Construction of multi-material interfaces from moment data

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Talk layout

1) brief look at the Volume-of-Fluid (VoF) algorithms

2) new Moment-of-Fluid (MoF) strategy

- two-material case
- Lagrangian remap: how to update the moment data
- multi-material case

Volume-of-fluid (VoF) technology

Two components of typical VoF method:

1) interface reconstruction (IR) algorithm

- input data:
  cell-wise material volume fractions
- preserves (volume=mass) of each material
- can separate two materials in a mixed cell
- uses one linear segment per mixed cell

2) volume tracking scheme:
   volume fractions are updated
   by calculating the material fluxes
   through the cell boundaries

VoF-IR resolution limit

The evaluation of the interface normal
from the volume fractions
can not be accomplished
without the data from the neighbors.

$\Downarrow$

Resulting interfaces
in adjacent mixed cells
are not independent

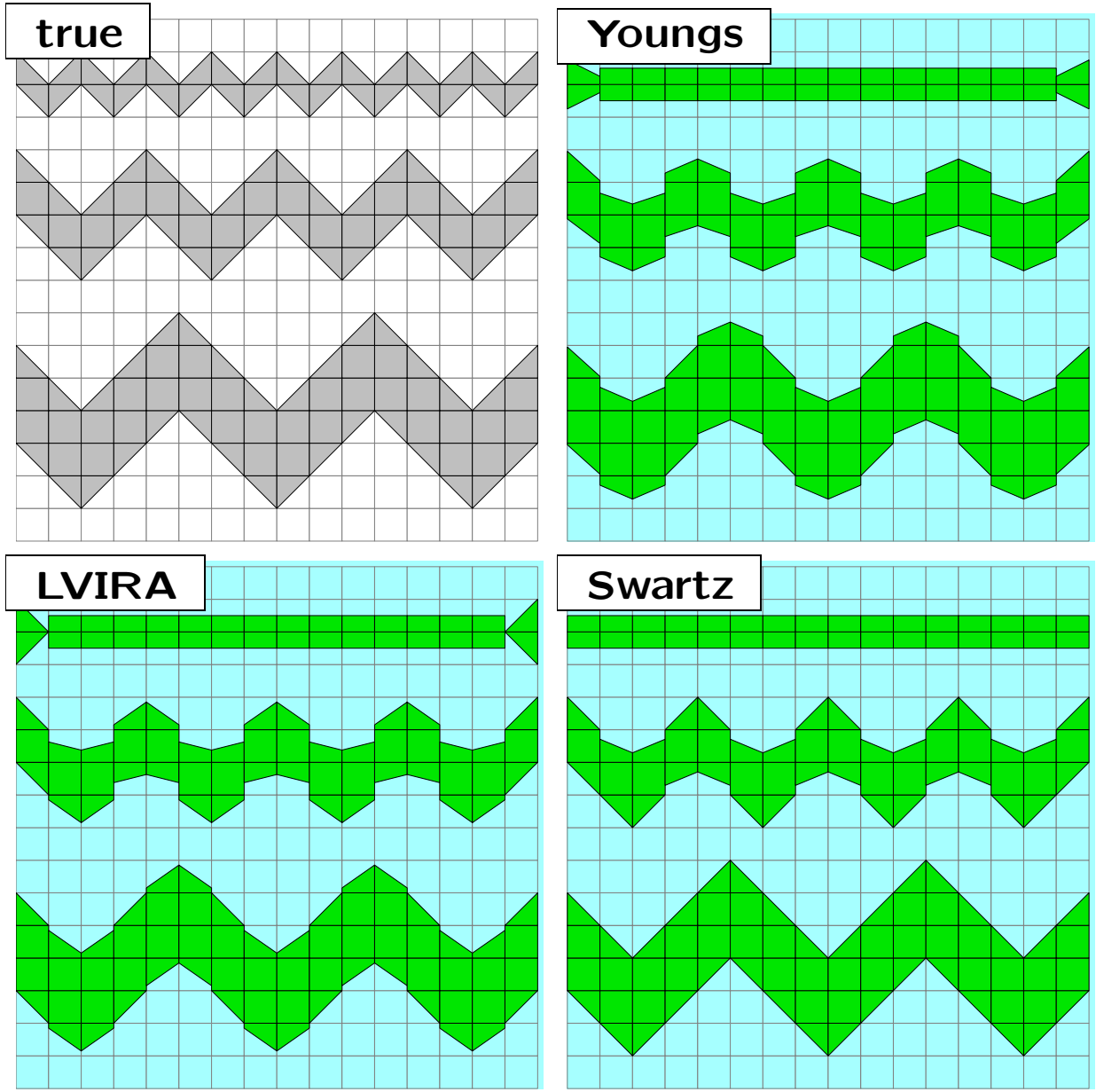
$\Downarrow$

VoF interface approximation
can not resolve interface details
smaller than the size of the cell cluster
participating in the interface normal evaluation.

$\Downarrow$

VoF-IR resolution limit $\approx$ 2 to 3 cell sizes

“zigzag” shape reconstruction

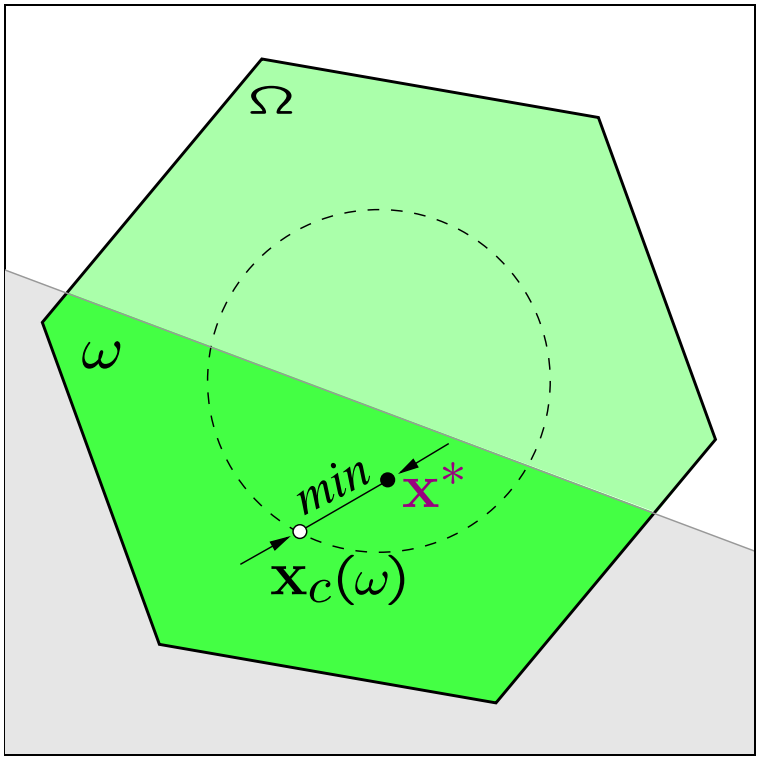


Moment-of-Fluid (MoF) reconstruction

assist VoF with material centroids

MoF interface reconstruction

input data: a volume fraction (μ^*) and a centroid ($\mathbf{x}^*$)



Among all the subcells ω with a linear interface and the prescribed volume

$$|\omega| = \mu^* \, |\Omega|,$$

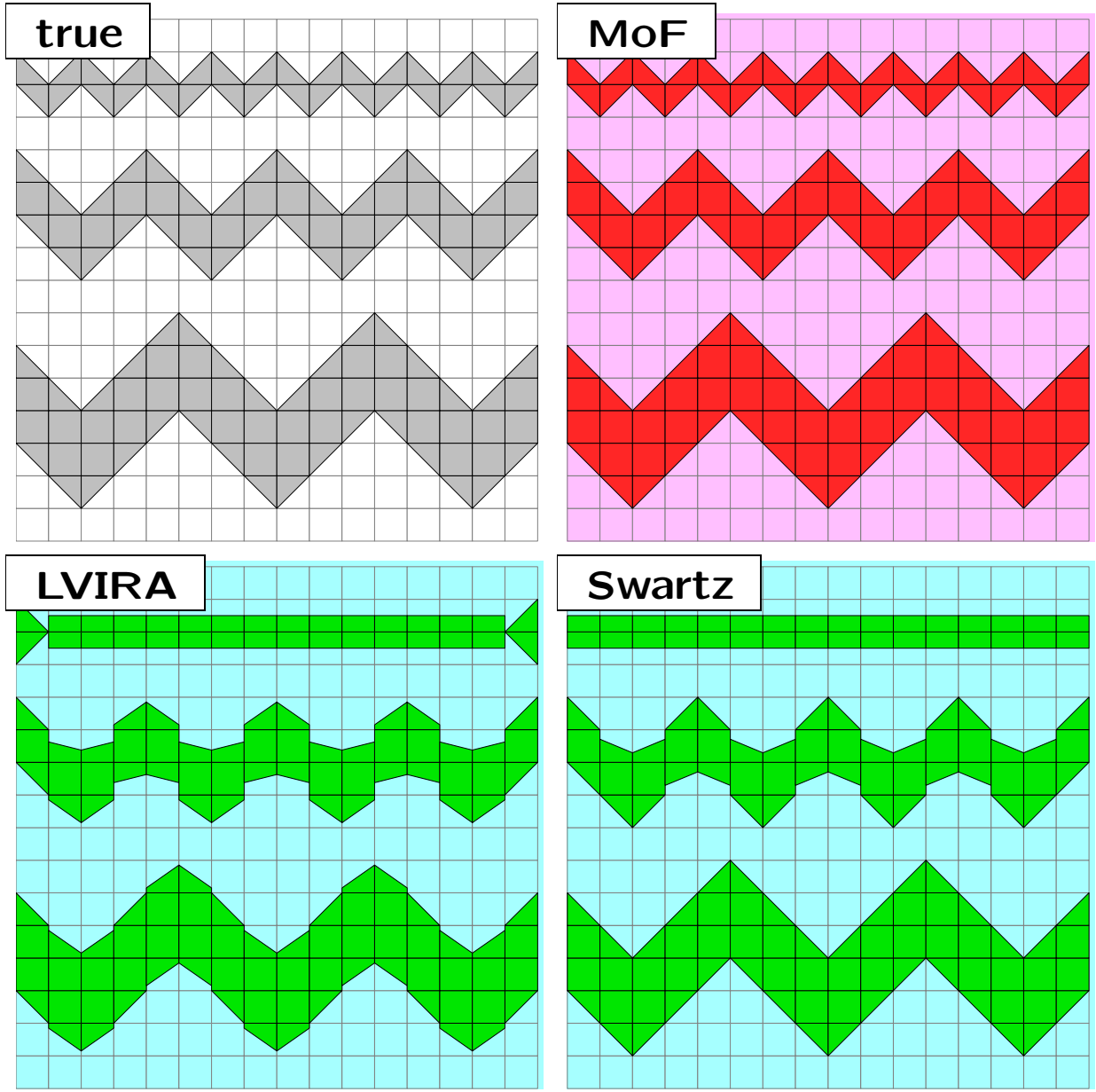
find the one whose centroid $\mathbf{x}_c(\omega)$ is closest to the given centroid:

