Numerical simulations with a mixing model for plasma interpenetration

R. Sentis

with C. Baranger, D. Paillard, G. Carré

CEA/Bruyères (France)

Standard hydrodynamics simulations lead to very high unphysical densities (not in agreement with experimental emissivity).

Aim : Modification radiative two-temperature hydrodynamics system.

When the temperature is very high, ion mean free paths are not small enough. Pure hydro is no more valid.

Outline:

A. The mixing model

B. Implementation (in a Lagrange/ALE code)

C. Numerical results

A. THE MIXING MODEL

Theoretical starting system

Six equations (+equation for electron temperature T_e). Two fluids ($q = g$ and l)

$$\rho_g, \rho_l, \qquad \mathbf{u}_g, \mathbf{u}_l, \qquad \varepsilon_g, \varepsilon_l$$

Assumption: The fluids fill the same volume when mixing occurs; $P_q = (\gamma - 1)\rho_q \varepsilon_q$.

$$\frac{\partial}{\partial t}\rho_q + \nabla\left(\rho_q \mathbf{u}_q\right) = 0,$$

$$\left(\frac{\partial}{\partial t} + \nabla\left(\mathbf{u}_q \cdot\right)\right)\left(\mathbf{u}_q\right) + \nabla P_q = \mathbf{\Xi}_q,$$

$$\left(\frac{\partial}{\partial t} + \nabla\left(\mathbf{u}_q \cdot\right)\right)\left(\rho_q \varepsilon_q + \frac{1}{2}\rho_q |\mathbf{u}_q|^2\right) + \nabla.(\mathbf{u}_q P_q) = \Omega_q + C_{e,q},$$

where $C_{e,q}$ is a coupling term with electron temperature, $\Omega_g = -\Omega_l$ and the drag term may reads as

$$\mathbf{\Xi}_g = -\mathbf{\Xi}_l = \nu^0 \rho_g \rho_l (\mathbf{u}_l - \mathbf{u}_g),$$

Principle of derivation: 1. a five-equations model / 2. Diffusion approximation

1. Intermediate model (5-equations):

Density, concentration, mean velocity, mean internal energy, relative velocity

$$\rho = \rho_g + \rho_l, \qquad c = \frac{\rho_g}{\rho}, \qquad \mathbf{u} = c\mathbf{u}_g + (1-c)\mathbf{u}_l, \qquad \varepsilon = c\varepsilon_g + (1-c)\varepsilon_l, \qquad \mathbf{V} = \mathbf{u}_g - \mathbf{u}_l,$$

Standard calculus. Set $P_* = (\gamma - 1)\rho\varepsilon$ and $\overline{\overline{P_b}} = \rho \mathbf{V}\mathbf{V} c(1-c)$ mixing pressure

$$\begin{aligned}
&\rho D_t \rho^{-1} - \nabla.\mathbf{u} = 0, \qquad D_t = \partial/\partial t + \mathbf{u}\nabla \\
&\rho D_t \mathbf{u} + \nabla P_* + \nabla.\overline{\overline{P_b}} = 0, \\
&\rho D_t c + \nabla\left(\rho c(1-c)\mathbf{V}\right) = 0,
\end{aligned}$$

1st Closure: $\frac{1}{\rho_g}\nabla P_g - \frac{1}{\rho_l}\nabla P_l \rightrightarrows \nabla(\Psi(c)\varepsilon)$ with $\Psi(1) = \gamma, \quad \Psi(0) = -\gamma,$ e.g. $\Psi(c) = \gamma(c - \frac{1}{2})$

$$\rho D_t(\frac{\mathbf{V}}{\rho}) - \nabla.\left(|\mathbf{V}|^2(c - \frac{1}{2})\right) + \nabla(\Psi\varepsilon) = -\sigma\mathbf{V}, \qquad \sigma = \rho\nu^0 \tag{1}$$

$$\rho D_t E + \nabla.(P_*\mathbf{u}) + \nabla.(\overline{\overline{P_b}}\mathbf{u}) + \nabla.\left(\mathbf{V}c(1-c)...\right) = C_{e,i}.$$

This is closed to the "mix model" of Scannapieco-Cheng (2002) [hyperbolic system for $|\mathbf{V}|$ small enough].

