Construction of multi-material interfaces from moment data

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Talk layout

1) brief look at the **Volume-of-Fluid (VoF)** algorithms

2) new **Moment-of-Fluid (MoF)** strategy

- two-material case
- Lagrangian remap: how to update the moment data
- multi-material case
Volume-of-fluid (VoF) technology

Two components of typical VoF method:

1) interface reconstruction (IR) algorithm
   - input data:
     cell-wise material volume fractions
   - preserves (volume=mass) of each material
   - can separate two materials in a mixed cell
   - uses one linear segment per mixed cell

2) volume tracking scheme:
   volume fractions are updated
   by calculating the material fluxes
   through the cell boundaries
VoF-IR resolution limit

The evaluation of the interface normal from the volume fractions can not be accomplished without the data from the neighbors.

Resulting interfaces in adjacent mixed cells are not independent

VoF interface approximation can not resolve interface details smaller than the size of the cell cluster participating in the interface normal evaluation.

VoF-IR resolution limit $\approx 2$ to $3$ cell sizes
“zigzag” shape reconstruction

true

Youngs

LVIRA

Swartz
Moment-of-Fluid (MoF) reconstruction

assist VoF with material centroids
MoF interface reconstruction

input data: a volume fraction ($\mu^*$) and a centroid ($x^*$)

Among all the subcells $\omega$ with a linear interface and the prescribed volume

$$|\omega| = \mu^* |\Omega|,$$

find the one whose centroid $x_c(\omega)$ is closest to the given centroid:

$$\| x_c(\omega) - x^* \| \rightarrow \min$$

local, dimension- and cell-shape- independent

unique, stable, 2nd-order accurate
"constellation" shape  reconstruction

true

MoF

LVIRA

Swartz
Approximation error: circle of radius $R$

\[
\log_{10} \Delta \Gamma / R = R - 18\%
\]
Approximation error: $2L \times 2L$ square
Lagrangian remap

how to update the moment data
Volume tracking

1) trace the cell vertices back in time; connect them in the proper order with the straight segments

2) intersect the Lagrangian preimage with the underlying pure sub-cells; calculate the total volume of the materials enclosed
The centroid of any parcel of incompressible fluid moves very much like a Lagrangian particle

\[ \frac{d}{dt} x_c(\omega) = v(x_c(\omega)) + O(\text{diam}^2 \omega) \]
Dynamic example

reversible “vortex-in-a-box” field:

\[ v((x, y), t) = \begin{bmatrix} \sin^2(\pi x) \sin(2\pi y) \\ -\sin^2(\pi y) \sin(2\pi x) \end{bmatrix} \cos(\pi t/T) \]

\[ t = 0, \quad t = T = 8 \]

\[ t = T/2 = 4 \]
“Vortex-in-a-box” test, $t = T/2$
“Vortex-in-a-box” test, t=T

Youngs | LVIRA
---|---

Swartz | MoF
The errors measured in the reversible vortex test

\[ \log_{10} \Delta \Gamma \] vs. \[ -\log_2 h \]

- Dashed line: Youngs
- Gray line: LVIRA
- Dotted line: Swartz
- Red line: MoF

\(-67\%\)
MoF coupled with incompressible Navier-Stokes solver

\[ \log_{10} \Delta \Gamma = -\log_2 h \]

\(1.6\)
Multi-material MoF

automatic material ordering
Multi-material MoF

a single mixed cell with $M \geq 3$ materials
Automatic material ordering

**Like multi-material VoF**
MoF uses the two-material algorithm to separate materials one by one.

**Unlike multi-material VoF**
MoF can determine the right material order automatically, by trying all possible material orders and selecting the one that results in the minimal defect of the 1st moment:

$$
\sum_{m} |\omega_{m}^{*}|^{2} \| x_{c}(\omega_{p},m) - x_{m}^{*} \|^{2}_{2} \rightarrow \text{min}
$$
Automatic material ordering

true partition

MoF approximations obtained with all possible material orders

\[ \Delta M_1 = 3.89 \times 10^{-3} \]

\[ \Delta M_1 = 7.74 \times 10^{-3} \]

\[ \Delta M_1 = 1.14 \times 10^{-2} \]
Examples of the MoF reconstruction

materials are separated one by one
Serial partitions

**Serial partition**: all materials can be separated one by one with twice-continuously-differentiable dissections.

**Theorem**: MoF approximation to any serial partition with sufficiently low interface curvature is 2nd-order accurate:

\[ \Delta \Gamma = O(h^2/R) \]
Automatic material aggregation

instead of separating materials one by one, one can recursively separate the groups of materials
B-tree partitions

B-tree partition: all materials can be separated with $M$ twice-continuously-differentiable nested dissections.

\[ M! (M-1)! \text{ trial partitions} \]

**Theorem:** MoF approximation to any B-tree partition with sufficiently low interface curvature is 2nd-order accurate:

\[ \Delta \Gamma = O(h^2/R) \]
Concluding remarks

Summary:

- new two-material interface reconstruction technique
- automatic processing of the multi-material cells

Ongoing research:

- Lagrangian remap with discrete velocities
- stable multi-segment interface approximation
- error-driven AMR for the MoF interface reconstruction

Publications & supplemental material:

http://math.lanl.gov/~vdyadechko/research
MoF Interface Reconstruction in 3D - Bolt-and-Nut
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