$$||\,\mathbf{x}_c(\omega) - \mathbf{x}^*|| \rightarrow \text{min}$$

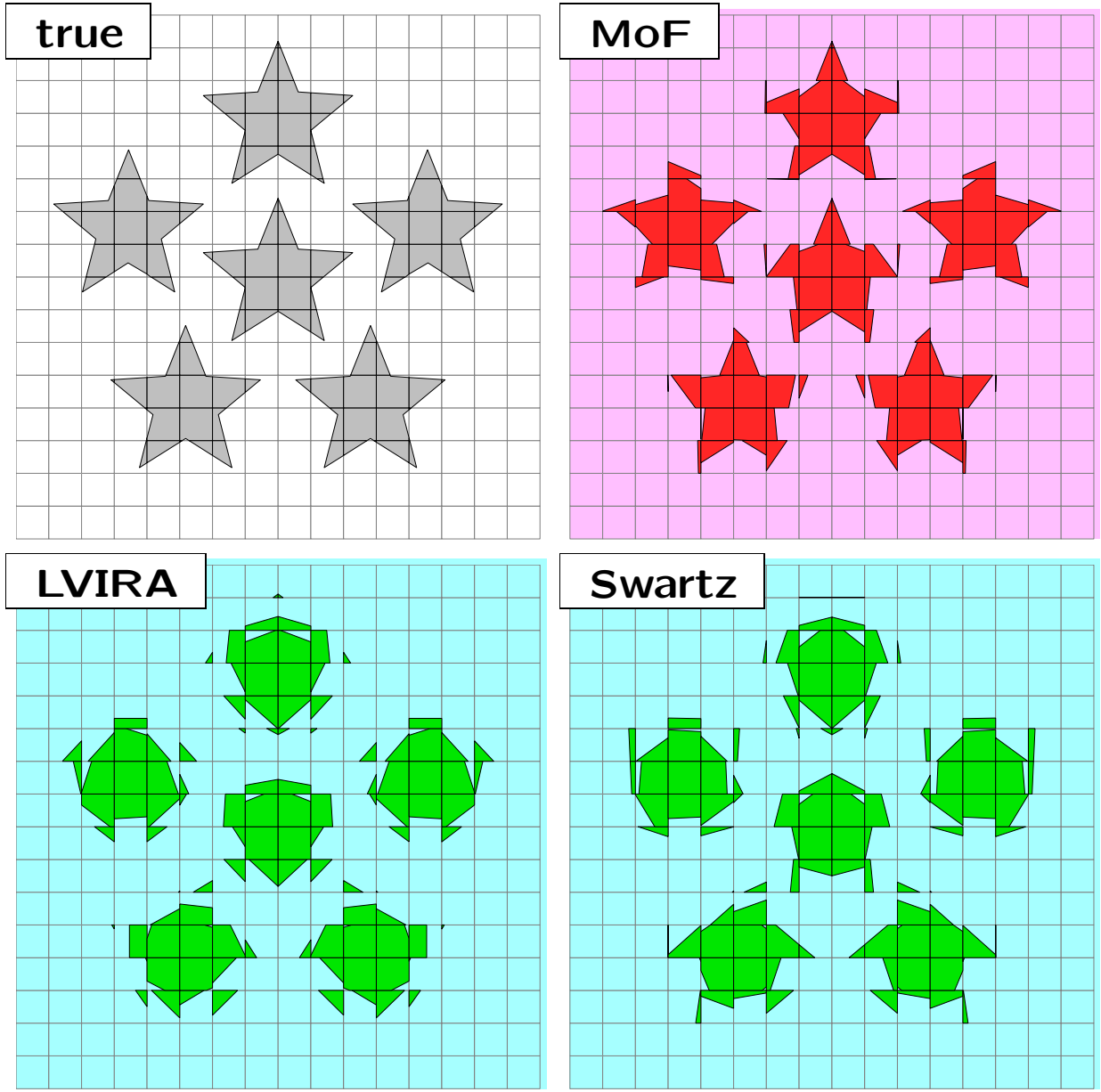
local, dimension- and cell-shape- independent

