Smoothed Particle Hydrodynamics as a tool for modeling material strength and failure

Numerical methods for multi-material fluid flows
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September 2007

This work was performed under the auspices of the U.S. Department of Energy by the University of California Lawrence Livermore National Laboratory under Contract No. W-7405-Eng-48.

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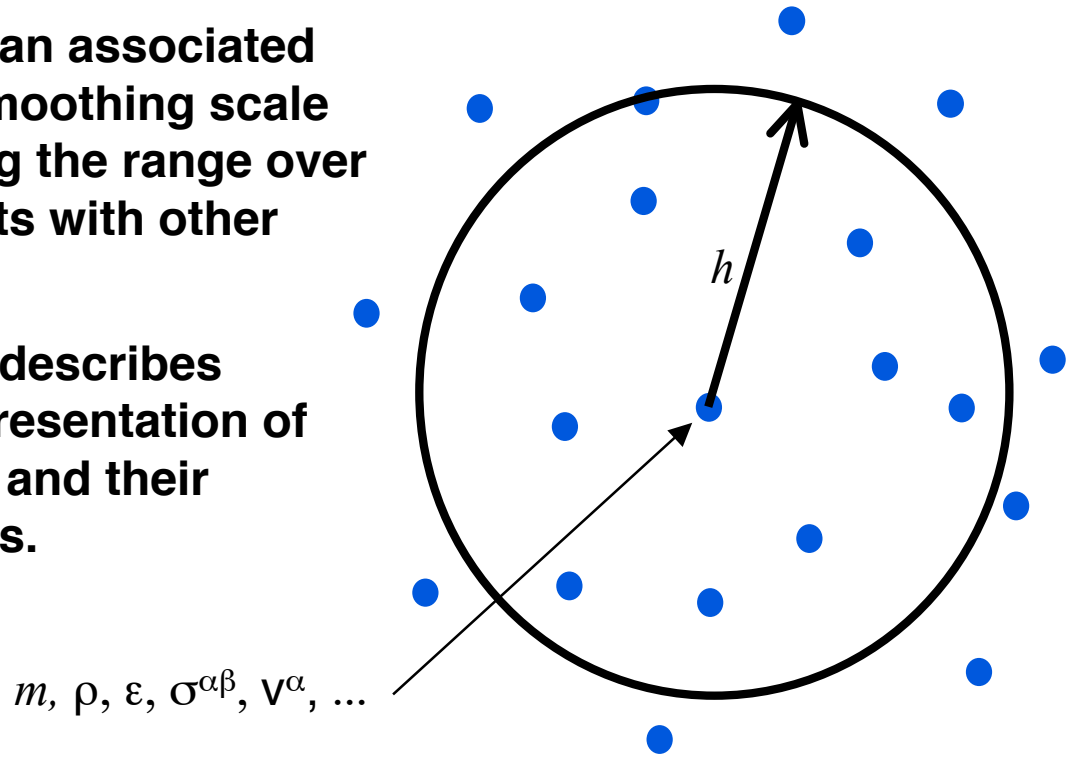
Why use a meshless technique?

- Meshless methods such as SPH have unique advantages (and difficulties) when trying to model problems involving material strength, failure, and fragmentation.
- Pros:
  - Lagrangian *and* robust (no mesh to tangle).
    - Well suited for problems with complex flows or large deformations.
    - No mixed material elements!
    - History variables (such as $S^{\alpha\beta}$ and $D^{\alpha\beta}$) are simple to handle.
  - Naturally allows for gaps in material to form -> fragmentation!
- Cons:
  - Interfaces difficult to represent accurately.
  - Current formulations can be inaccurate as node distribution is distorted and near surfaces.
  - Often more expensive than mesh based approaches (at least for simple problems).

Cartoon view of how SPH works.

- Physics variables defined at an arbitrary set of points in space.
- Points move with material velocity, arbitrarily reconnecting with new neighbors as simulation proceeds.
- Each point has an associated resolution or smoothing scale (*h),* representing the range over which it interacts with other points.
- SPH formalism describes continuous representation of nodal variables and their spatial gradients.



SPH equations with material strength.

