A genuinely multidimensional non-dissipative finite volume scheme for transport in compressible fluids (Vofire)

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Outline

- Context
- Geometric formulation for pure advection on simplices
- Algebraic generalization (general cells)
- Application in the ALE framework
- Results in 2/3 D
- Conclusions

Context

- Euler equations
- Physical constraints (for instance for the hohlraum calculation)
    - Several materials and phases
    - Materials with very different properties (equations of state. . .)
- Numerical features
    - Millions of cells in 3D
    - Million(s) of mixed cells (with a dissipative scheme!)
    - Unstructured (Lagrangian or ALE) meshes
    - Parallel Calculation
- Need: a robust, quick and efficient 2/3D reconstruction method

Subjective review of interface tracking

	locally conservative in mass	interface geometry	cheap	robust	parallel
Front-T. [a]	- -	++	-	- -	-
Level-Set [b]	- -	+	+	+	+
VOF [c]	+ [d]	+	-	+	+

[a] S.O. Unverdi - G. Tryggvasen (1992) J. Comput. Phys.

[b] C.W. Hirt - B.D. Nichols (1981) J. Comput. Phys.

[c] B.J. Parker - D.L. Youngs (1992) UK Atomic Weapon Establishment report

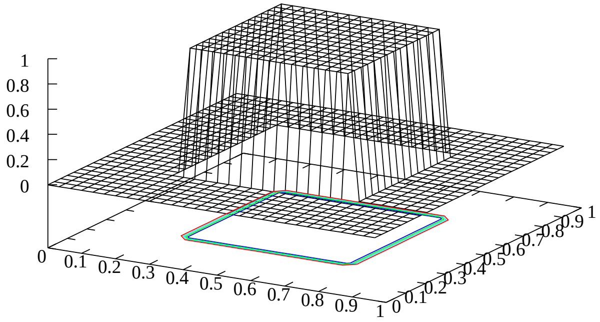
[d] Controled by the Newton convergence

Strategy

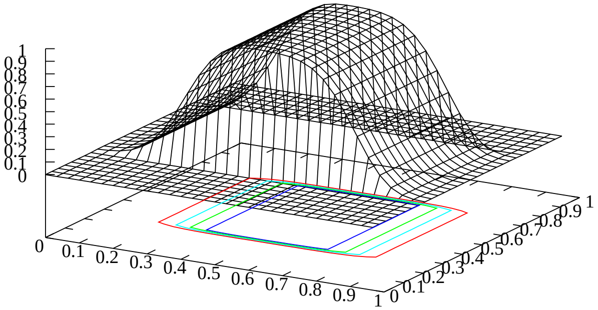
- Finite volume scheme
- General meshes
- Strong anti-dissipative mechanism for mass fractions (ALE/remap compatible)
- Basis: geometric reconstruction

Initial observation

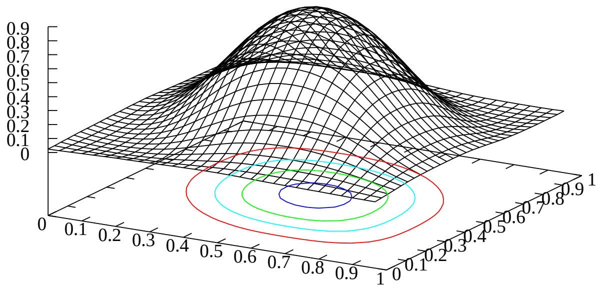
Numerical diffusion in elementary advection test cases: made up of two distinct phemomena.



initial value



aligned mesh



unaligned mesh

Distinction between

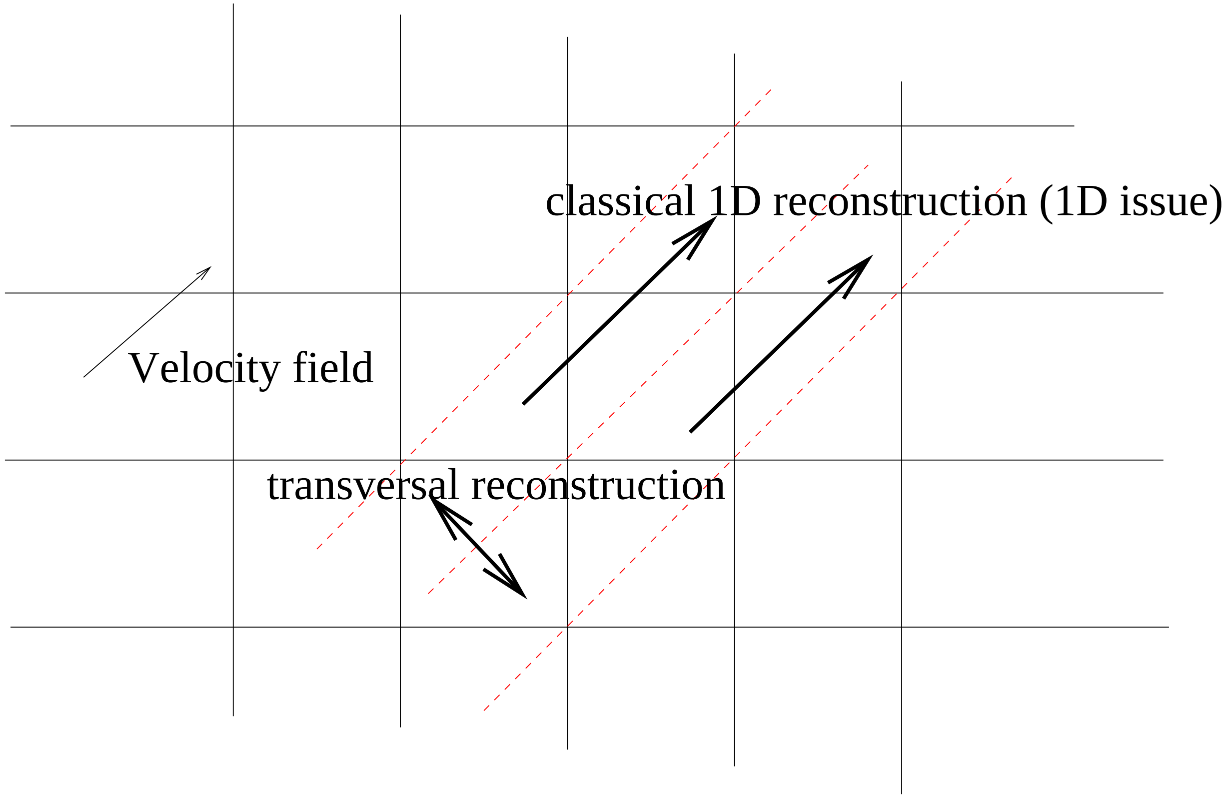
- the diffusion parallel to the transport direction (1D),
- and the transversal diffusion (multi-D)

should be done.

How to deal with this issue?

