The Application of Multi-Phase Flow Models in Simulations of Fluid Structure Interaction

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September 2007
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This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.
Fluid-structure interaction problem arises in numerous engineering applications:

- Aerospace (e.g. airplane design, bird strike scenarios, engine blade containment).
- Automotive (e.g. airbag design, tire performance and hydroplaning).
- Medical equipment design (e.g. artificial heart valves, lithotryptors).
- Military and defense (e.g. weapons design, armor design, personnel protection, underwater shock explosion, blast resistance).
- Failure analysis of structures (e.g. blast loading due to pressure vessel rupture).
Motivation

- Examples of simulation techniques for fluid-structure interaction:
  - Coupled Lagrangian-Eulerian methods.
  - Eulerian methods.
  - ALE methods.
  - Particle methods (e.g. SPH).
  - ...

- In the context of Eulerian methods:
  - Need to deal effectively with different materials-interface tracking difficult.
  - Materials include solids, fluids or gases.
  - Chemical reactions or phase transitions may also be involved.

- Finite volume Godunov methods well suited for handling multiple materials - **Discrete Equation Method** (Chinnayya et al., 2004).

Cirak and Radovitzky (2005)

Chinnayya et al. (2004)
**Key idea** (Abgrall and Saurel, 2003):
- Discretize Euler equations at the microscopic level for all pure phases via a Godunov scheme.
- Average the discretization over the set of all possible realizations to obtain a scheme for the averaged multiphase flow equations.

- Treatment of each phase is Eulerian.
- The original DEM formulation allowed for the treatment of fluids.
- Extended to reactive fluid flows by Chinnayya *et al.* (2003).
- Would like to model materials with strength (solids): need to extend the treatment of the mechanical response to include the deviatoric stress component.
Conservation Equations

mass:
\[ \frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = 0 \]

momentum:
\[ \frac{\partial \rho \mathbf{v}}{\partial t} + \text{div}(\rho \mathbf{v} \otimes \mathbf{v} - \mathbf{\sigma}) = 0 \]

energy:
\[ \frac{\partial \rho(e + \frac{1}{2} \mathbf{v}^2)}{\partial t} + \text{div}[\rho(e + \frac{1}{2} \mathbf{v}^2)\mathbf{v} - \mathbf{\sigma} \mathbf{v}] = 0 \]

or
\[ \rho \dot{e} = \mathbf{\sigma} \cdot \text{grad}(\mathbf{v}) \equiv \mathbf{\sigma} \cdot \mathbf{L} \]

Need closure: material constitutive relations
Material Constitutive Equations

- Relate the stress to some measure of deformation and temperature.
- Must satisfy certain constraints imposed by
  - Thermodynamics (e.g. Clausius-Duhem inequality).
  - Locality.
  - Material frame indifference.
- May also depend on a number of internal (history) variables to track micro-structure development-inelastic material response.
- Existence of a Helmholtz free energy functional from which the material response derives is often assumed.
- Commonly the material response split into volumetric (EOS) and isochoric (strength) components.
- Example: \( \sigma^{VJ} = C \cdot \frac{1}{2}[L + L^T] \) - hypoelastic constitutive relation.
- Need a solution of the Riemann problem to evaluate fluxes of conserved variables.
- Use conservation equations for a solid with linear elastic material constitutive relation:

\[ \sigma = \lambda \text{tr}(\varepsilon) + 2\mu \varepsilon \]

where: \( \lambda, \mu \)-elastic constants and \( \varepsilon \)-the infinitesimal strain tensor.
- Non-linear (geometric) effects due to finite kinematics neglected.
- Riemann solvers taking into account finite kinematics available (e.g. Garaizar, 1991; Miller and Colella, 2001).
Acoustic Riemann Solver

\[
\begin{align*}
c_L &= \sqrt{\frac{\lambda + 2\mu}{\rho}} \\
c_T &= \sqrt{\frac{\mu}{\rho}}
\end{align*}
\]

- Two elastic waves: longitudinal \((c_L)\) and tangential \((c_T)\).
- Treatment of tangential waves analogous to longitudinal waves.
- Quantities discontinuous across shear waves: \((\sigma_{12}, v_2)\) and \((\sigma_{13}, v_3)\).
- Rod: $l = 10$ cm and $h = 1$ cm.
- Discretization: 1 zone per $h$ and 100 zones per $l$.
- Periodic boundary conditions in $y,z$-directions.
- Outflow boundary conditions at both ends.
- Initial conditions: velocity of 1 m/s in both $x$ and $y$-directions applied to left half of the rod ($x < 0$).
- Material: linear elastic steel $\lambda = 87$ GPa and $\mu = 80$ GPa.
- Elastic wave velocities: $c_L = 0.563$ cm/μs and $c_T = 0.320$ cm/μs.
- Velocity solutions at 10 μs.
- Two-elastic-wave structure present in the solution.
- Elastic wave velocities match exactly those computed from the elastic constants.
Taylor Impact Test

- Fundamental test for characterizing the inelastic mechanical response of structural materials (plastic deformation, shear band formation and fracture).
- Experimental data: House et al. (1995).
- Material: 4340 low strength steel.
- Sample dimensions: $d = 7.595$ mm, $h = 11.39$ mm.
- Impact velocity: 285 m/s.
- Final length: 9.3 mm.
- Mushroom diameter: 10.9 mm.

Wang et al. (2003)
- 4340 steel material model:
  - EOS: 7-term polynomial.
  - Strength: conventional $J_2$-plasticity, no strain hardening.
- Air material model: $\gamma$-law gas EOS. No strength.
- Quarter configuration modeled (3D).
- 139,194 total number of zones.
Final simulated cylinder length: 7.02 mm (experiment: 9.3 mm).
Mushroom diameter: 12.1 mm (experiment: 10.9 mm).
Very good agreement with the experimental data.
Need to include strain hardening effects to obtain a better match!
- Investigate interaction of a shock wave with a solid body.
- $d = 20\ \text{mm}, \ h = 5\ \text{mm}$.
- 5 Mach shock.
- Steel modeled as:
  - EOS: 7-term polynomial.
  - Strength: $J_2$-plasticity, no strain-hardening.
- Air modeled as:
  - EOS: $\gamma$-law gas.
  - No strength.
- Quarter configuration modeled (3D).
- Discretization: 31,164 zones.
Summary

- Extended the DEM-based multi-phase flow approach of Chinnayya *et al.* to materials with strength.
- The necessary changes confined to an acoustic Riemann solver.
- The validity of our multi-phase flow model for solids verified on a simple 1D dimensional problem.
- Multi-phase flow model applied to a Taylor problem involving a steel cylinder impacting a rigid anvil. Good agreement with experimental measurements.
- The effort on modeling an interaction of a shockwave with a steel plate ongoing.
- Future work: ballistic impact and penetration, modeling of the effects of blast-waves on solids and structures.