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The novel discretization of the Continuum Surface Force model and improved numerical modelling of surface tension effects via a...
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The method is a member of the class of balanced force algorithms. A numerical technique is presented in which the only potential source of spurious currents lies in curvature estimation. The imbalance between surface tension forces and associated pressure gradients in the CFS model generates spurious velocities, at fluid interfaces. Force acting over the fluid interface, due to surface tension, as a smoothly varying volume fraction, reformulates the discontinuous jump conditions at fluid interfaces.

Motivation
\[
\begin{align*}
\frac{\partial p}{\partial x} + \frac{\partial h}{\partial y} + \frac{1}{\rho} \frac{\partial h}{\partial \gamma} &= 0, \\
\text{Orthogonal curvilinear metric:} &
\end{align*}
\]

Numerical method includes Young's volume of fluid method [5] used to track interfaces. Remap multi-component, single pressure model, hydrocode. The method is currently implemented in a 2D compressible Lagrange-
ease of description the 2D analogue is given here. CFS discretization itself is fully applicable to 3D problems, though for evolved for each fluid component. Simplified compressible flow model: an internal energy equation is • Directionally split Van Leer remap method • Unsplit Lagrange phase • General numerical method overview cont
In the presence of surface tension there is a pressure jump at the interface such that fluids have pressures $p_1$, $p_2$.

Consider a four cell stencil, centered on vertex $(i,j)$, containing two fluids, with volume fractions $f_1$, $f_2$, separated by an interface.

Novel CFS discretization
\[ \iiint d \left( \frac{\zeta}{\hat{t}} + f \frac{\zeta}{\hat{t}} + i \right) + \ii d \left( \frac{\zeta}{\hat{t}} + f \frac{\zeta}{\hat{t}} + i \right) = \frac{\zeta}{\hat{t}} + f \frac{\zeta}{\hat{t}} + i \]

\[ \iiint d \left( \frac{\zeta}{\hat{t}} - f \frac{\zeta}{\hat{t}} + i \right) + \ii d \left( \frac{\zeta}{\hat{t}} - f \frac{\zeta}{\hat{t}} + i \right) = \frac{\zeta}{\hat{t}} - f \frac{\zeta}{\hat{t}} + i \]

\[ \iiint d \left( \frac{\zeta}{\hat{t}} + f \frac{\zeta}{\hat{t}} - i \right) + \ii d \left( \frac{\zeta}{\hat{t}} + f \frac{\zeta}{\hat{t}} - i \right) = \frac{\zeta}{\hat{t}} + f \frac{\zeta}{\hat{t}} - i \]

\[ \iiint d \left( \frac{\zeta}{\hat{t}} - f \frac{\zeta}{\hat{t}} - i \right) + \ii d \left( \frac{\zeta}{\hat{t}} - f \frac{\zeta}{\hat{t}} - i \right) = \frac{\zeta}{\hat{t}} - f \frac{\zeta}{\hat{t}} - i \]

of fluid, \( f \) in the cell stencil used are given as Cell centered pressures, in terms of fluid pressures and volume traction

Novel CSE discretization cont
\begin{align}
(8) \quad & f^\tau_{\tau^\tau}(\exp z \exp y \exp y \exp y \exp y) = f^\tau_{\tau^\tau}(\exp y) \quad f^\tau_{\tau^\tau}(\exp x \exp y \exp y \exp y) = f^\tau_{\tau^\tau}(\exp y) \\
& \text{Where cell face areas are given by} \quad \bullet
\end{align}

\begin{align}
(7) \quad & \left(\frac{\tau}{\tau^\tau} - f^\tau_{\tau^\tau} \right) \left( f^\tau_{\tau^\tau} + \frac{\tau}{\tau^\tau} \right) \frac{\tau}{\tau^\tau} \left( \frac{\tau}{\tau^\tau} \right) \frac{\tau}{\tau^\tau} + \left( \frac{\tau}{\tau^\tau} - f^\tau_{\tau^\tau} \right) \left( f^\tau_{\tau^\tau} + \frac{\tau}{\tau^\tau} \right) \frac{\tau}{\tau^\tau} \left( \frac{\tau}{\tau^\tau} \right) \\
& \text{In the radial direction} \quad \bullet
\end{align}