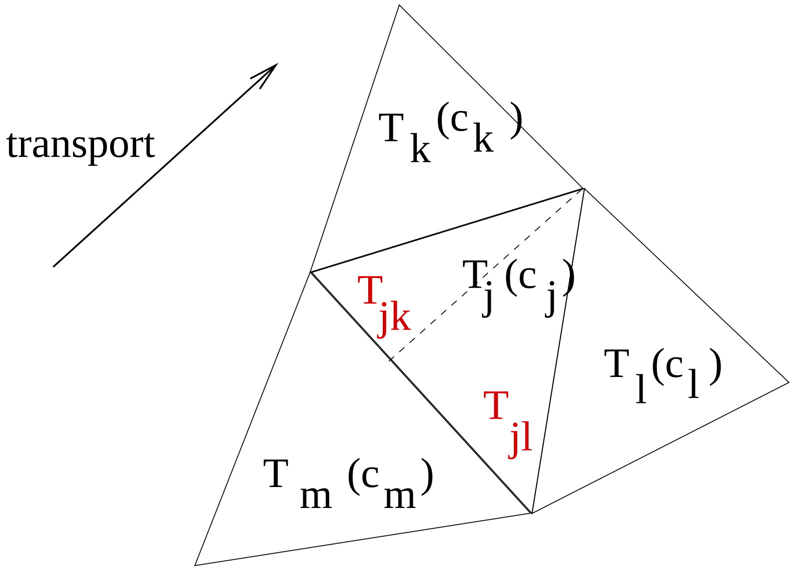
The anti-diffusive process will act

- in the direction normal to the velocity field (transversal anti-diffusion),
- in the direction of the velocity field (1D anti-diffusion).



Geometric formulation (1/4)

Focus on the pure advection equation: $\partial_t c + a \cdot \nabla c = 0$,
on triangles (2D).
First stage: the transversal reconstruction

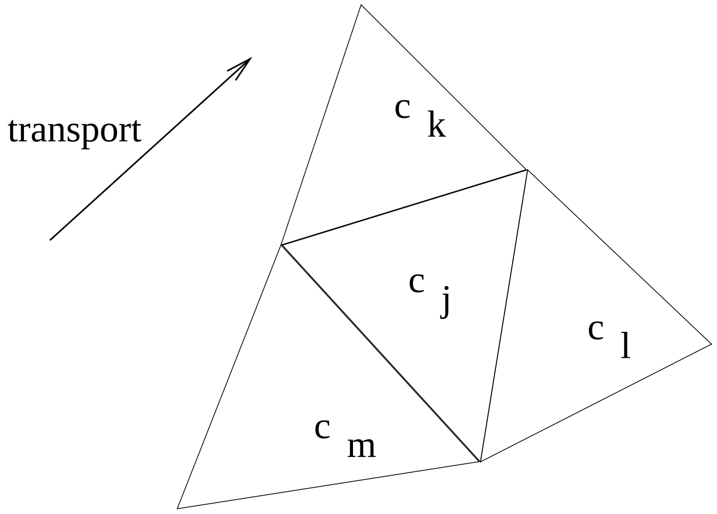


s_j: surface of T_j.
T_j is geometrically split into two subgrid triangles T_{jk} and T_{jl}, with $s_{jk} + s_{jl} = s_j$.
How to reconstruct c_{jk} and c_{jl} (subgrid values of c in T_{jk} and T_{jl})?
Goal: c_{jk} as close as possible of c_k, c_{jl} as close as possible of c_l.
Constraints: conservation, maximum principle.

Geometric formulation (2/4)

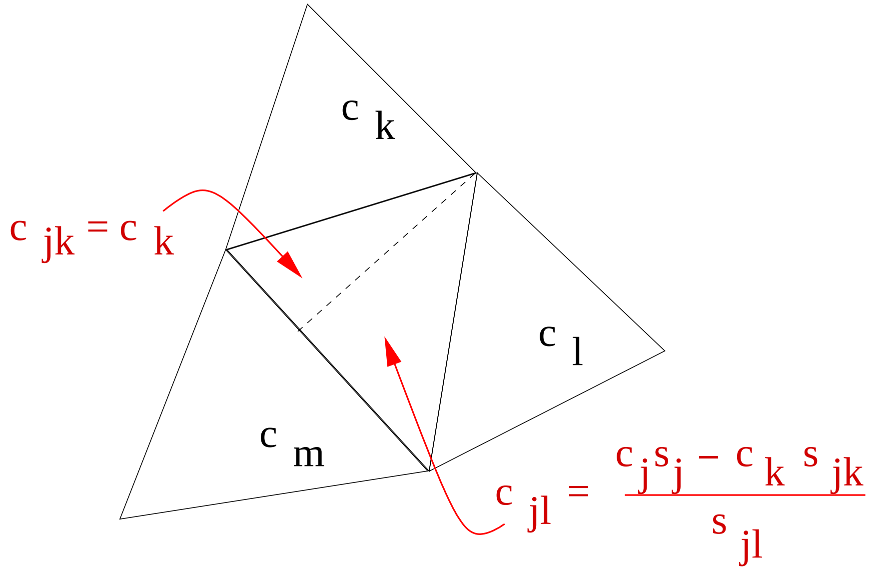
Assuming for instance $c_k \leq c_l$, 3 configurations can occur:

a)



$c_j \notin (c_k, c_l)$
no transversal reconstruction

b)

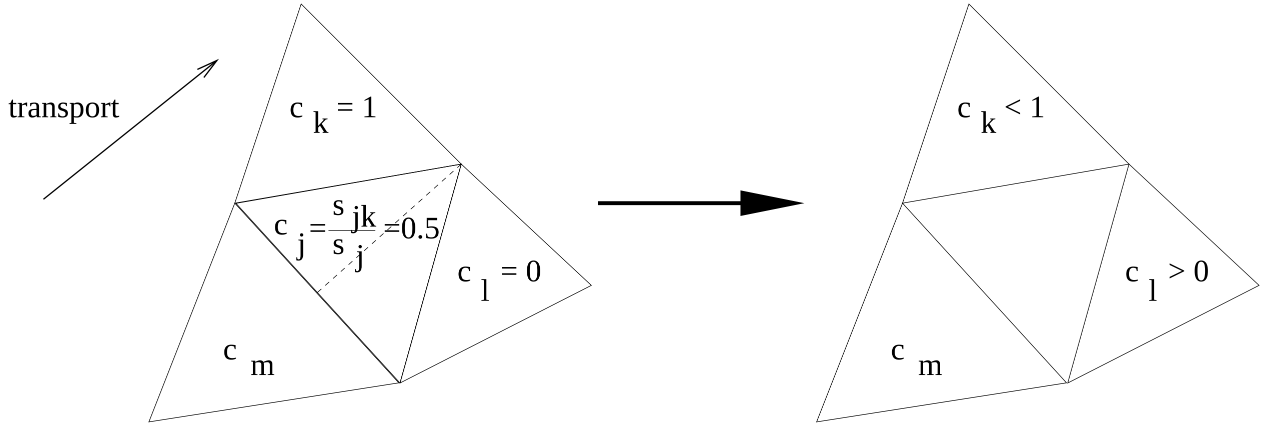


$c_j \in (c_k, c_l)$
$(c_j - c_k)s_{jk} \leq (c_l - c_j)s_{jl}$
sharp, conservative reconstruction

c) The other case is similar.