unique, stable, 2nd-order accurate

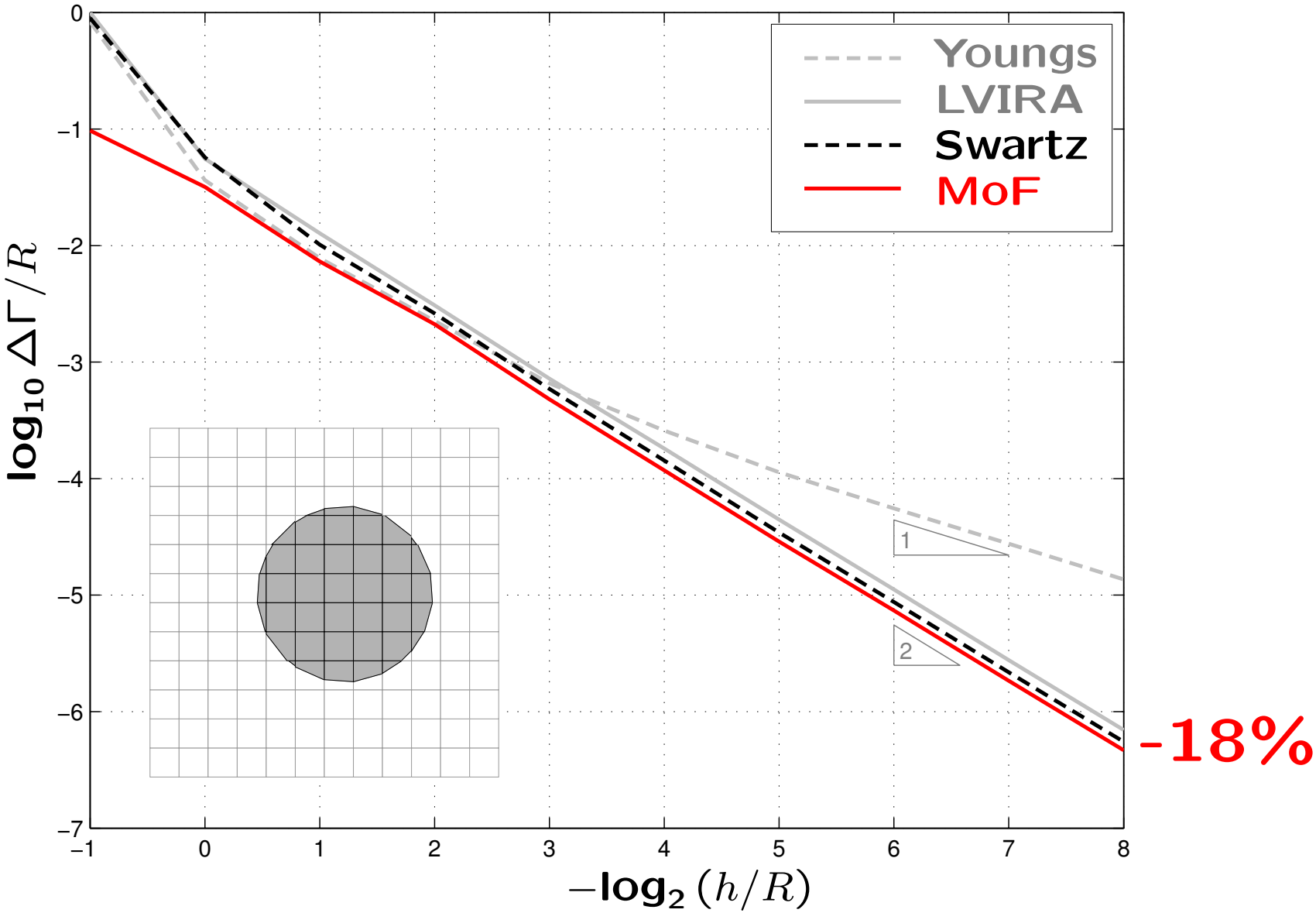
“zigzag” shape reconstruction



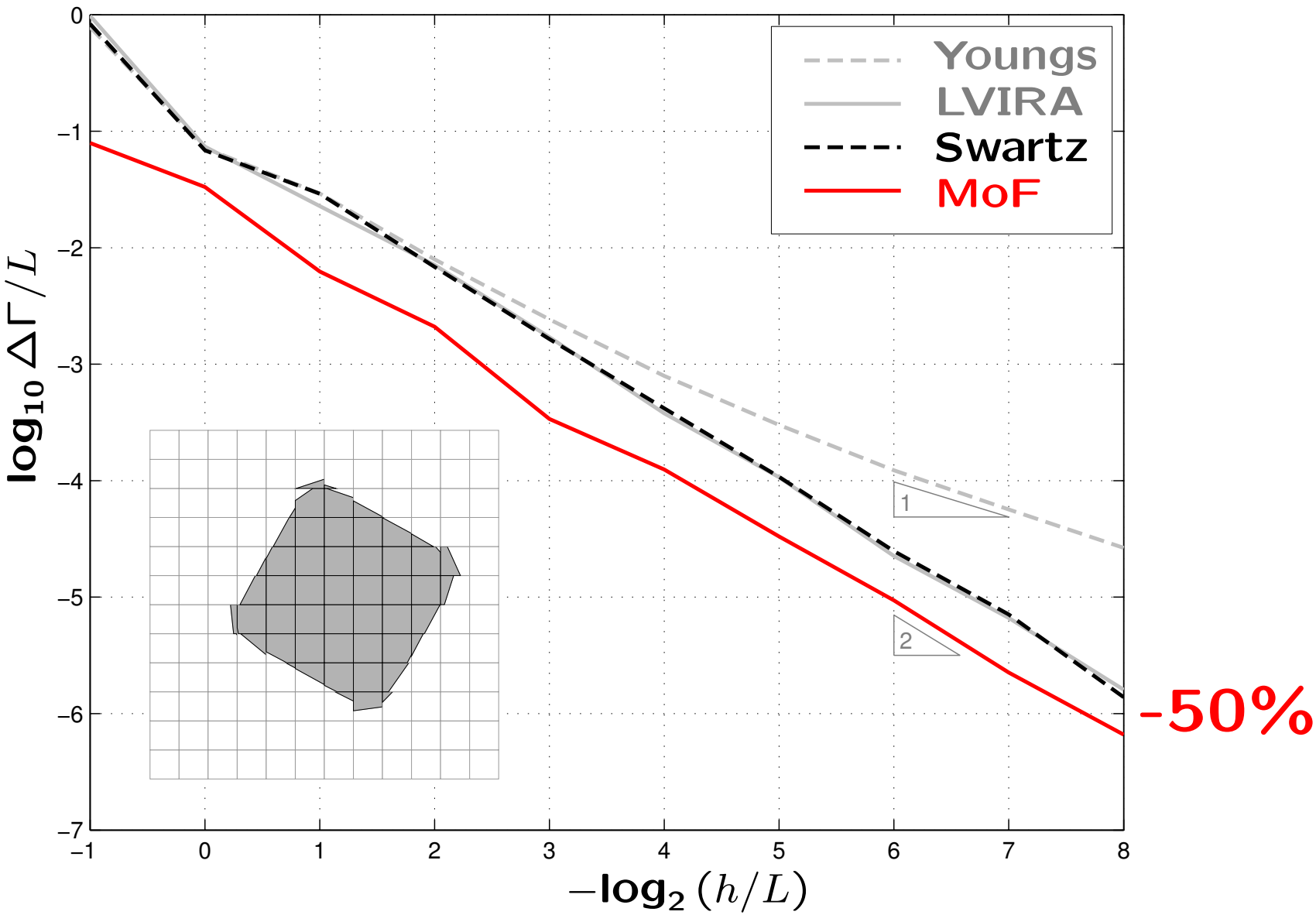
“constellation” shape reconstruction



Approximation error: circle of radius R



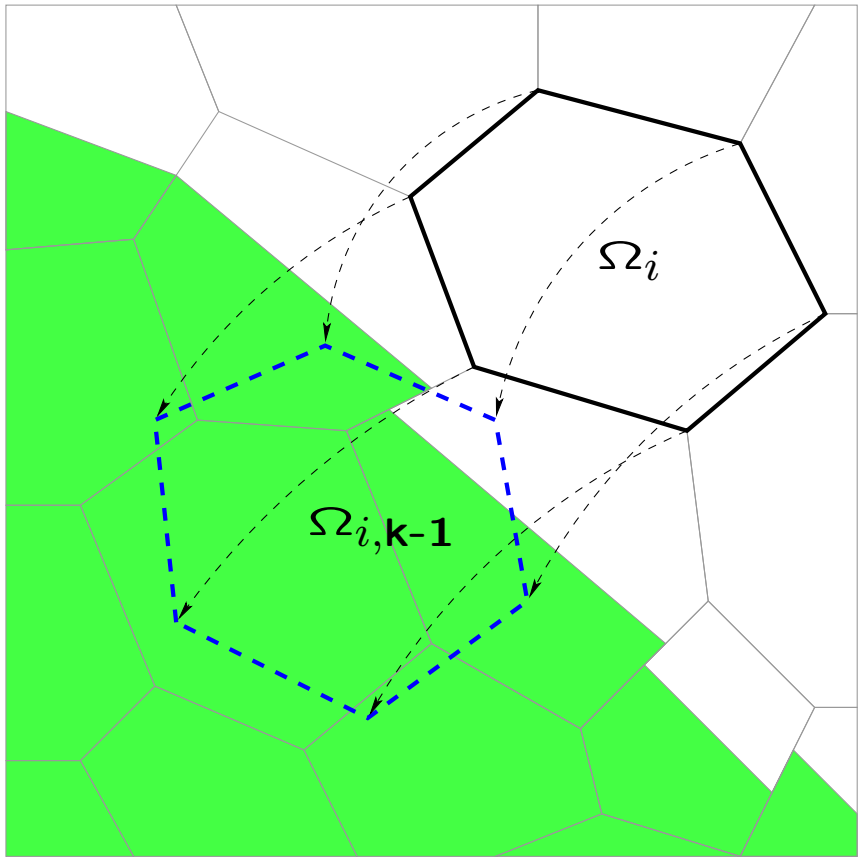
Approximation error: 2Lx2L square



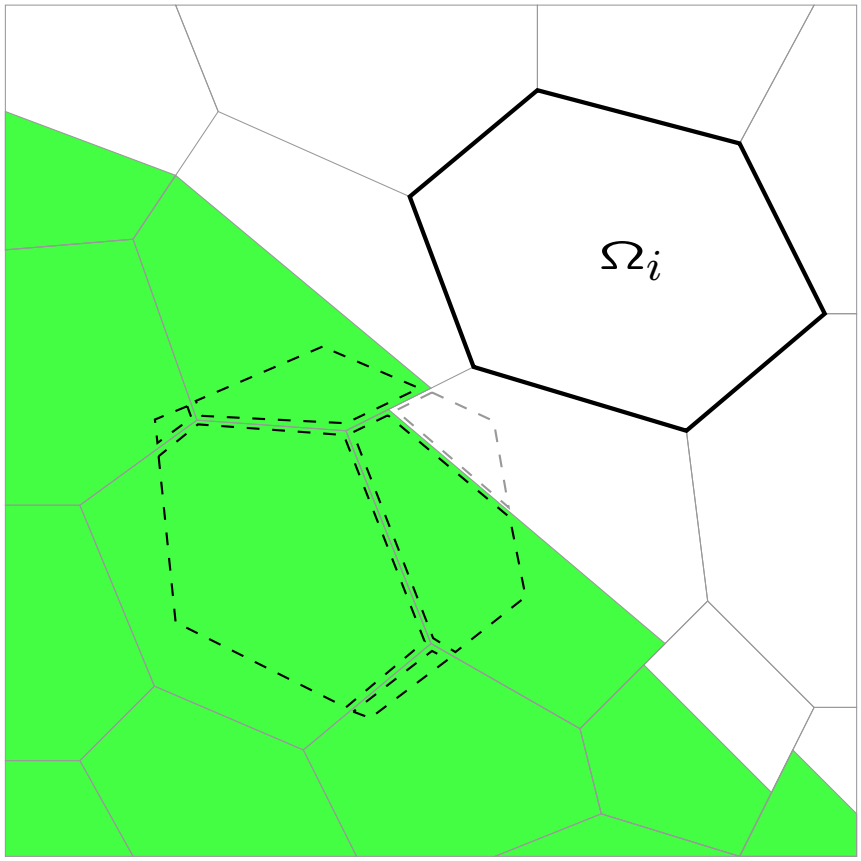
Lagrangian remap

how to update the moment data

Volume tracking

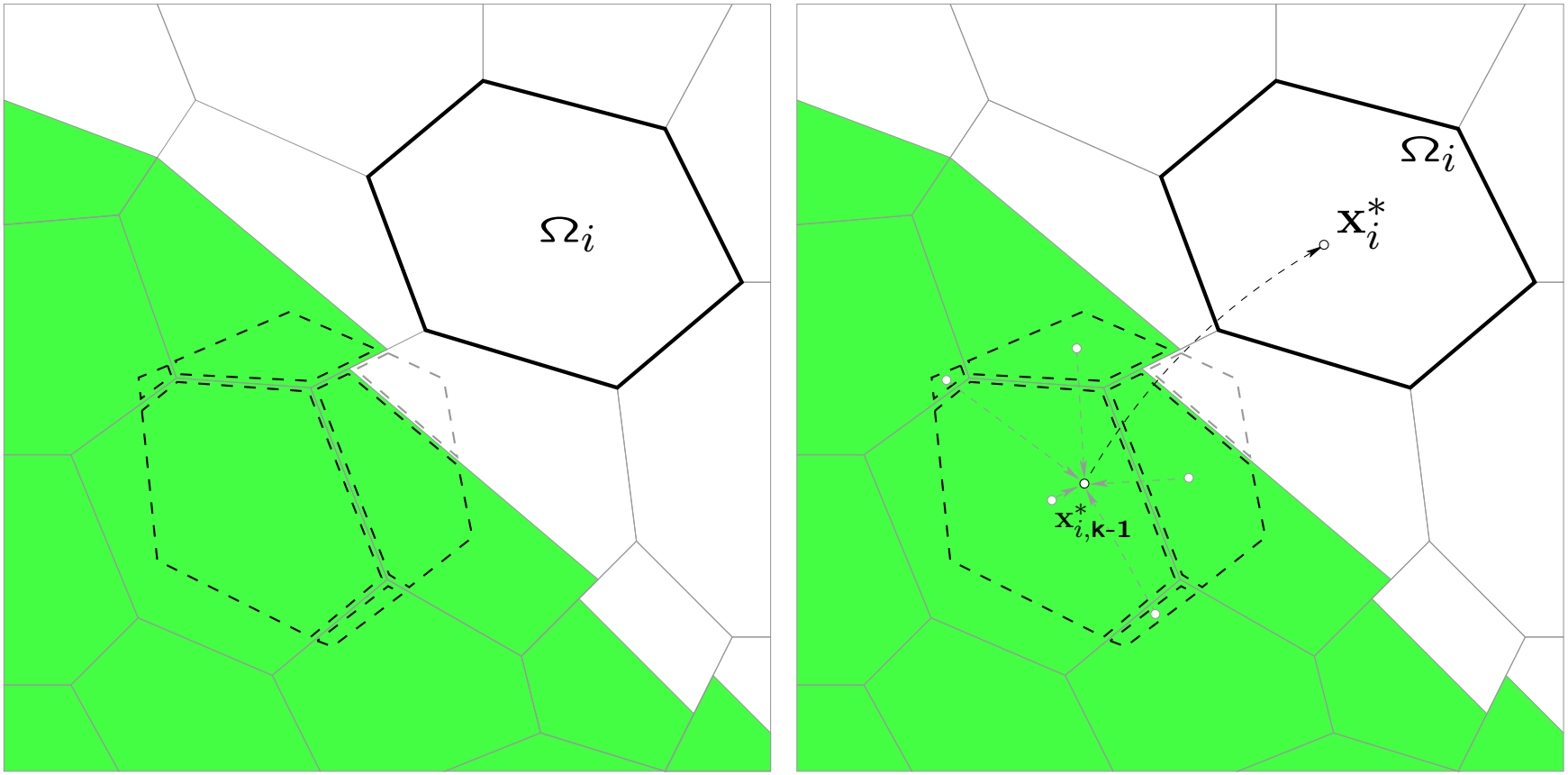


1) trace the cell vertices back in time; connect them in the proper order with the straight segments