2. **Diffusion approximation**

Second Closure: (1) is replaced by

$$\mathbf{V} = -\frac{1}{\sigma}\psi' \varepsilon \nabla c \equiv -D\nabla c.$$

Then, one gets a diffusion eq.

$$\rho D_t c - \nabla.\left(\rho c(1-c) D \nabla c\right) = 0,$$

Mixing kinetic energy

$$K = \frac{1}{2} c(1-c)|\mathbf{V}|^2 \ , \qquad \text{Then mixing pressure} \qquad \overline{\overline{P_b}} = 2\rho K \text{ or } 2\rho K \frac{\overline{\overline{\nabla c \nabla c}}}{|\nabla c|^2}$$

$$\rho D_t K + \overline{\overline{P_b}}.\nabla \mathbf{u} + 2\sigma\rho K = ...$$

Since $\varepsilon = E - \frac{1}{2}|\mathbf{u}|^2 - K$, one gets a simple equation for $\rho D_t \varepsilon = ...$

3. Final system.

Besides the eq. for the electron temperature T_e,

$$\begin{aligned}
\rho D_t \rho^{-1} - \nabla.\mathbf{u} &= 0, && (2)\\
\rho D_t \mathbf{u} + \nabla P_* + \nabla.\overline{\overline{P_b}} &= 0 && (3)\\
\rho D_t c - \nabla.(c(1-c)D\nabla c) &= 0, && (4)\\
\rho D_t \varepsilon + P_* \nabla.\mathbf{u} - 2\sigma\rho K &= C_{e,i} + \Psi\nabla.(\rho\varepsilon c(1-c)D\nabla c), && (5)\\
\rho D_t K + \overline{\overline{P_b}}.\nabla\mathbf{u} + 2\sigma\rho K &= \sigma\rho c(1-c)|D|^2|\nabla c|^2 - \nabla\left((2c-1)\rho K D\nabla c\right). && (6)
\end{aligned}$$

It is conservative. Classical form.

Without the terms at the right hand side, it is the same than the simplest compressible turbulence model, see Wilcox (1998), Gavrilyuk-Saurel (2006).

B. IMPLEMENTATION.

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- Remark : the Heaviside function is a trivial solution to (4), (no mixing occurs).
- It is unstable: one has to give an initialization time for the mixing (for instance by setting $c = .9999999$ in only one cell, instead of $c = 1$)
- Due to the expression of the friction coef. the diffusion coefficient D reads as

$$D(c) \simeq \frac{C_v T_*}{\beta} \frac{\Psi'(c)}{\rho} \left(\frac{T_*}{m_*} + V_r^2 \right)^{\frac{3}{2}}$$

  For the corrector term V_r^2, we have choosen

$$V_r{}^2 = T_*^5 (C_v / L\rho\beta)^2$$

  the constant L is a characteristic mean free path at the begining of the mixing process.

One checks that the influence of L on the results is small.

Arbitrary-Lagrangian-Eulerian hydro code.

ρ, c, ε, K are evaluated in the center of the cells ; $\mathbf{u}$ at the nodes

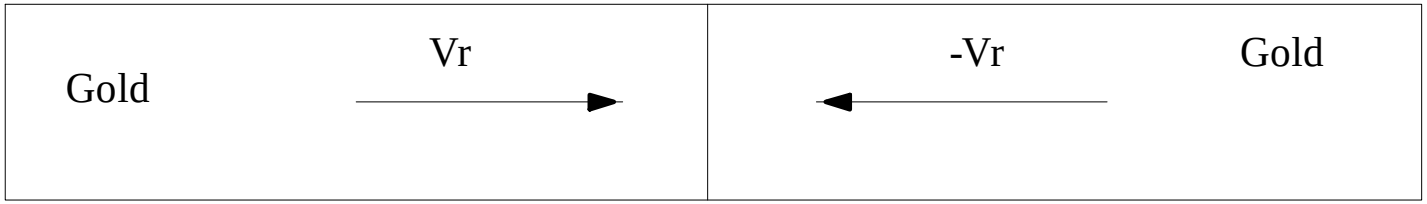
At each time step

- Move the nodes of the mesh
- Solve non-linear diffusion equation for c: iterative method (due to the non-linearity)
- Solve the internal energy equations for ions and electrons
  Electron thermal conduction is taken into account by using the classical Spitzer-Härm formula.
- Solve the mixing energy equation (localy with respect to the spatial variable).
- Mesh rezoning. Remapping all the quantities.