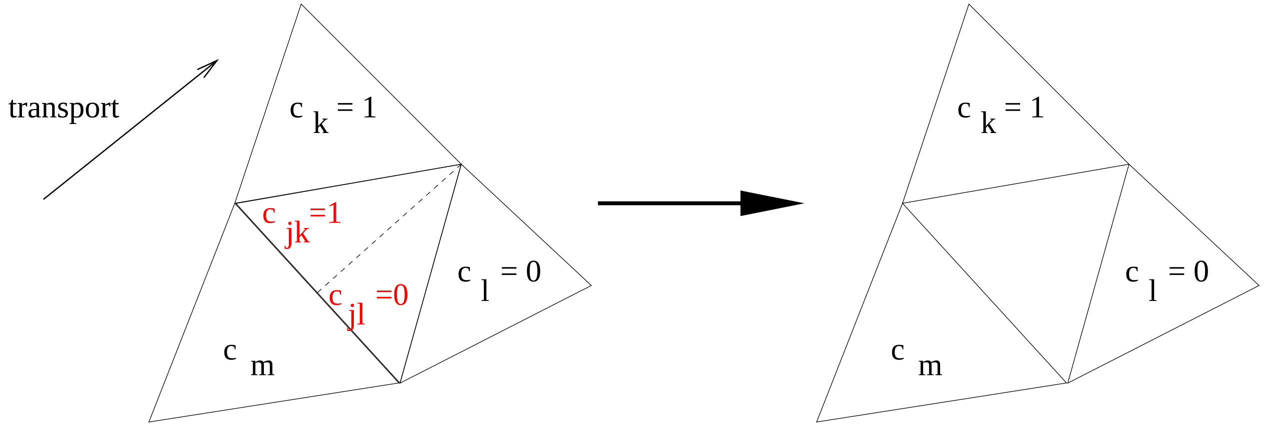
Example of reconstruction (1/2)

No reconstruction: the advection phase is diffusive:



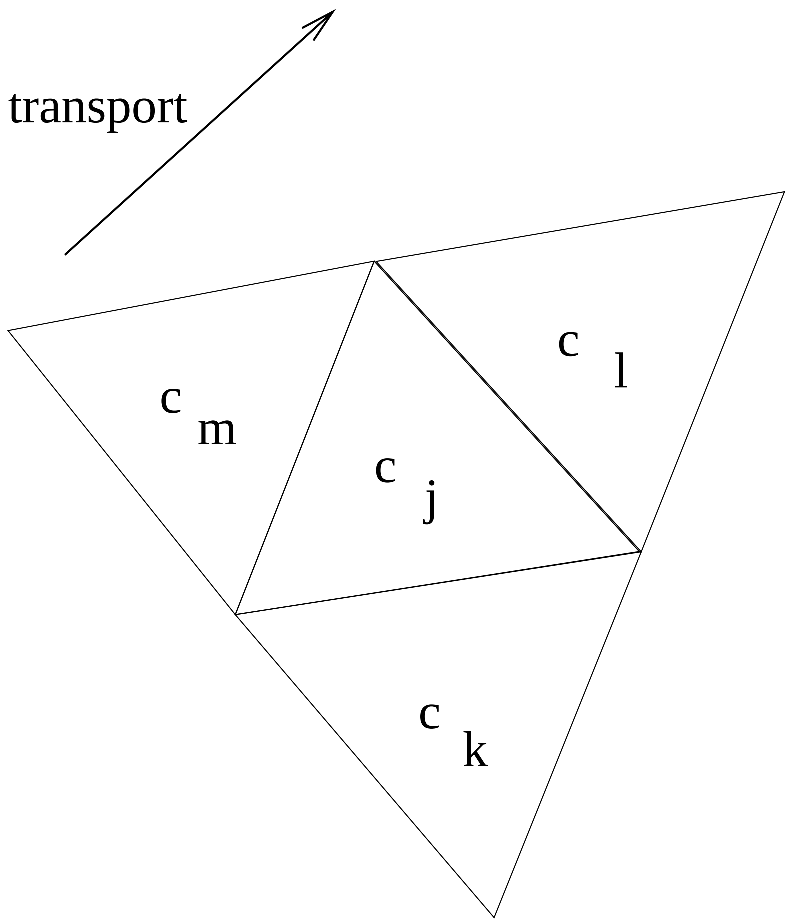
Diffusion occurs

With transversal reconstruction: no more diffusion:



No diffusion

Example of reconstruction (2/2)



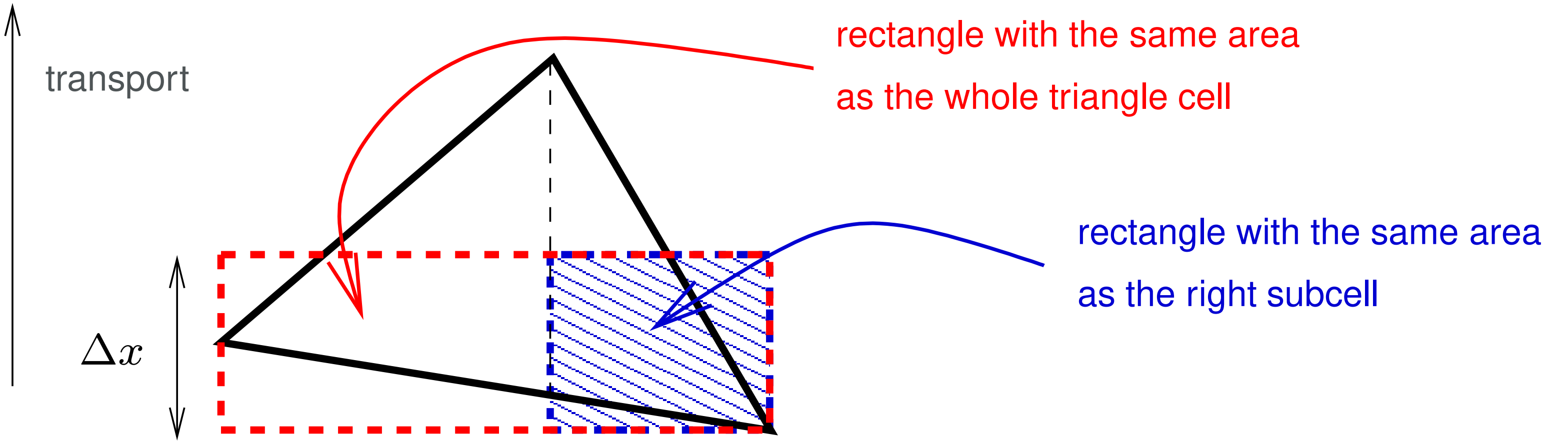
We cannot reconstruct (in cell T_j)

No transversal reconstruction possible, but no “transversal” diffusion anyway, because there is only one downwind cell.