- **We discretize the usual Lagrangian equations for conservation of mass, momentum, and energy.**

$$\frac{D\rho_i}{Dt} = -\rho_i(\partial_\alpha v^\alpha)_i \approx \rho_i \sum_j \frac{m_j}{\rho_j}\left(v_i^\alpha - v_j^\alpha\right)\partial_\alpha W_i$$

$$\frac{Dv_i^\alpha}{Dt} = \rho_i^{-1}(\partial_\beta \sigma^{\alpha\beta})_i \approx \sum_j m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2} + R_{ij}^{\alpha\beta} - \Pi_{ij}^{\beta\alpha}\right)\partial_\beta W_{ij}$$

$$\begin{aligned}\frac{Du_i}{Dt} &= \rho_i^{-1}(\sigma^{\alpha\beta}\partial_\beta v^\alpha)_i \\ &\approx -\sum_j m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + R_i^{\alpha\beta} - \frac{1}{2}\Pi_{ij}^{\beta\alpha}\right)\left(v_i^\alpha - v_j^\alpha\right)\partial_\beta W_{ij}\end{aligned}$$

- **As written here, these equations exactly conserve mass and linear momentum.**

Total Energy Conserving SPH

- One important enhancement we use over standard SPH is a total energy conserving form based on compatible differencing ideas described in Caramana, Burton, Shashkov, & Whalen (1998).
- We replace the energy evolution equation from the previous slide with a form that exactly describes the discrete work done by node *j* upon *i*.

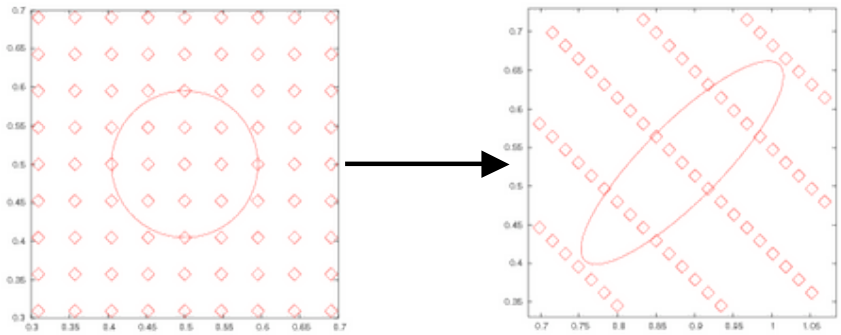
$$\Delta u_{ij} = f_{ij}\left[(v_j^{\alpha})^{1/2} - (v_i^{\alpha})^{1/2}\right](a_{ij}^{\alpha})^{0}\Delta t$$

- The pair-wise work is distributed between each node pair according to f_{ij}, subject to the constraint $f_{ij} + f_{ji} = 1$.
- We do not go into the details of this algorithm in this talk.
  - A preprint manuscript on this subject is available if anyone is interested.

ASPH is important for modeling anisotropic evolution.

- Adaptive SPH (ASPH) replaces the scalar smoothing scale with a tensor transformation, which allows adaptation in arbitrary elliptical (2-D) or ellipsoidal (3-D) volumes.
  - SPH: $\eta_i^\alpha = x^\alpha / h_i$
  - ASPH: $\eta_i^\alpha = H_i^{\alpha\beta} x^\beta$

$$H_i^{\alpha\beta} \propto \sum_j \frac{(W_i^H)^2}{r_{ij}^{\nu-1}} \, \hat{x}_{ij}^\alpha \, \hat{x}_{ij}^\beta$$



- The hourglassing problems with directly measuring $H^{\alpha\beta}$ we described previously have now been dealt with.
  - Hourglassing is still a concern, but we have been been able to mitigate the effect on $H^{\alpha\beta}$ by using an interpolation kernel that is not sensitive to nearest neighbors.

$$(W_i^H)^2 = \partial_\alpha W_i \, \partial_\alpha W_i$$

The tensor damage variable.