2) intersect the Lagrangian preimage with the underlying pure subcells; calculate the total volume of the materials enclosed

Centroid tracking



The centroid of any parcel of incompressible fluid moves very much like a Lagrangian particle

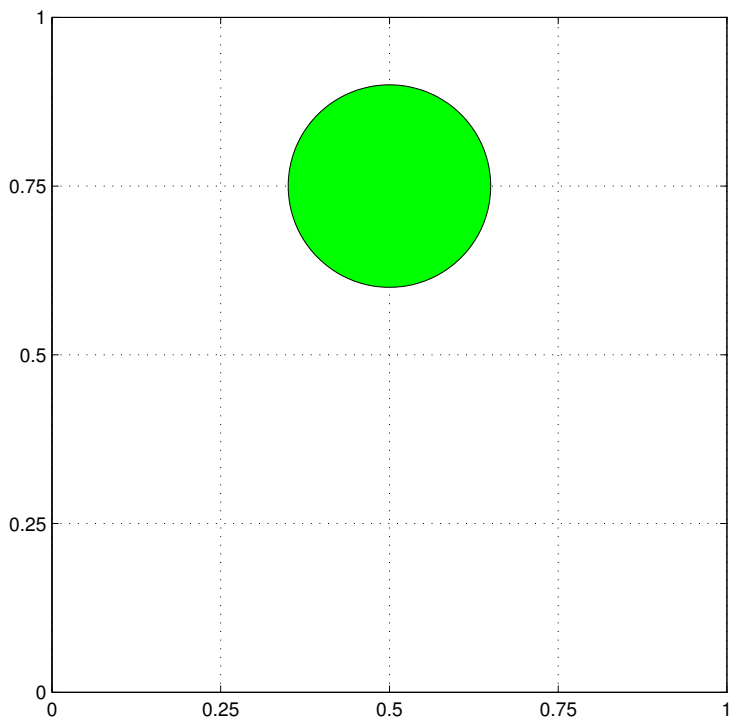
$$\frac{d}{dt}\mathbf{x}_c(\omega) = \mathbf{v}(\mathbf{x}_c(\omega)) + O(\mathbf{diam}^2\omega)$$

Dynamic example

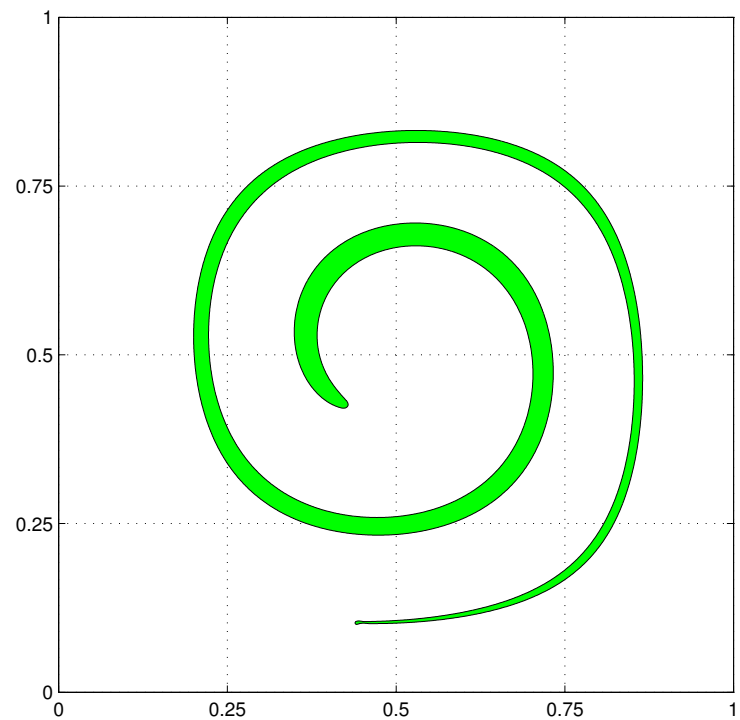
reversible "vortex-in-a-box" field:

$$\mathbf{v}((x,y),t) = \begin{bmatrix} +\sin^2(\pi x)\,\sin(2\pi y) \\ -\sin^2(\pi y)\,\sin(2\pi x) \end{bmatrix} \cos(\pi t/T)$$