Geometric formulation (3/4)

Important remarks

1. The transversal reconstruction does not modify the CFL stability condition.



Classical CFL condition: $|a|\Delta t/\Delta x \leq 1$.

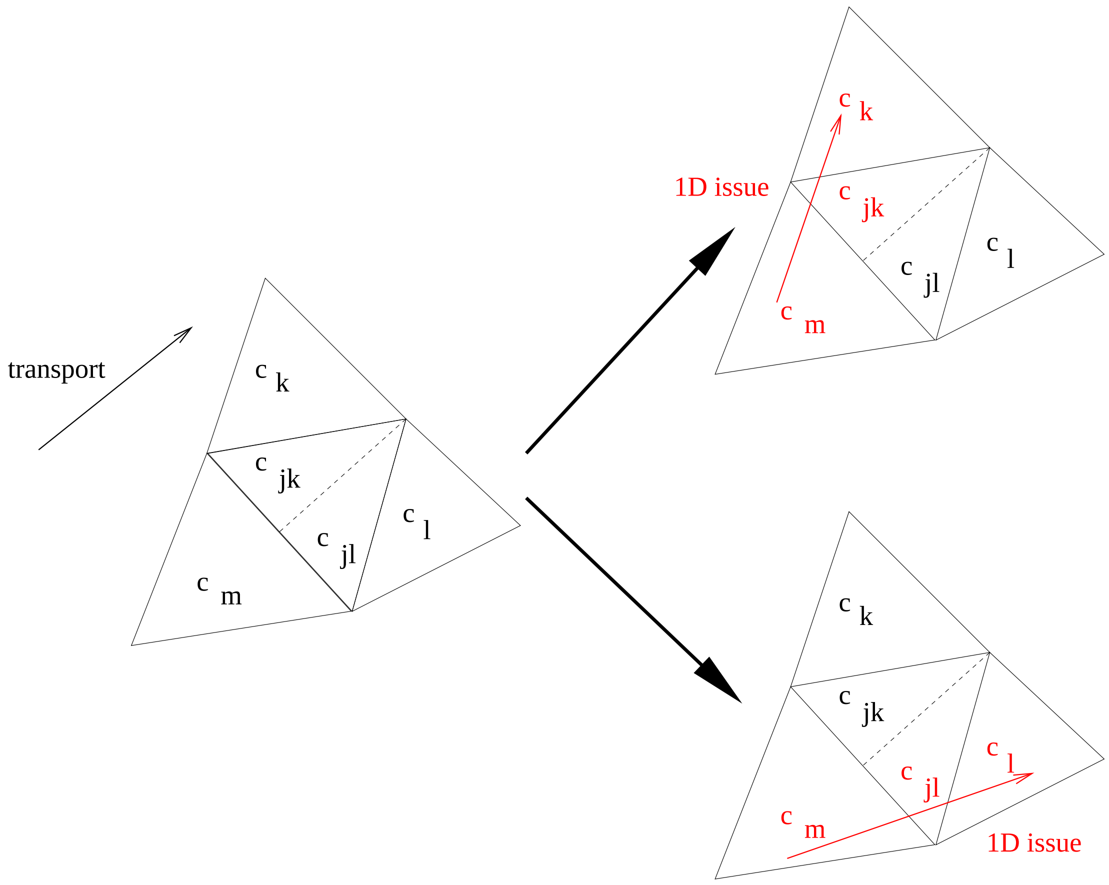
Δx does not change in the reconstruction process!

2. Each subcell has now
   - *one* upwind neighbor
   - and *one* downwind neighbor.

Geometric formulation (4/4)

Second stage: transport direction

After the transversal reconstruction, 1D problems remain.



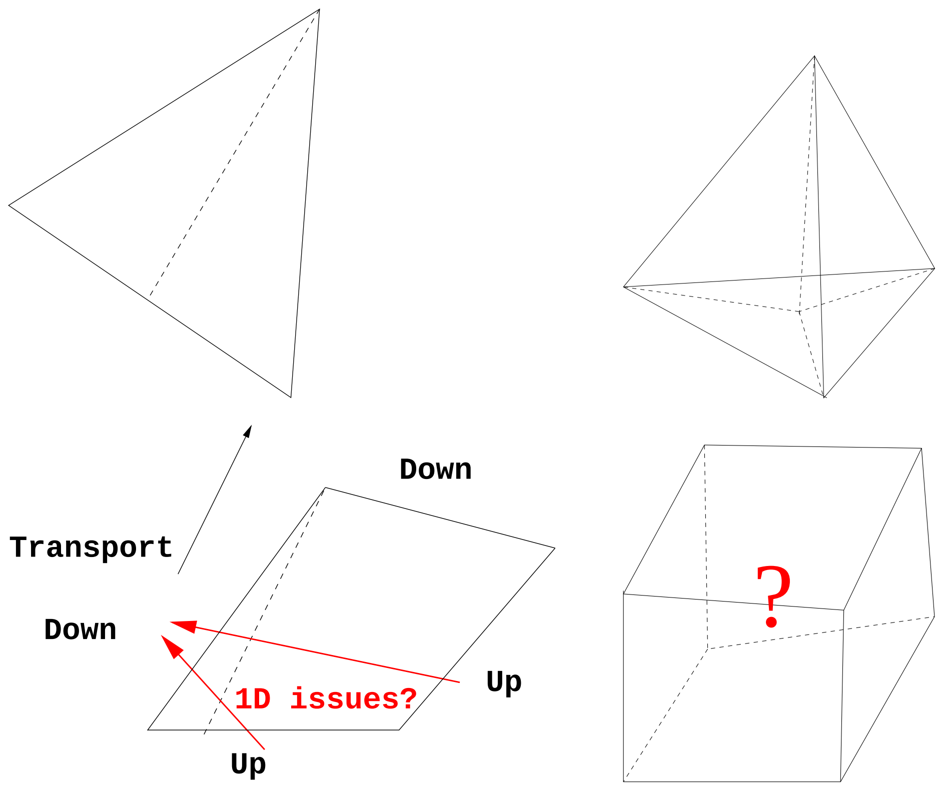
construction of two 1D problems

To be as anti-diffusive as possible, these problems can be solved using the Ultrabee, or limited downwind, scheme.

Other cell shapes?

a)

b)



geometric formulation is adapted

geometric formulation is not adapted

(Same problems with a non-constant velocity. . .)
This *geometrical* formulation can be recast as an *algebraic* formulation on 3D and on any cells, in minimizing fonctionals.
For the case a), the algebraic formulation is equivalent to the geometric formulation.
(A bit technical).