\begin{align}
(6) \quad & \left(\frac{\tau}{\tau^\tau} - f^\tau_{\tau^\tau} \right) \left( f^\tau_{\tau^\tau} + \frac{\tau}{\tau^\tau} \right) \frac{\tau}{\tau^\tau} \left( \frac{\tau}{\tau^\tau} \right) \frac{\tau}{\tau^\tau} + \left( \frac{\tau}{\tau^\tau} - f^\tau_{\tau^\tau} \right) \left( f^\tau_{\tau^\tau} + \frac{\tau}{\tau^\tau} \right) \frac{\tau}{\tau^\tau} \left( \frac{\tau}{\tau^\tau} \right) \\
& \text{In the axial direction} \quad \bullet
\end{align}

The forces on cell vertex (i,j), due to cell pressure are thus,
If the exact curvature is known (11), ensures exact equilibrium, elimi-

\[
\frac{\tan \theta}{f \partial_1} \approx \frac{\Lambda}{f \partial V}
\]

Where, in terms of the metric

\[
\left. \right|_{r=1,2} \frac{\Lambda}{f \partial_1 V}
\]

In equilibrium, the force on vertex \(i\) due to surface tension

\[
V \partial = (2d - 1d) = d \nabla
\]

surface tension coefficient and interfacial curvature, tension is described by the Laplace-Young law, where \( \partial \) denote

In a static frame the balance of forces due to pressure and surface


Finite differenciating of a convoluted volume fraction: CVF

Numerical curvature estimation

Three main techniques investigated for estimating interfacial curvature
\[
\begin{cases}
0 & \text{otherwise} \\
(\varepsilon (\frac{\nu}{a} - 1) a)^2 & \nu > \nu_r \\
(\varepsilon (\frac{\nu}{a}) - 6 + (\frac{\nu}{a}) - 1)^2 & \nu > 2 \nu_r
\end{cases}
\]

\[= (\nu, a) R \]

Kernel

Where \( \nu \) is a smoothing scale and \( R(\nu, a) \) is the following splitting cubic

\[(12) \]

\[
\frac{\varepsilon x^p x^p (\nu, |w|^2 X - \chi^p, X|) X^{|w|^2}}{\varepsilon x^p x^p (\nu, |w|^2 X - \chi^p, X|) X^{|\chi^p, f | w|^2}} = \chi^p, f
\]

Volume fraction field convolved using smoothing kernel

\textbf{Numerical curvature estimation: CVE}
Due to smoothing momentum will not be fully conserved.

\[ u \cdot \Delta - \gamma \]  

mechanical curvature

Conservative divergence of normal vectors provides estimate of nu-
Numerical curvature estimation: Youngs
Numerical curvature estimated using (13).

Simple differencing is used to calculate normal vector to the interface.

Corner values \( \{3, 5, 7, 9\} \) are used to detect cases where procedure fails.
1. Calculate simple estimate of normal vector: \( \mathbf{n} = -\Delta \mathbf{f} \)

2. Orient height function in direction of maximal normal

3. Sum volume fractions to locally evaluate height function

\( \alpha_{ij} \) in cell \((j,k)\) calculate \( \xi_{i,j}^{k} \) + 1 k

\( \lfloor \alpha_{i,j}^{k} \rfloor \) \( \alpha_{i,j}^{k} \) 

\( \alpha_{i,j}^{k} \)
5. \( \vec{\lambda} \cdot \vec{X} \), \( \dddot{\vec{X}} \) estimated using central differences

\[
\frac{z/\varepsilon}{x^{x_{\lambda}}} \left( \frac{x_{\lambda} + 1}{x_{\lambda}} \right) |\vec{\lambda}| = y
\]