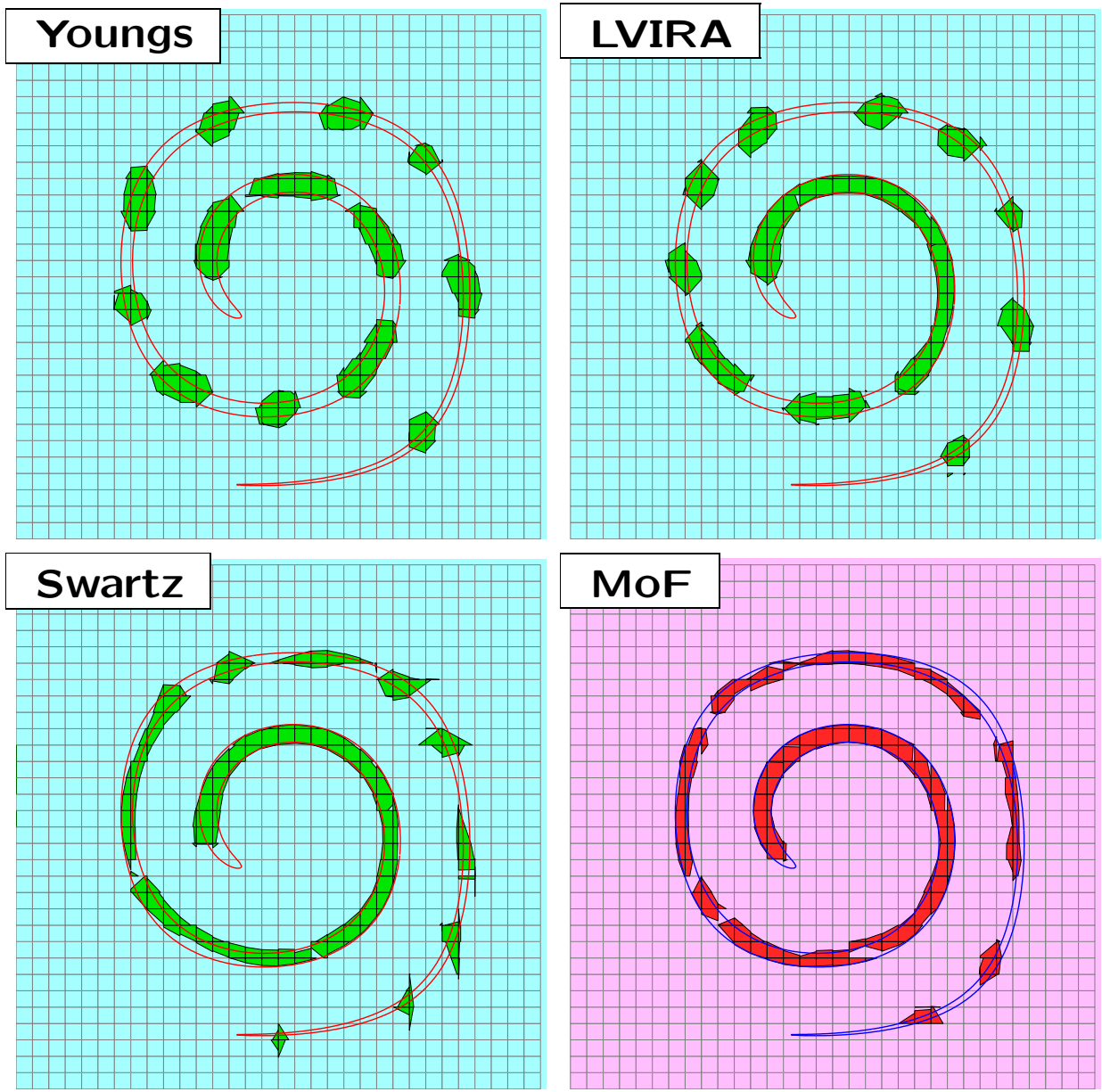
$t = 0, \qquad t = T = 8$



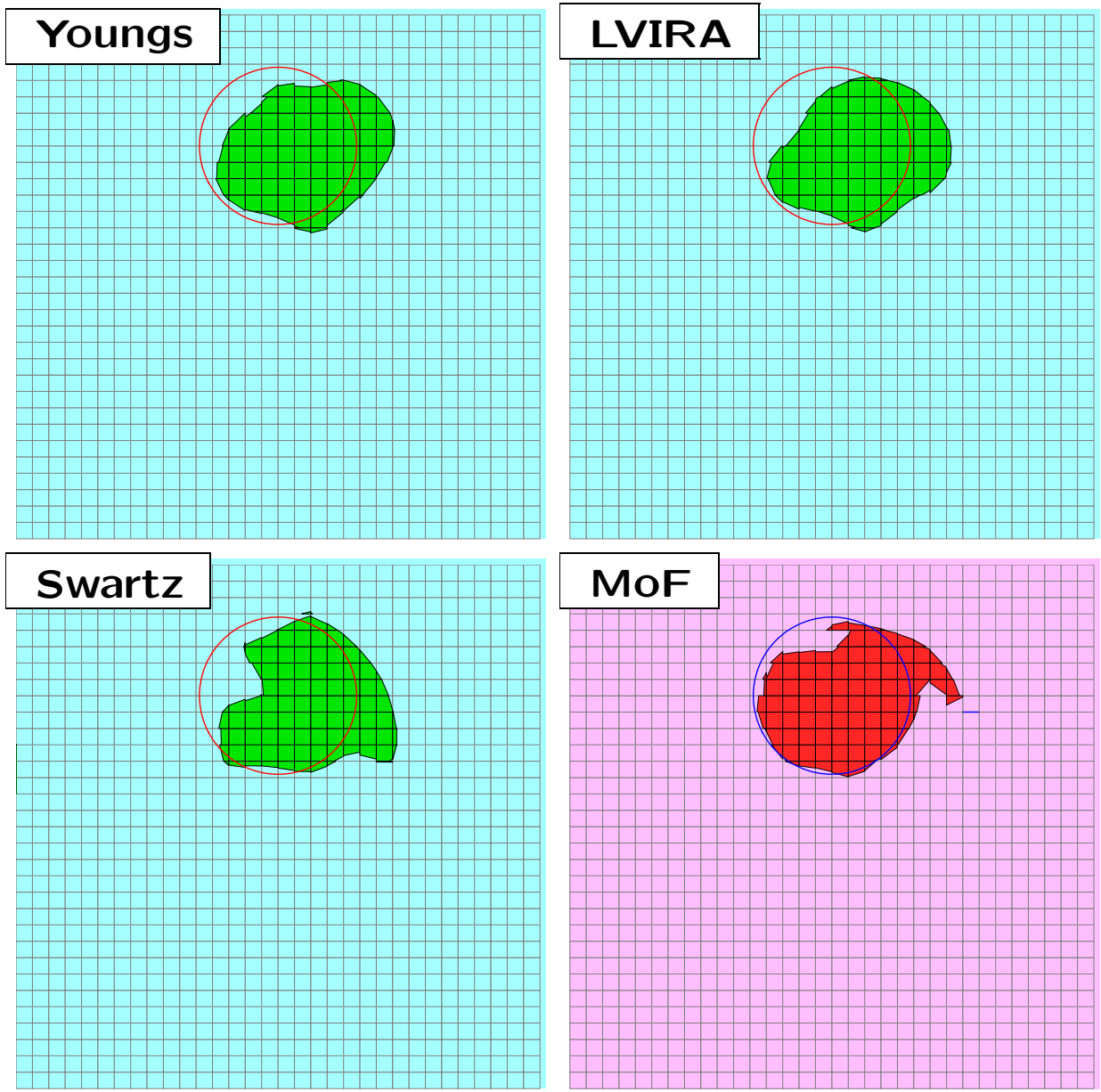
$t = T/2 = 4$



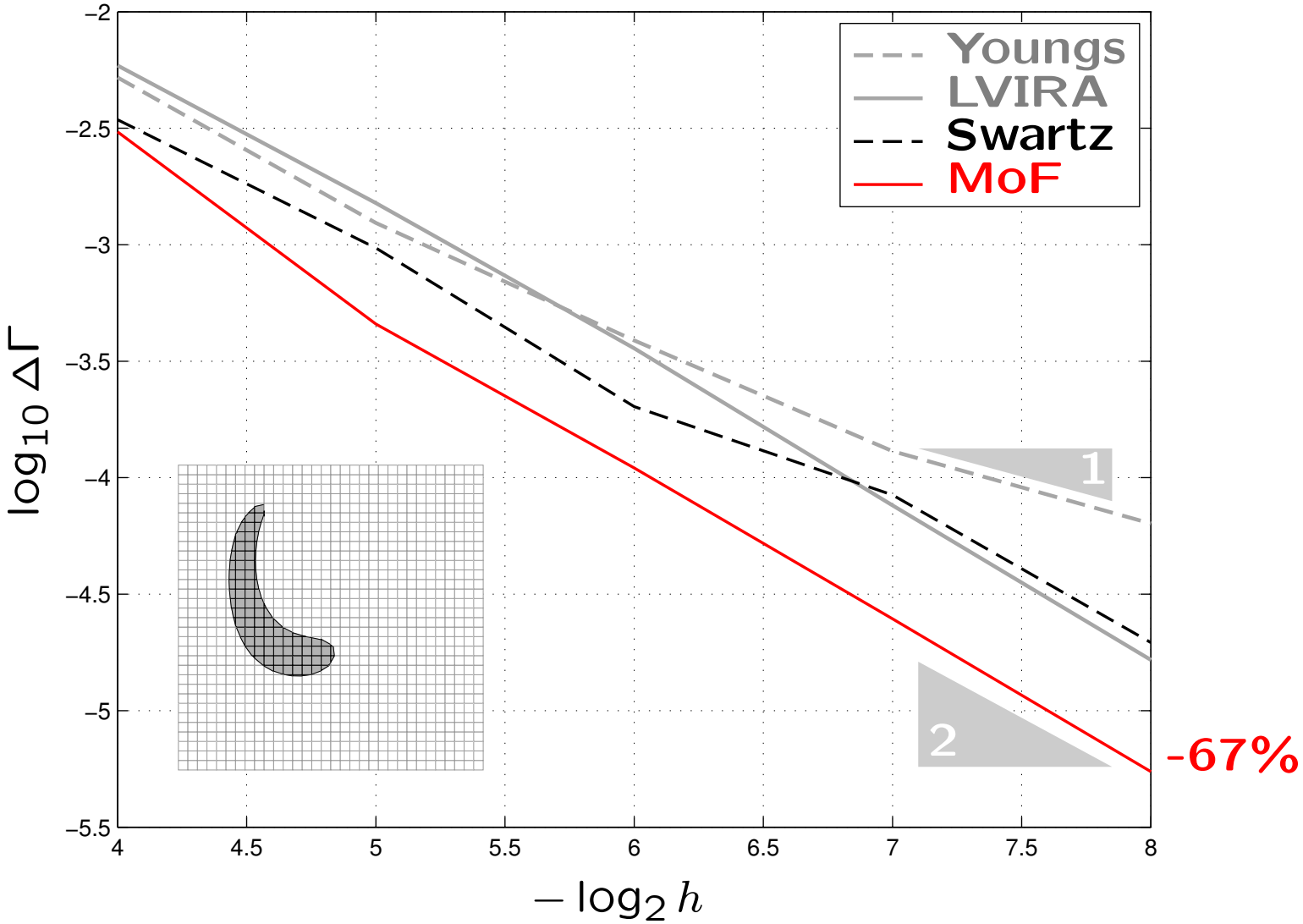
“Vortex-in-a-box” test, t=T/2



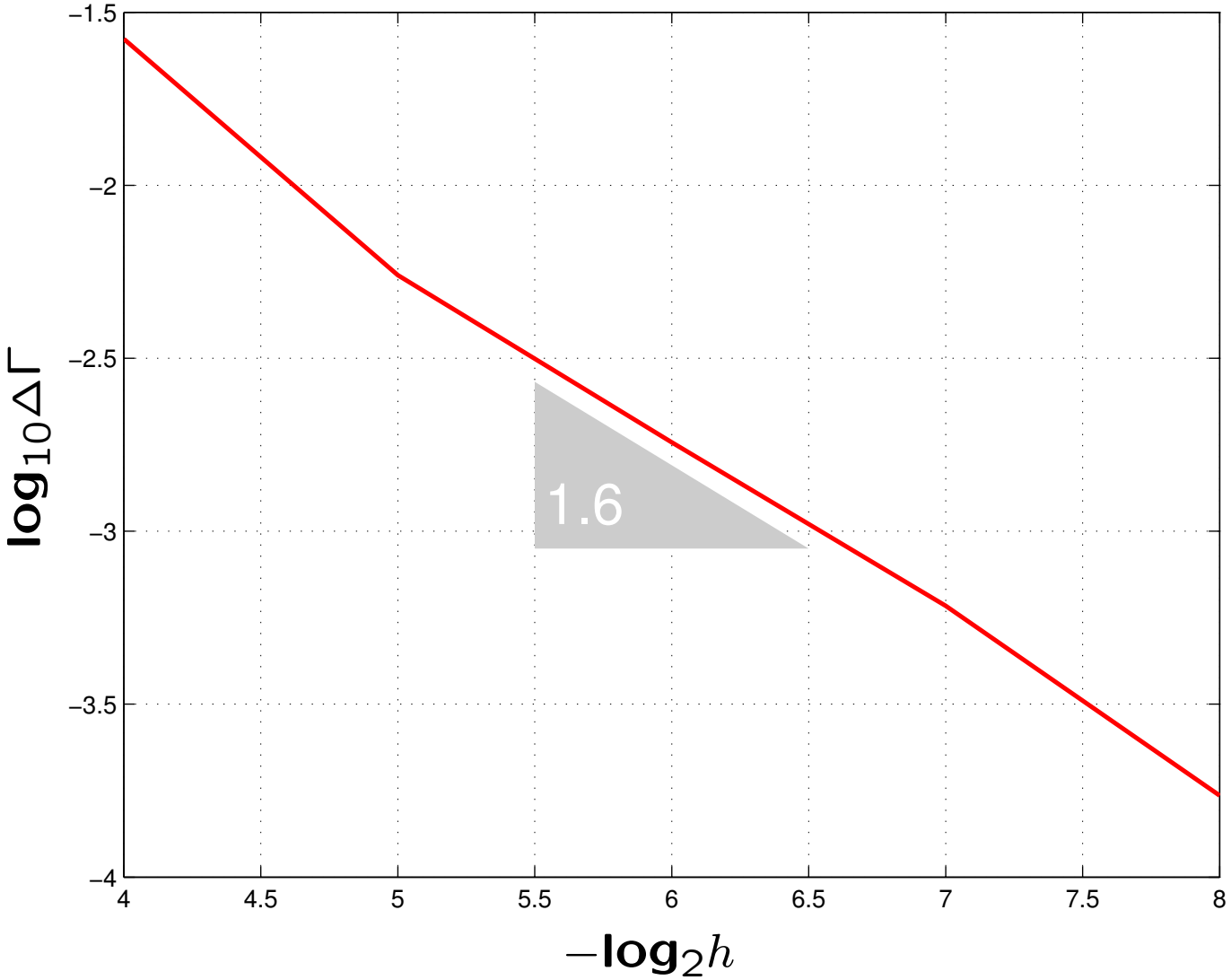
“Vortex-in-a-box” test, t=T



The errors measured in the reversible vortex test



MoF coupled with incompressible Navier-Stokes solver

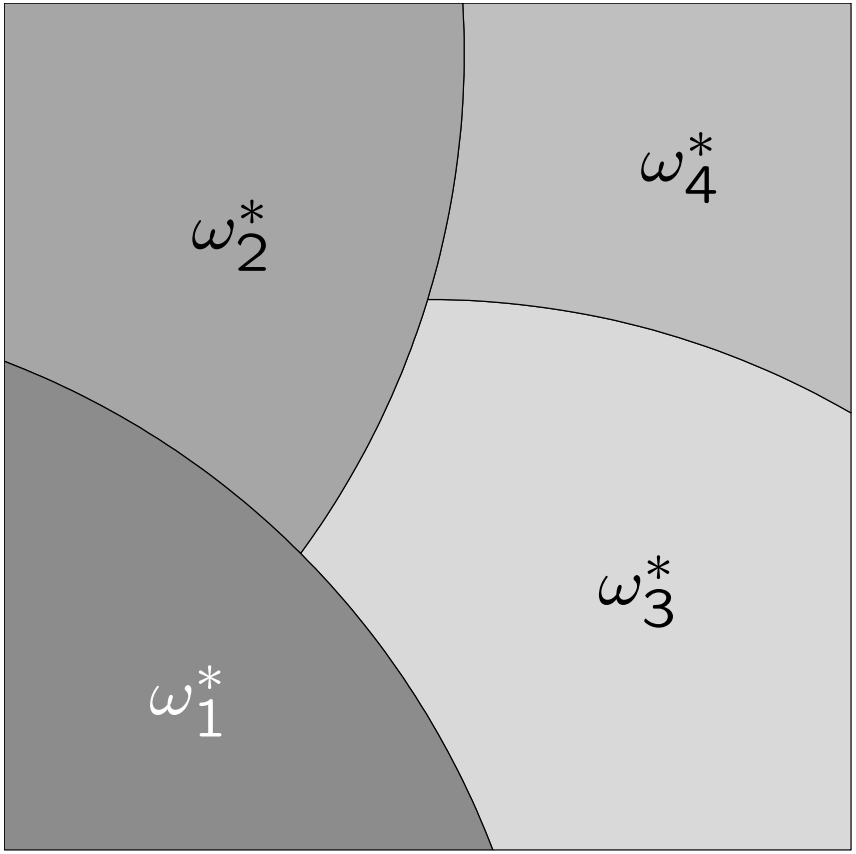


Multi-material MoF

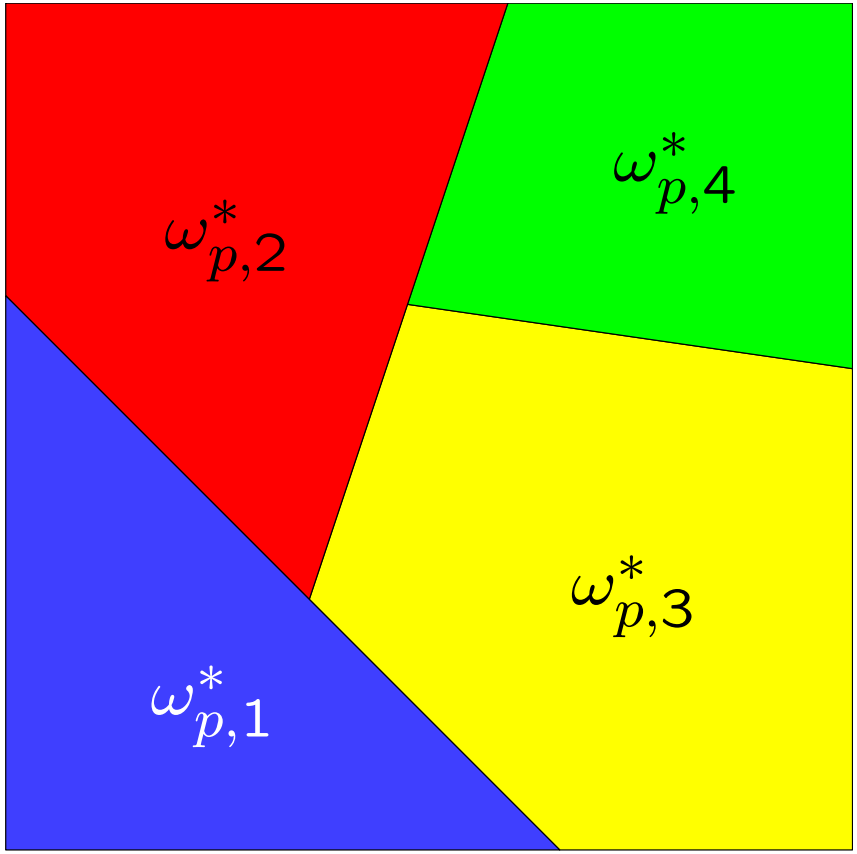
automatic material ordering

Multi-material MoF

a single mixed cell with $M \geqslant 3$ materials



true partition



polygonal approximation

Automatic material ordering

Like multi-material VoF

MoF uses the two-material algorithm
to separate materials one by one.

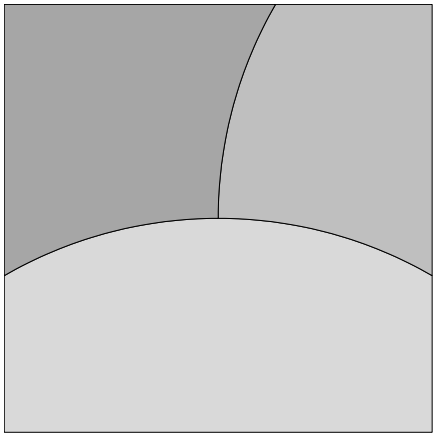
Unlike multi-material VoF

MoF can determine the right material order automatically,
by trying all possible material orders
and selecting the one
that results in the minimal
defect of the 1st moment:

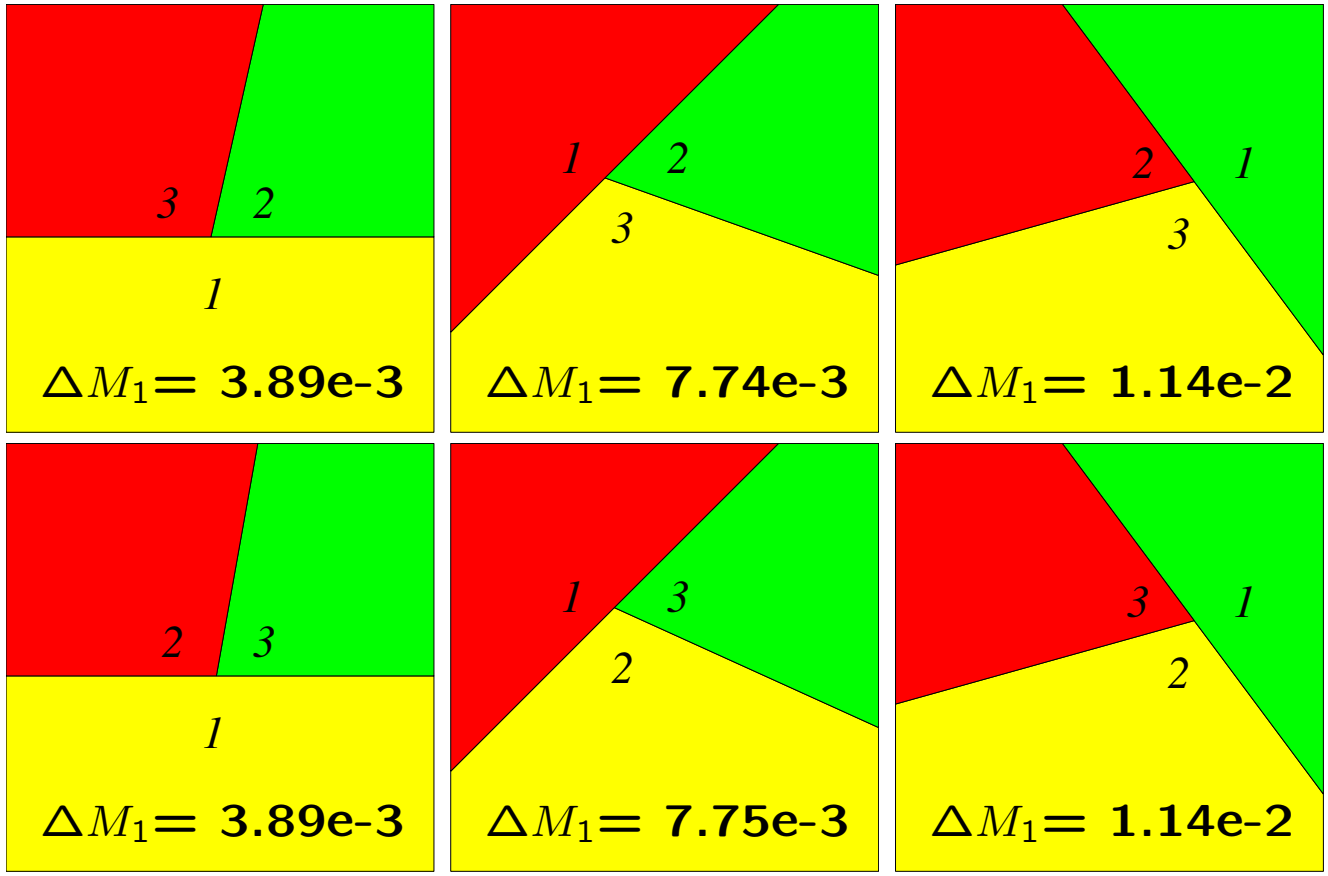
$$\sum_m |\omega_m^*|^2 ||\mathbf{x}_c(\omega_{p,m}) - \mathbf{x}_m^*||_2^2 \longrightarrow \textbf{min}$$