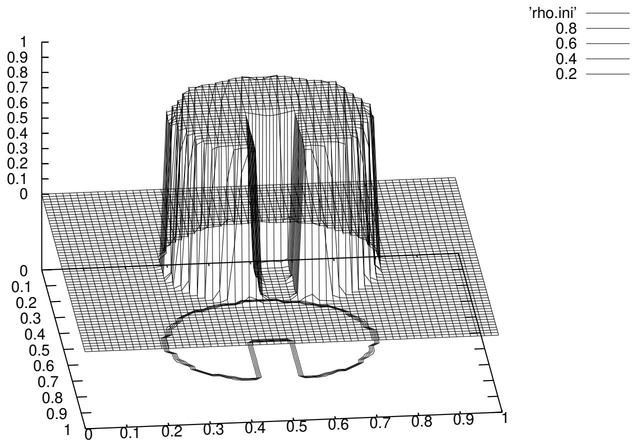
Summary

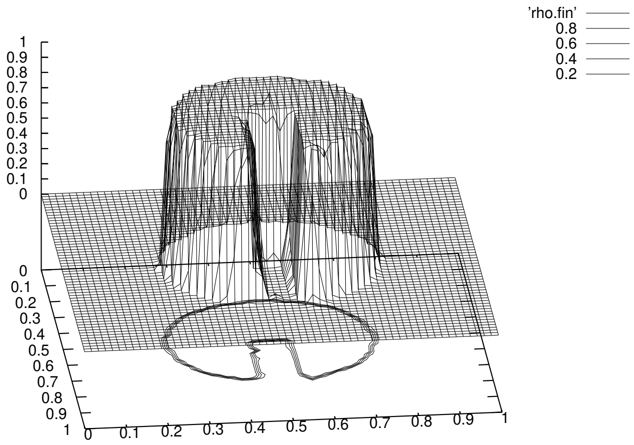
The scheme (on simplices) is made up of three stages:

- *the transversal reconstruction stage* -
  The constant by cell, discontinuous at faces function
  is recast as a
  discontinuous in cell, as continuous as possible at faces
  function,
- *the transport stage* -
  Using the previous function, compute the fluxes with the
  UltraBee calculation,
- *the projection stage* -
  Compute the resulting *constant by cell, discontinuous at faces*
  function.

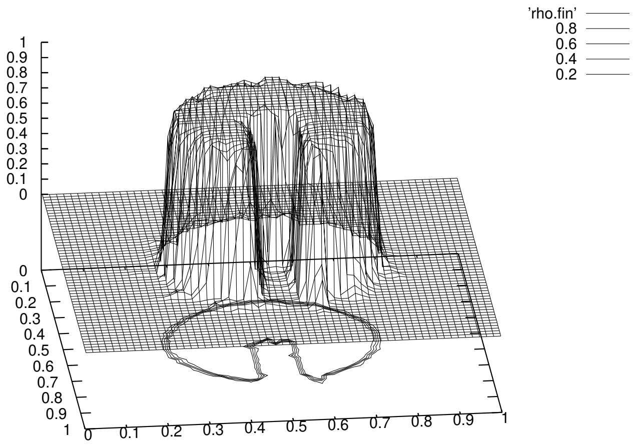
Zalesak's test-case

quad



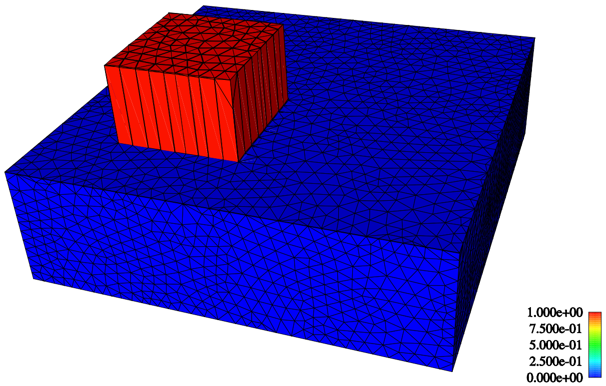


tri

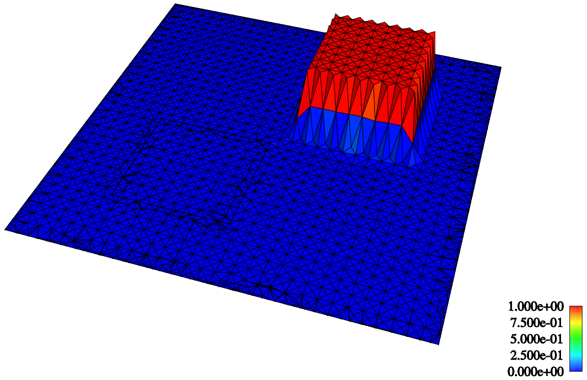


A 3D pure advection test case

Pure advection in the diagonal direction (on tetraedra: 2D solution, 3D mesh).