4. Curvature follows from

\[
(1 - y_{\bar{h}} - y_{\bar{h}}) \int_{y_{\bar{h}} + y_{\bar{h}}}^{\varepsilon - y} \int_{y_{\bar{h}} + y_{\bar{h}}}^{\varepsilon + y} = y_{\bar{h}} \vec{\lambda}
\]
(16) \[ \left( \max \left| y - y_{\text{exact}} \right| \right)_\infty = \left( \max \left| y - y_{\text{exact}} \right| \right)_1 \]

Accuracy of methods determined via consideration of the following:

Exact curvature is $y_{\text{exact}} = \frac{1}{R}$.

Domain: A circle of radius $R = 2.0$ is centered at $(1/2, 1/2)$ in a unit square.

Numerical curvature estimation: accuracy norm.
Method of Youngs implemented.
Computed pressure through the center of the static drop.

\[ \tau = \rho \left( 1 + \frac{p}{\rho g} \right) = 1000 \quad \text{and} \quad \int_0^{1000} \frac{d\omega}{\rho g} \]

Applications of the method: Static cylindrical drop.
Evolution profile of 2D viscous droplet

Fluid properties are \( \rho = 1 \), \( \mu = 5 \times 10^{-5} \), and an ellipsoidal drop is placed in a 20 x 20 domain.

Applications of the method: 2D viscous droplet
% > 0 \times \frac{L_{\text{theory}}}{L_{\text{numerical}}} - L_{\text{theory}} 

Kinetic energy vs time profile for a 2D viscous droplet.
bulbous end to swell.

push fluid from the end towards the central stem of fluid, causing the

High curvature at rounded edges generates pressure gradients, which

of air.

column of water with a bulbous elliptical end, sitting in a background

An axisymmetric domain of dimensions 7.0 mm x 1.0 mm contains a

Applications of the method: Relaxation of a water filament
The necking region creates a region of high pressure gradients, which
mum pressure, creating a necking region.

The bubbles end lead to a region of negative curvature and a mini-

region. A sudden increase in radius of the neck, and eventual break of the bubbles
pushes fluid away from the neck in both directions, causing the re-

The necking region creates a region of high pressure gradients, which

region.
\[(17) \quad \left( \frac{Y}{VZ} \right) \phi - \left( 2d + \frac{1}{2} \right) s \right) \left( \frac{2d + \frac{1}{2}}{VZ} \right) \frac{\eta}{u} = \frac{\nu}{u} \]

Linear mode analysis predicts an interfacial growth rate, \( \eta \), given by

is applied at the interface between the two fluids.

A sinusoidal perturbation, wavenumber \( k \) cm and amplitude 6.25 cm,

\( \frac{\partial^2 \phi}{\partial x^2} \)

gradient.

gravitational field, \( s = 0.35 \), supported by a counterbalancing pressure

A heavy fluid, \( \eta = 3.0 \), rests on top of a light fluid, \( \eta = 1.0 \), in a

Applications of the method: Rayleigh–Taylor instability.
Interface plots at $t = 60$ for $\phi = 0, 0.25, 0.5, 0.75, 0.75$ respectively.

(18) \[ \frac{\partial \theta}{\partial t} = (u) \phi \]

that $u = 0$. A surface tension stability parameter is defined for a fixed wave-number a critical surface tension, $\sigma^c$, exists such
Growth rate: Numerical vs Theoretical for RT instability.
• The method copes well with a range of flows at varying complex...

• Problems:
  viscous oscillating droplets, capillary-driven and fluid instabilities.

• The method is validated against the standard static drop test.

• Curvature estimation:
  Only potential source of unphysical spurious currents lies within...

• A new discretization of the CSE model has been presented...

Conclusions


Bibliography