C. Numerical Results

1. Toy problem in a 1D configuration

Initially, two Gold sheets with opposite velocity $V_r = 10^8 cm/s$ which are separated by a vacuum.

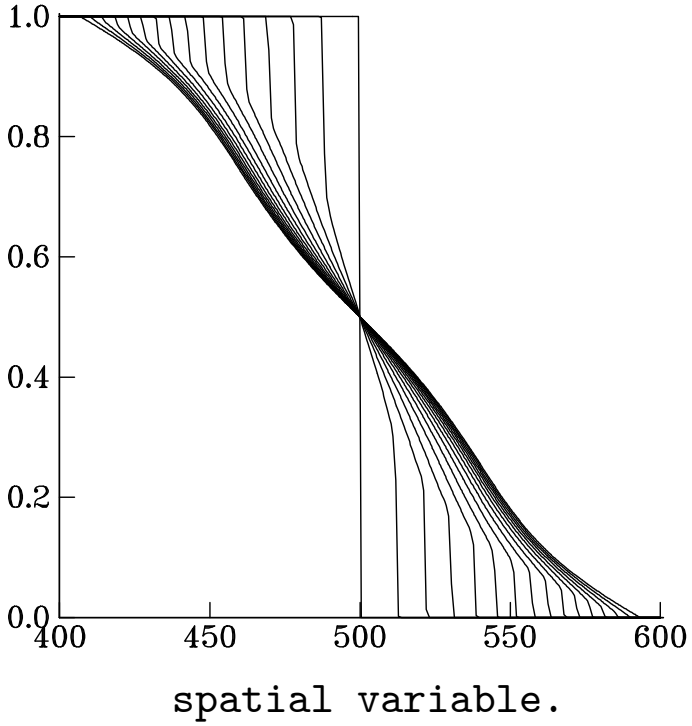


The initial density of both plates is equal to 3. 10^{-3} g/cm^3.

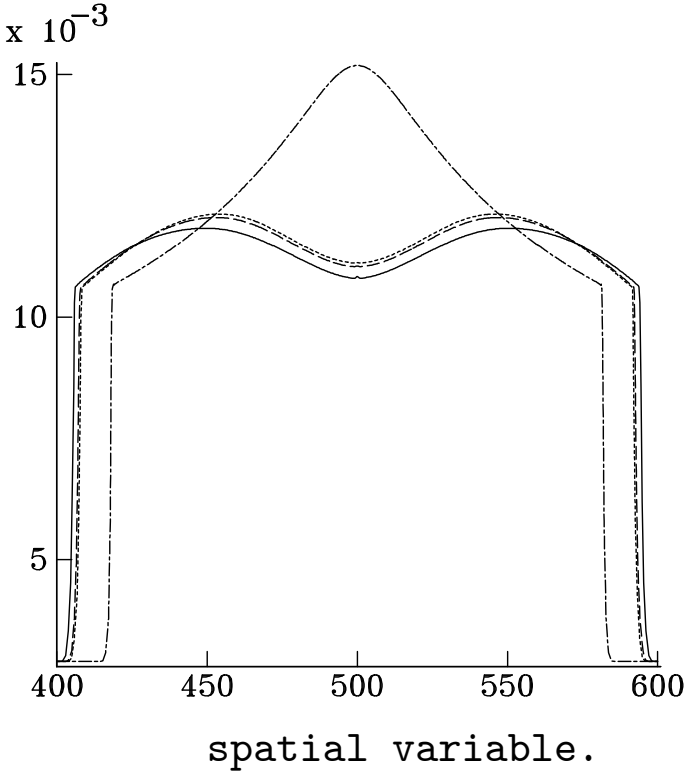
The initial ion and electron temperatures are equal to 1 keV and 2 keV.

The spatial domain is 1000 μm long.

Profiles of the concentration (from 0 up to 300 ps).



Profiles of density with/without mixing (at 300 ps)



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Results with different values of the corrector parameter L.