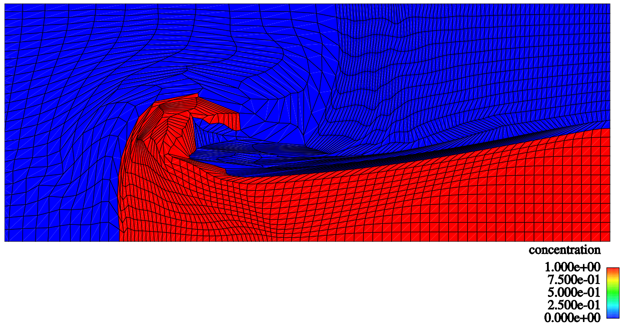
mesh & extruded initial value

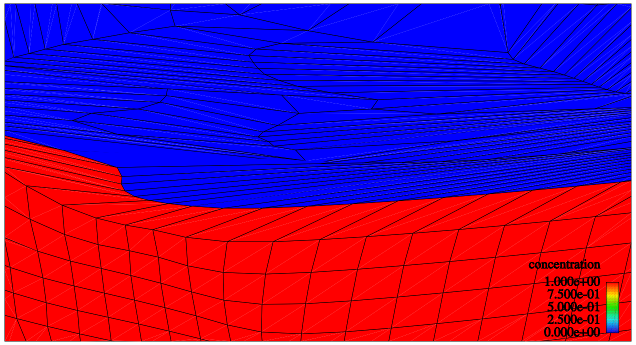


extruded solution for section $z = z_{ave}$

Issues in Lagrangian framework

Lagrangian: the mesh follows the material interfaces

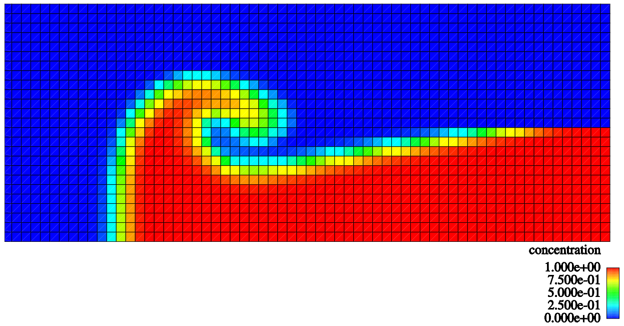


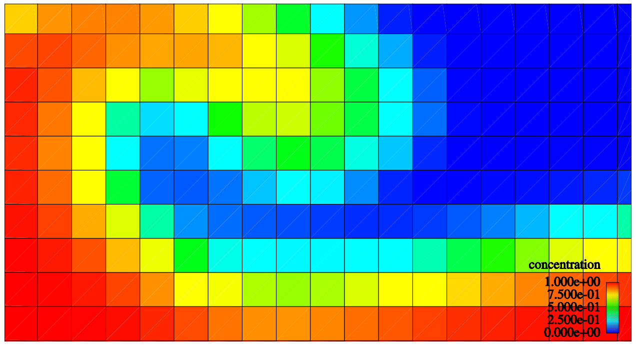


+ interfaces match the mesh

- the mesh is deformed with the interfaces

Eulerian: the mesh is fixed





+ mesh is free from the interfaces

- interfaces are diffused

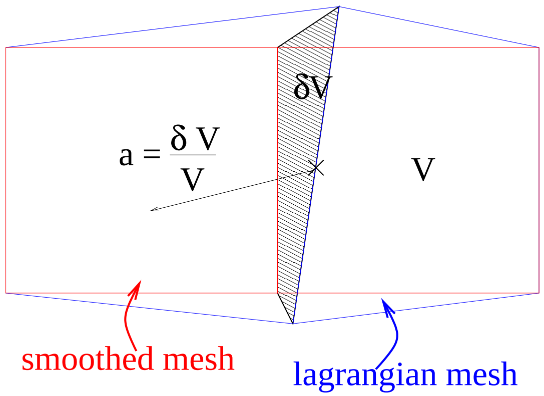
ALE method

- Lagrangian stage (Després-Mazeran scheme)

$$\frac{d}{dt}\int_{V_{lag}(t)} \rho dV = 0$$
$$\frac{d}{dt}\int_{V_{lag}(t)} \rho u dV + \int_{S(t)} P dS = 0$$
$$\frac{d}{dt}\int_{V_{lag}(t)} \rho e dV + \int_{S(t)} P u.n dS = 0$$

- EOS: $(p,T) = f(\rho, e)$
- smoothing of the resulting mesh
- Projection stage

$$\frac{D}{Dt}\int_{V_{ale}(t)} \rho_i dV + \int_{S(t)} \rho_i (u - u_g).n dS = 0$$
$$\frac{D}{Dt}\int_{V_{ale}(t)} \rho u dV + \int_{S(t)} \rho u (u - u_g).n dS = 0$$
$$\frac{D}{Dt}\int_{V_{ale}(t)} \rho e dV + \int_{S(t)} \rho e \underbrace{(u - u_g)}_{\vec{a}}.n dS = 0$$



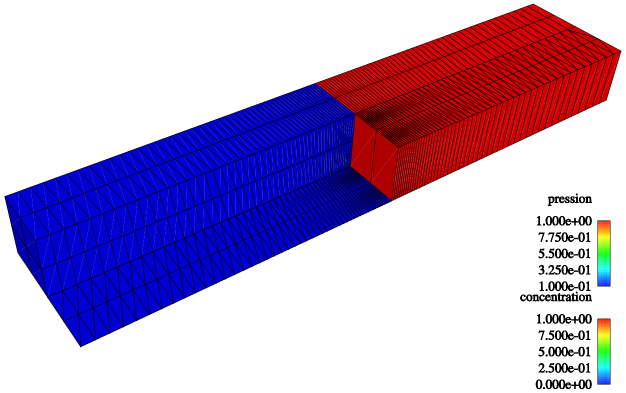
Swept used for the projection

with i the material, $\rho_i = \rho c_i$
and $\sum_i c_i = 1$

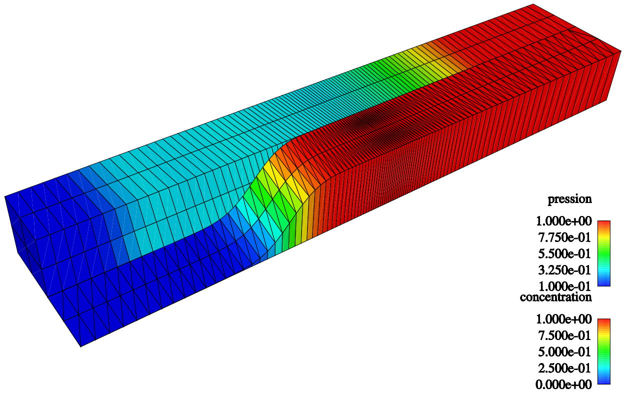
- EOS: $(p,T) = f(\rho, e)$

Results: two-phase shock tube

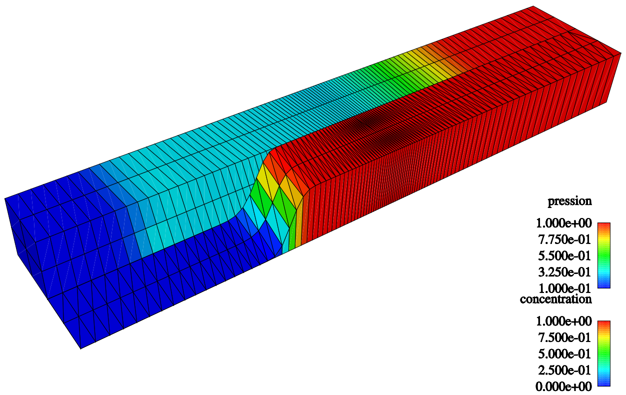
Sod's shock tube on distorted mesh: 1D solution on a 3D mesh



