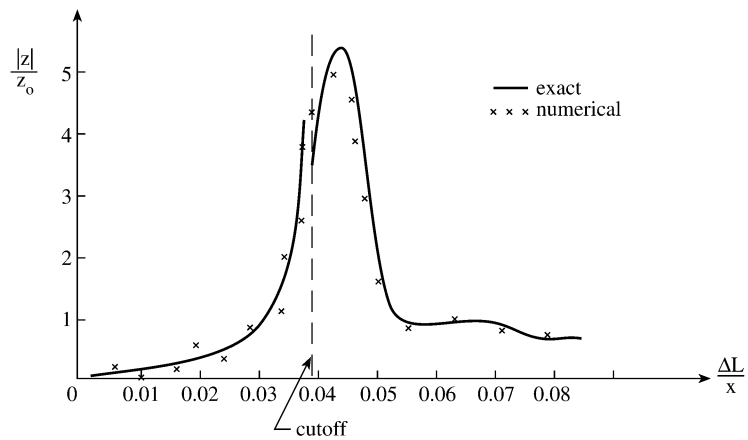

Figure E.9
For Problem 7.7.

7.11 Proof.

7.13 $\frac{1}{6}$ ns.

7.15 See Table below.

	$\lvert Z \rvert$		Arg(Z)	
$\Delta\ell/\lambda$	TLM	Exact	TLM	Exact
0.023	4.1981	6.1272	−0.2806	−0.0106
0.025	2.382	2.4898	1.2546	1.0610
0.027	0.3281	0.3252	−0.7952	−0.8554
0.029	5.2724	5.1637	0.8459	0.8678
0.031	0.2963	0.3039	−1.1340	−1.1610
0.033	1.8117	1.8038	1.3408	1.3385
0.035	0.8505	0.8529	−1.3820	−1.4025
0.037	0.4912	0.4838	1.3914	1.3932
0.039	5.3772	5.4883	−1.1022	−1.1155
0.041	0.2115	0.2179	−1.2795	−1.3174

7.17 For $\epsilon_r = 2$, $k_c a = 1.303$; for $\epsilon_r = 8$, $k_c a = 0.968$.

7.19 See Fig. E.10.

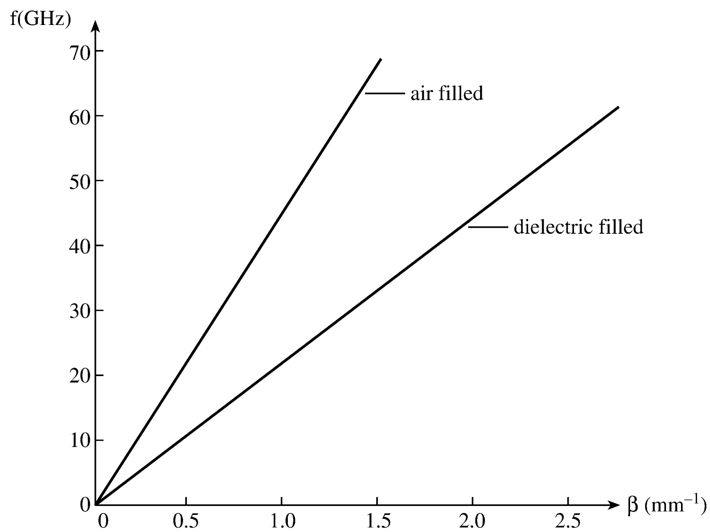

Figure E.10
For Problem 7.19.

Chapter 8

8.3 (a) 16, 187, 170, 429, 836, 47, 950, 369, 456, 307

(b) 997, 281, 13, 449, 277, 721, 133, 209, 757, 761.

8.7 $M = 5, a = 0, b = 1$. Generate the random variable as follows:

(1) Generate two uniformly distributed random variables U_1 and U_2 from (0, 1).

(2) Check if $U_2 \leq f_X(U_2)/M = U_2$.

(3) If the inequality holds, accept U_2.

(4) Otherwise, reject U_1 and U_2 and repeat (1) to (3).

8.9 (a) 3.14159 (exact), (b) 0.4597 (exact), (c) 1.71828 (exact), (d) 2.0.

8.11 0.4053 (exact).

8.13 2.5 (exact).

8.15 $V(0.4, 0.2) = 1.1$, $V(0.35, 0.2) = 1.005$, $V(0.4, 0.15) = 1.05$, $V(0.45, 0.2) = 1.15$, $V(0.4, 0.25) = 1.15$.

8.17 $2.991V$.

8.19 1.2 V.

8.21 $V(2, 10) = 65.85$, $V(5, 10) = 23.32$, $V(8, 10) = 6.4$, $V(5, 2) = 10.23$, $V(5, 18) = 10.34$.

8.23 (a) 0.33, 0.17, 0.17, 0.33,

(b) 0.455, 0.045, 0.045, 0.455.

8.25 12.11 V.

8.27 10.44 V.

8.29 25.0 V.

8.31 See Table below.

Node (ρ, z)	Markov chain	Exodus method	Finite difference
(5, 18)	11.39	11.44	11.47
(5, 10)	27.47	27.82	27.87
(5, 2)	12.31	12.18	12.13
(10, 2)	2.448	2.352	2.342
(15, 2)	0.4684	0.3842	0.3965

Chapter 9

9.1 Proof.

9.3 $$T_{ij} = \sqrt{\frac{2}{N + 1/2}} \cos \frac{(i - 0.5)(j - 0.5)\pi}{N + 0.5}$$

$$\lambda_k = 2 \sin \left(\frac{k - 0.5}{N + 0.5} \right) \pi$$

9.9 Proof.