Automatic material ordering

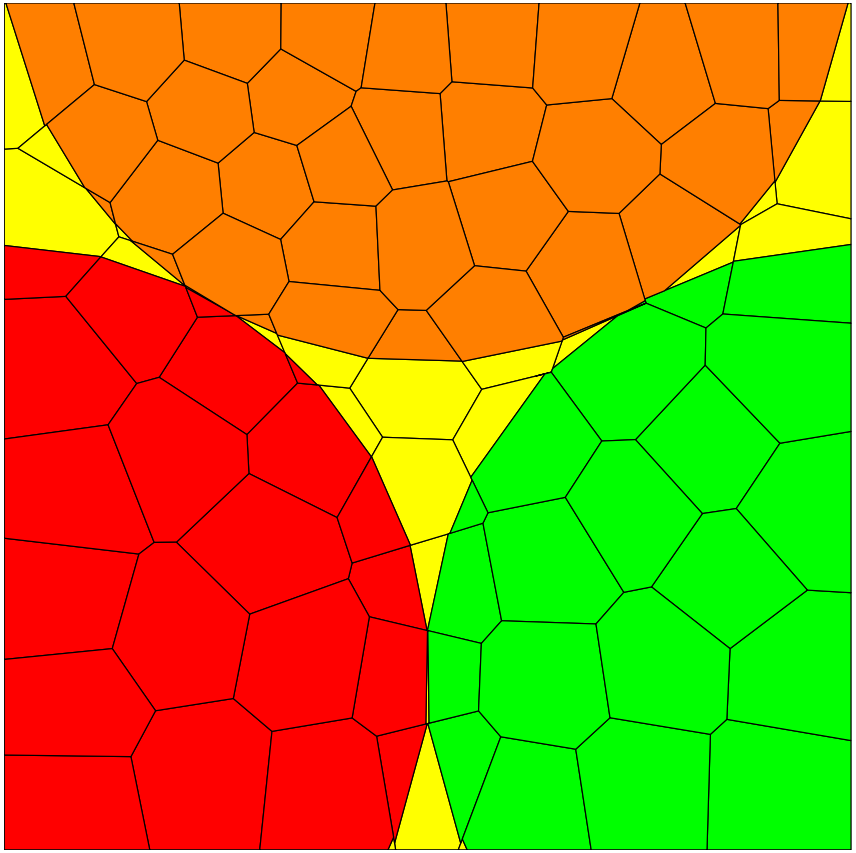
true partition

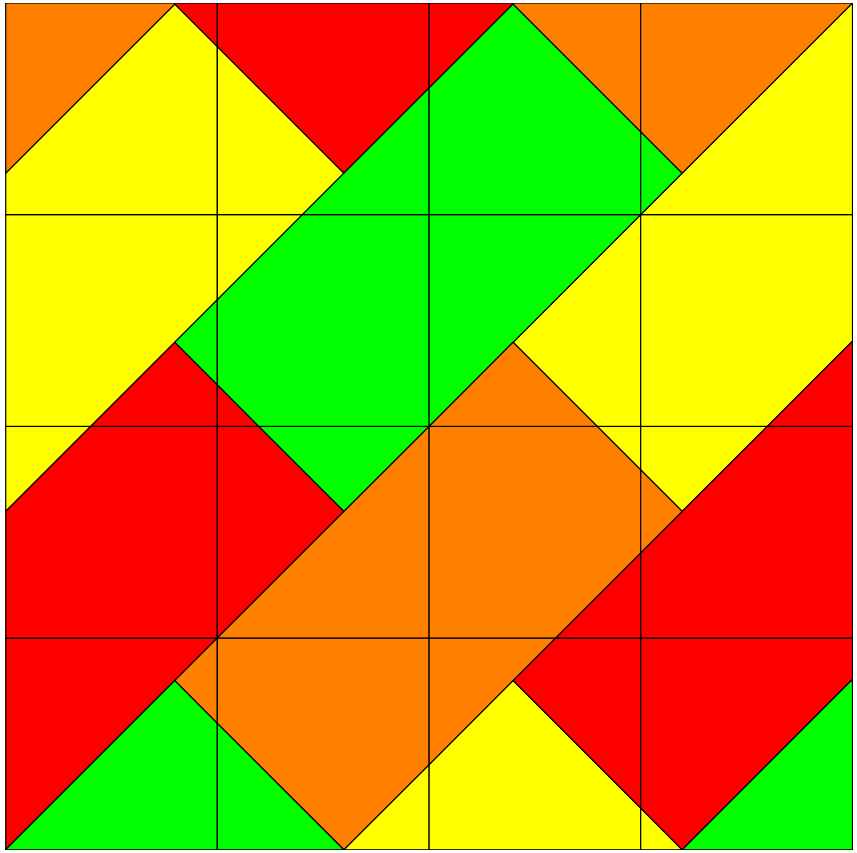


MoF approximations obtained with all possible material orders



Examples of the MoF reconstruction

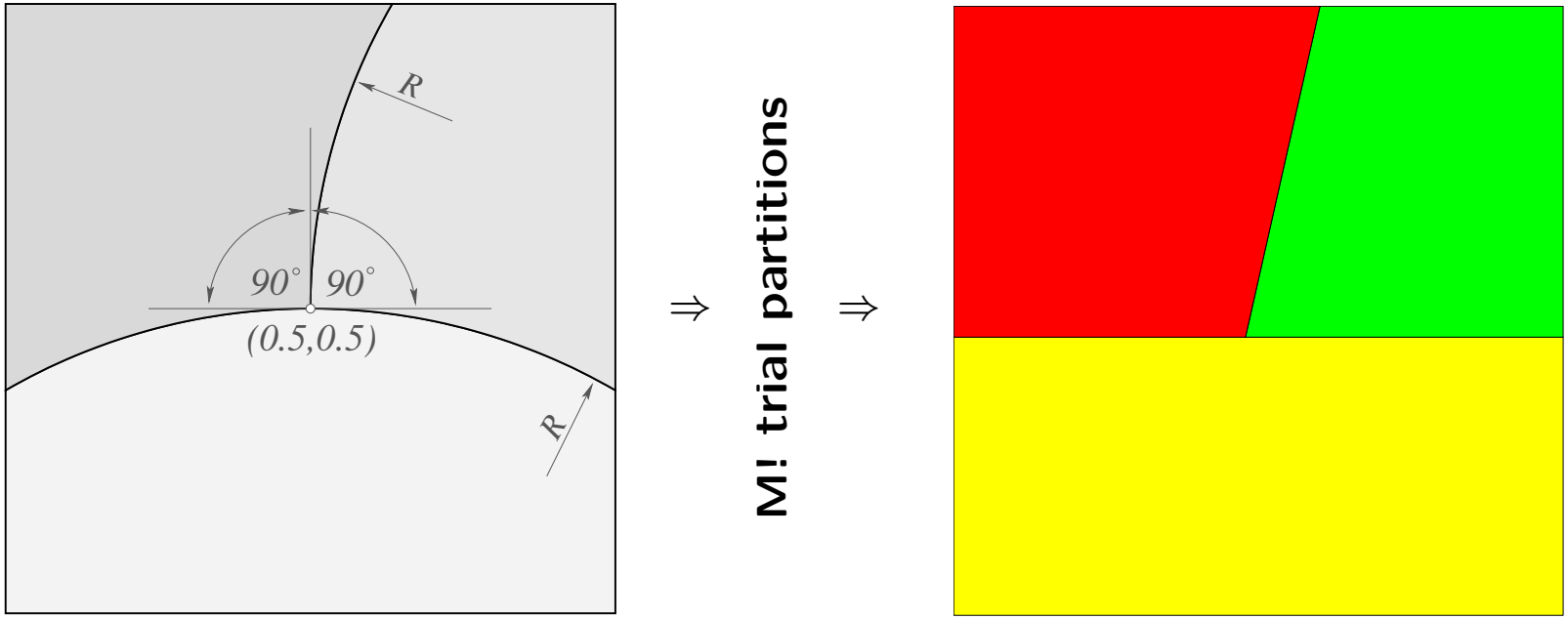




materials are separated one by one

Serial partitions

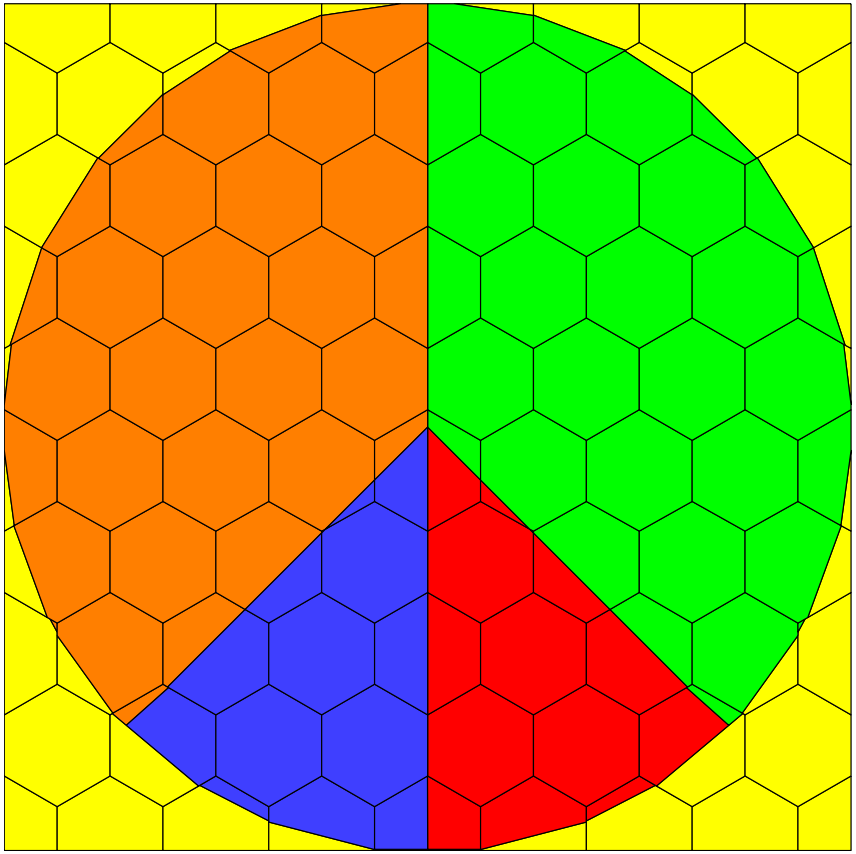
Serial partition: all materials can be separated one by one with twice-continuously-differentiable dissections.

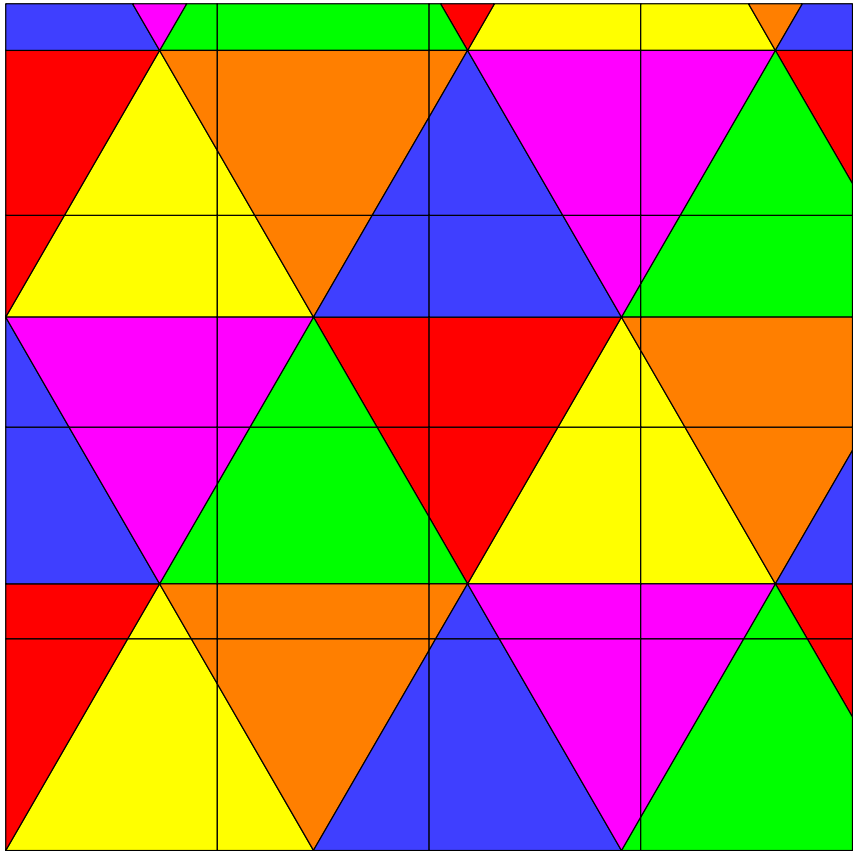


<u>Theorem:</u> MoF approximation to any serial partition with sufficiently low interface curvature is 2nd-order accurate:

$$\Delta\Gamma = O(h^2/R)$$

Automatic material aggregation



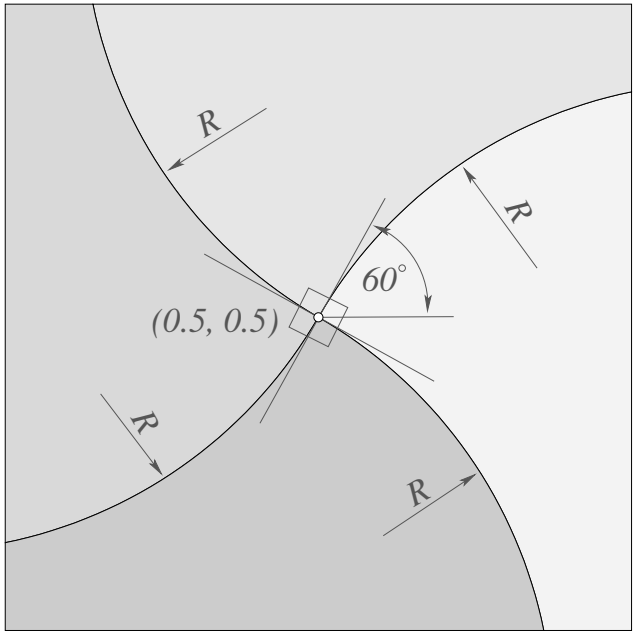


instead of separating materials one by one,

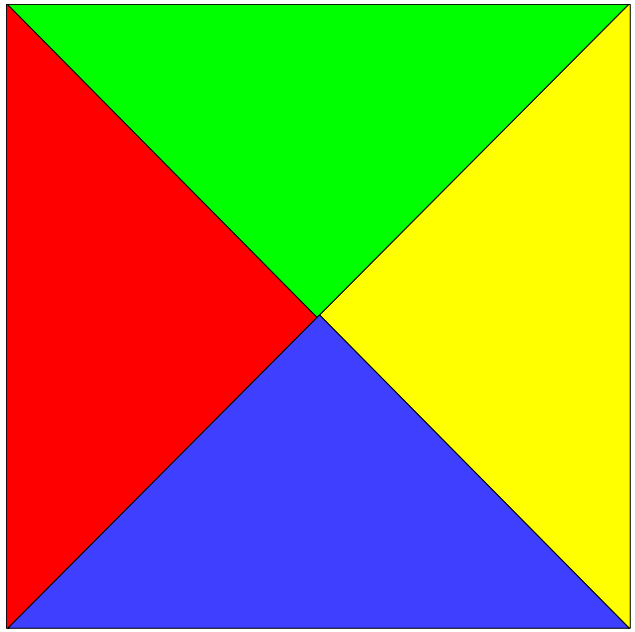
one can recursively separate the groups of materials

B-tree partitions

B-tree partition: all materials can be separated with M twice-continuously-differentiable nested dissections.



$\Rightarrow$ **M!(M-1)! trial partitions** $\Rightarrow$



<u>Theorem:</u> MoF approximation to any B-tree partition with sufficiently low interface curvature is 2nd-order accurate:

$$\Delta\Gamma = O(h^2/R)$$

Concluding remarks

Summary:

- new two-material interface reconstruction technique
- automatic processing of the the multi-material cells

Ongoing research:

- Lagrangian remap with discrete velocities
- stable multi-segment interface approximation
- error-driven AMR for the MoF interface reconstruction

Publications & supplemental material:

http://math.lanl.gov/∼vdyadechko/research

MoF Interface Reconstruction in 3D - Bolt-and-Nut

H. Ahn and M. Shashkov, T-7, LANL