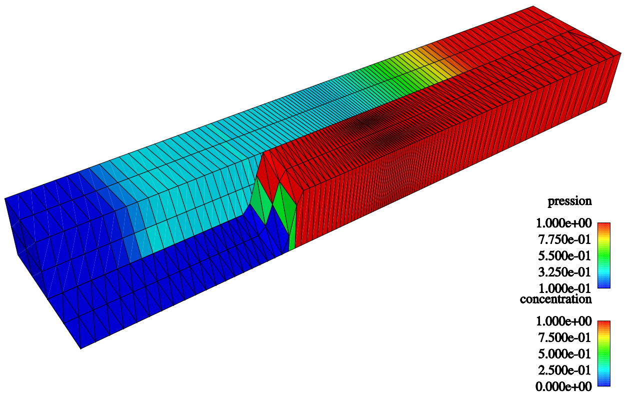
initial conditions



order 1 advection
113 mixed cells



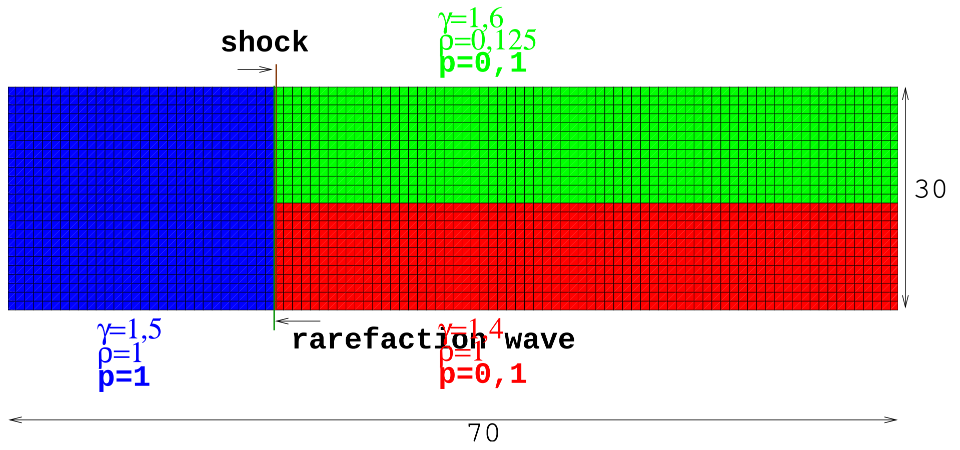
order 2 advection
47 mixed cells



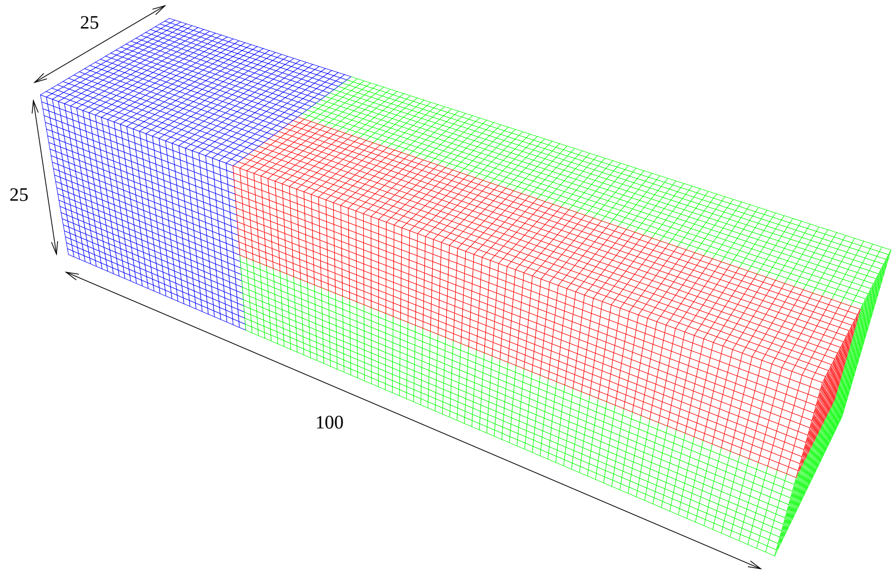
Vofire advection
6 mixed cells

Three materials instability

A shock hits a triple line: difficult benchmark for interface reconstruction (3 interfaces, 1 triple line).



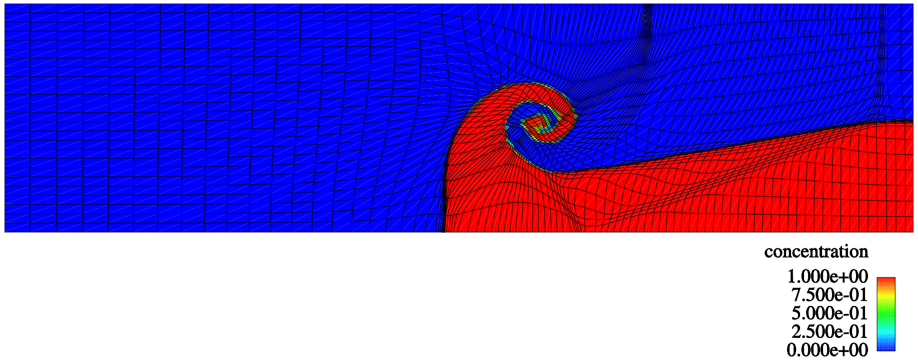
2D

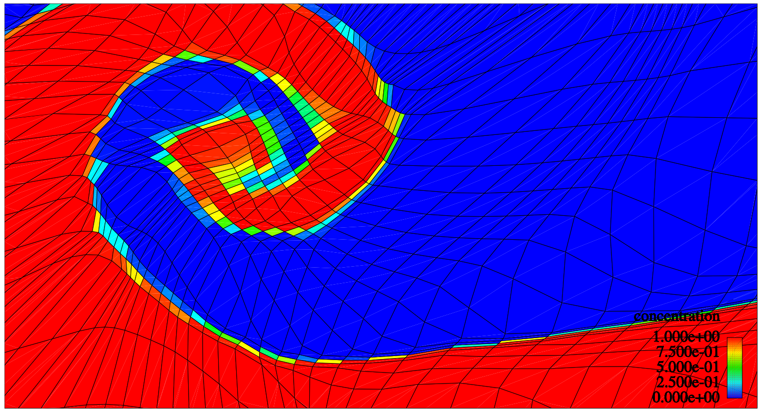


3D

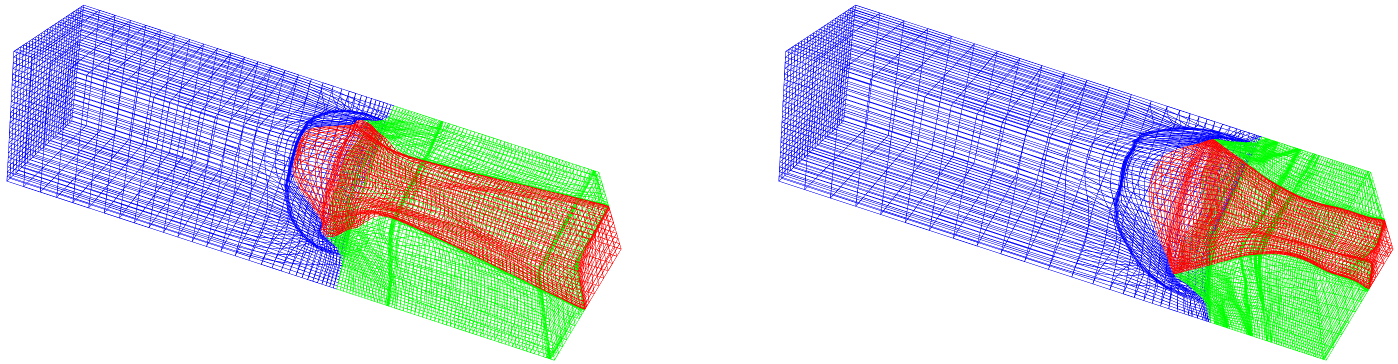
Results: 3 materials instability

- Equipotential smoother (Tipton-Jun), explicit (Jacobi),
- Metrics: intersection of gradient lenghtscales (internal energy, concentrations),
- Swept advection extended at the 2nd order with MUSCL,
- Vofire reconstruction for the concentrations.