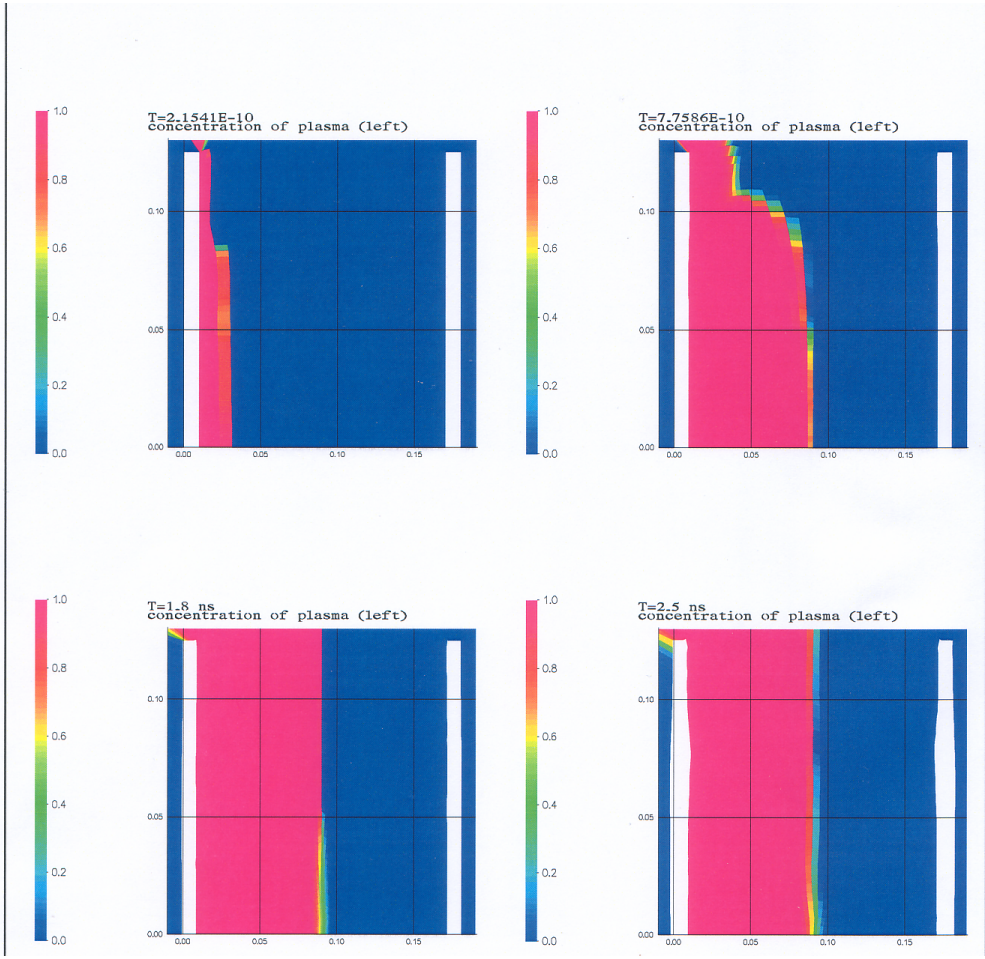
One checks that the behavior of c is characteristic of a non linear diffusion eq.

Notice that with the mixing model, there is a strong descreasing of the density where the two fluids collide.