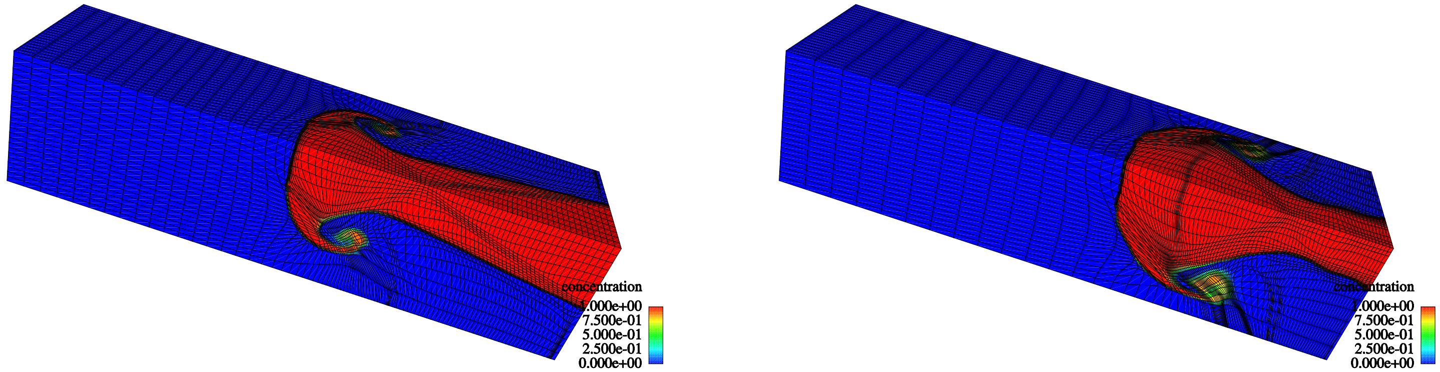




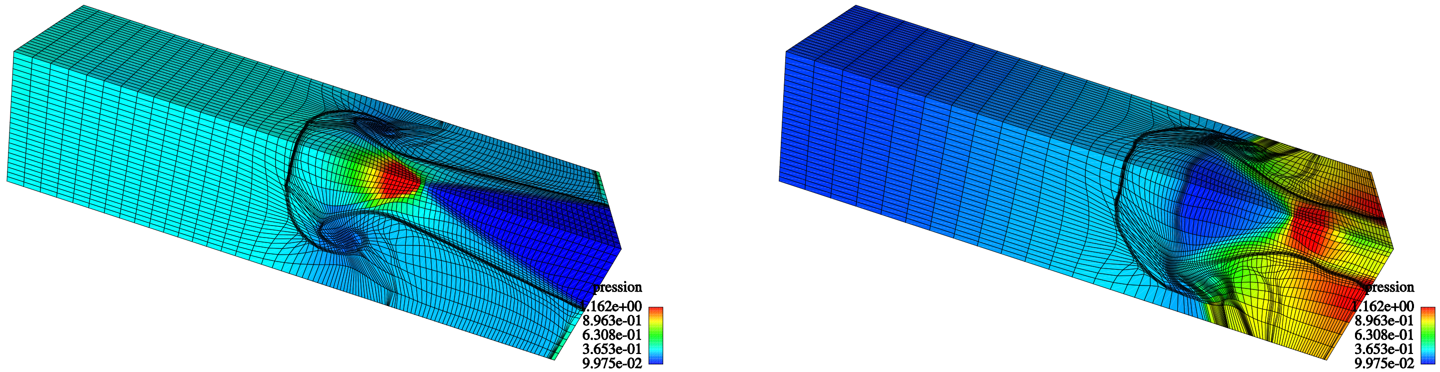
3D



mesh



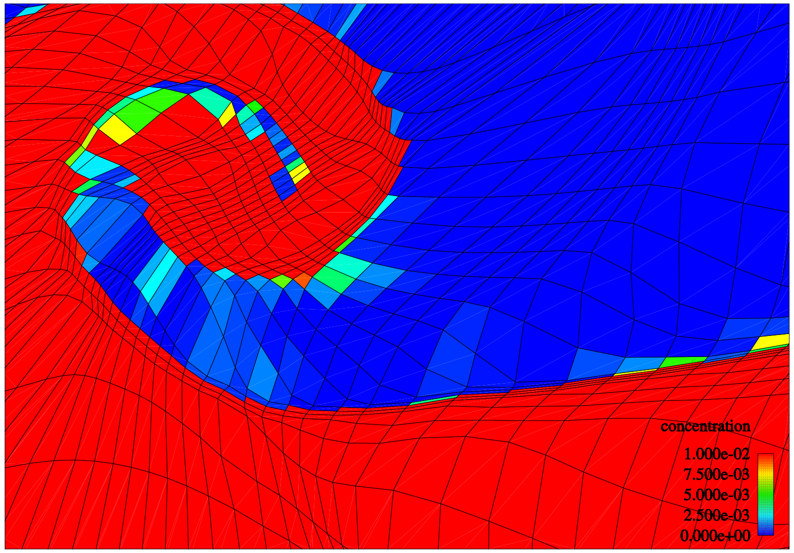
c_1



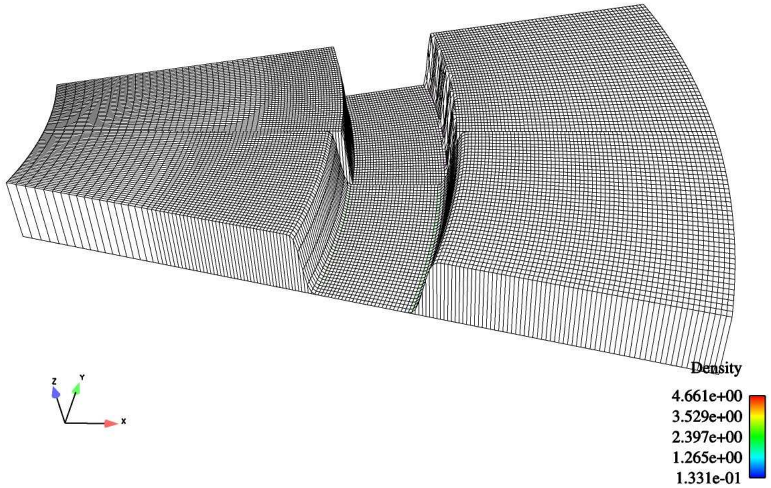
pressure

Limitations

Low level numerical diffusion remains (but the interfaces are compact),



Interface reconstruction can initiate a Rayleigh-Taylor (or Richtmyer-Meshkov) instability.



Subjective comparison in 3D

	conserv.	interface geometry	cheap	robust	parallel
Front-T.	- -	++	-	- -	-
Level-Set	- -	+	+	+	+
VOF	+	+	-	+	+
Vofire	++	-[a]	+	++	+

[a]cf. previous slide: limitations

Conclusion

- We have presented the Vofire method applied to the material interfaces in 3D configuration on unstructured meshes,
- We obtain a very favourable ratio cost/efficiency: *Vofire cost < 0.5 advection step cost* whereas *Youngs-PLIC cost > 10 advection step cost*,
- Discontinuities remain compact (the diffusion zone is finite),
- Parallel implementation is not difficult.