2. A 2D configuration

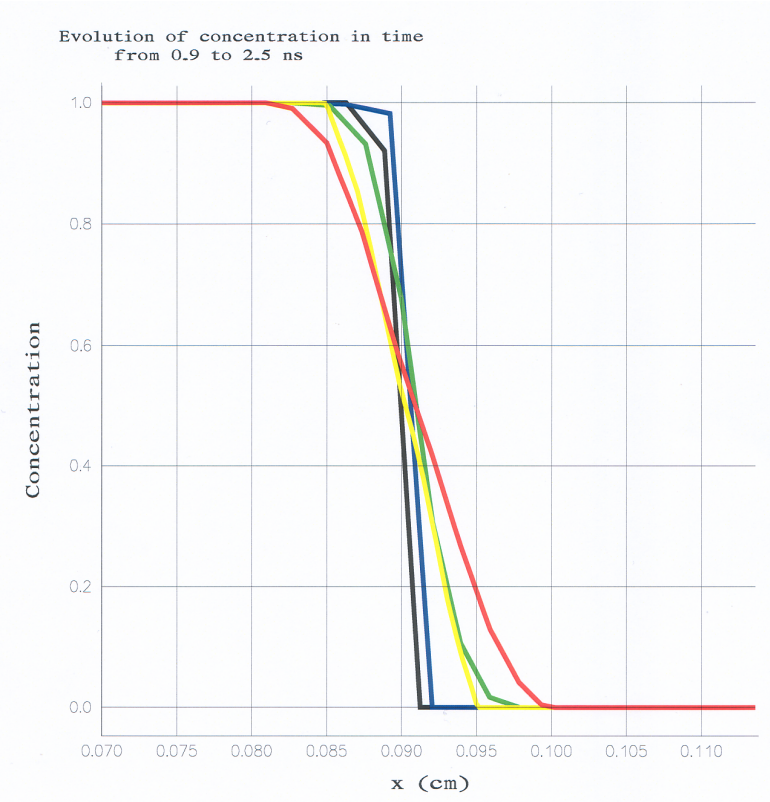
2D axisymmetric simulation : collision of two Titanium plasma disks; the relative velocity is obtained by a laser ablation.

Map of the concentration (at different times: 0.2, 0.8, 1.8, 2.5 ns).

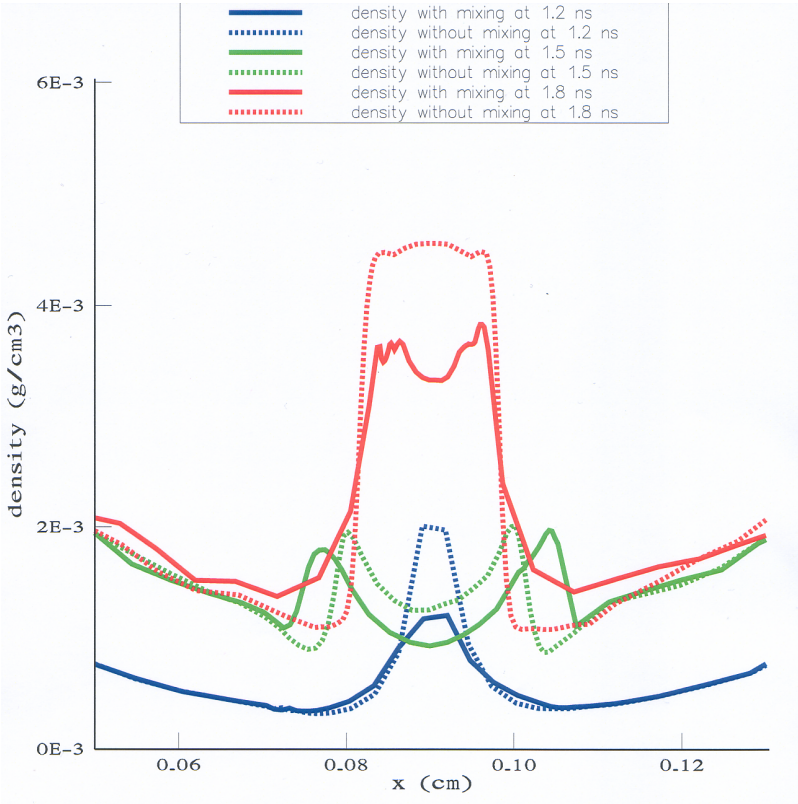


On the symmetry axis

Profile of concentration at different times.



Profile of density at different times



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Simulation with (dotted lines) and without the mixing model (plain lines)

Notice, as request, smaller density in the center of the simulation domain.

Since the modification of T_e is very moderated, the radiative emission (which is roughly speaking proportionnal to ρ^2) is weakened.

Conclusions

- The model may be implemented in a classical 2D hydro-radiative code wich uses a ALE technique.
- One has to introduce a diffusion eq. for the concentration c.

  and a supplementary eq. for the mixing kinetic energy K (which is of the same type of the turbulence one).
- For accurate results, the mesh size has to be fine enough.

  It is necessary to give an initialization criterion for the mixing model.

  But in any case, when compared to the classical simulation (without the mixing model) the profiles of the density are modified : its maximum is lower.