- We employ a symmetric tensor damage variable $D^{\alpha\beta}$ in order to account for material failure.
  - The tensor damage captures the directional nature of the strain and failure, allowing failure in particular directions for each node.
- We can build this tensor formlism from a scalar damage model based on the strain.
  - Given a scalar damage evolution law as a function of strain:

$$\frac{DD_S}{Dt}(\epsilon)$$

  - the tensor damage evolution is

$$\frac{DD_i^{\alpha\beta}}{Dt} = \Omega_i^{\alpha\gamma} D_i^{\gamma\beta} - D_i^{\alpha\beta}\Omega_i^{\alpha\beta} + \frac{DD_S}{Dt}(\epsilon_i^{\gamma}) e_i^{\alpha\gamma} e_i^{\beta\gamma}$$

  - where ε_i^{γ}, $e_i^{\alpha\beta}$ are the eigenvalues/vectors of the strain $\varepsilon_i^{\alpha\beta}$, and $\Omega_i^{\alpha\beta}$ is the spin tensor,

$$\Omega_i^{\alpha\beta} = \frac{1}{2}\left(\partial_\beta v_i^\alpha - \partial_\alpha v_i^\beta\right)$$

The scalar damage model.

- There are a variety of models we can choose from to determine the form of $DD_S(\varepsilon)/Dt$.
  - We are implementing a number of these models for comparison.
- The results in this presentation use a model based the statistical continuum theory of Grady & Kipp, formulated appropriately for a numerical scheme by Benz & Asphaug (1994, 1995, 1999).
  - Seed a population of flaw activation strains at each SPH node according to a Weibull distribution:

$$n(\epsilon) = k\epsilon^m$$

  - For the fraction of activated flaws on a node f_{act}:

$$\frac{DD_{S_i}^{1/3}}{Dt} = f_{\mathrm{act}_i}\frac{c_{g_i}}{R_{S_i}} = f_{\mathrm{act}_i}\frac{0.4c_{l_i}}{\eta_{\max}|H_i|^{-1/\nu}} \qquad f_{\mathrm{act}_i} = \frac{N_{\mathrm{flaws}_i}(\epsilon_f \le \epsilon_i)}{N_{\mathrm{flaws}_i}}$$

Defining the strain on a node.

- We currently support three options for identifying the instantaneous tensor strain on a node:
  - Direct history of the strain:

$$\epsilon_i^{\alpha\beta}(t) = \int_0^t \dot{\epsilon}_i^{\alpha\beta} dt = \int_0^t \frac{1}{2}\left(\partial_\alpha v^\beta + \partial_\beta v^\alpha\right)_i dt$$

  - Benz & Asphaug definition:

$$\epsilon_i^{\alpha\beta} = \frac{\sigma_i^{\alpha\beta}}{E_i}, \quad E_i = \frac{9K_i\mu_i}{3K_i + \mu_i}$$

  - Melosh, Ryan, & Asphaug (J. Geo. Res. 1992):

$$\sigma_i^{\alpha\beta} = \frac{(K_i - 2\mu_i)\epsilon_i^{\mathrm{V}}\delta^{\alpha\beta} + 2\mu_i\epsilon^{\alpha\beta}}{E_i}, \; \epsilon_i^{\mathrm{V}} = \epsilon_i^{\alpha\beta}\delta^{\alpha\beta}$$

Application of the damage.

- **We expect the damage to enter into our evolution as a one minus the damage softening of the stress,**

$$f_{D_i}^{\alpha\beta} = \delta^{\alpha\beta} - D_i^{\alpha\beta}$$

- **We reduce the stress and the velocity gradient by the damage,**

$$\sigma_i^{\alpha\beta} \rightarrow \begin{cases} f_{D_i}^{\alpha\gamma} \sigma_i^{\gamma\phi} f_{D_i}^{\phi\beta} & : \quad P_i < 0 \\ f_{D_i}^{\alpha\gamma} S_i^{\gamma\phi} f_{D_i}^{\phi\beta} - P_i & : \quad P_i \geq 0 \end{cases}$$

$$\partial_\beta v_\alpha = \rho_i^{-1} \sum_j \frac{1}{2} \left(f_{D_i}^{\alpha\gamma} + f_{D_j}^{\alpha\gamma} \right) \left(v_j^\gamma - v_i^\gamma \right) \partial_\beta W_{ij}$$

Problem: cleanly decoupling nodes

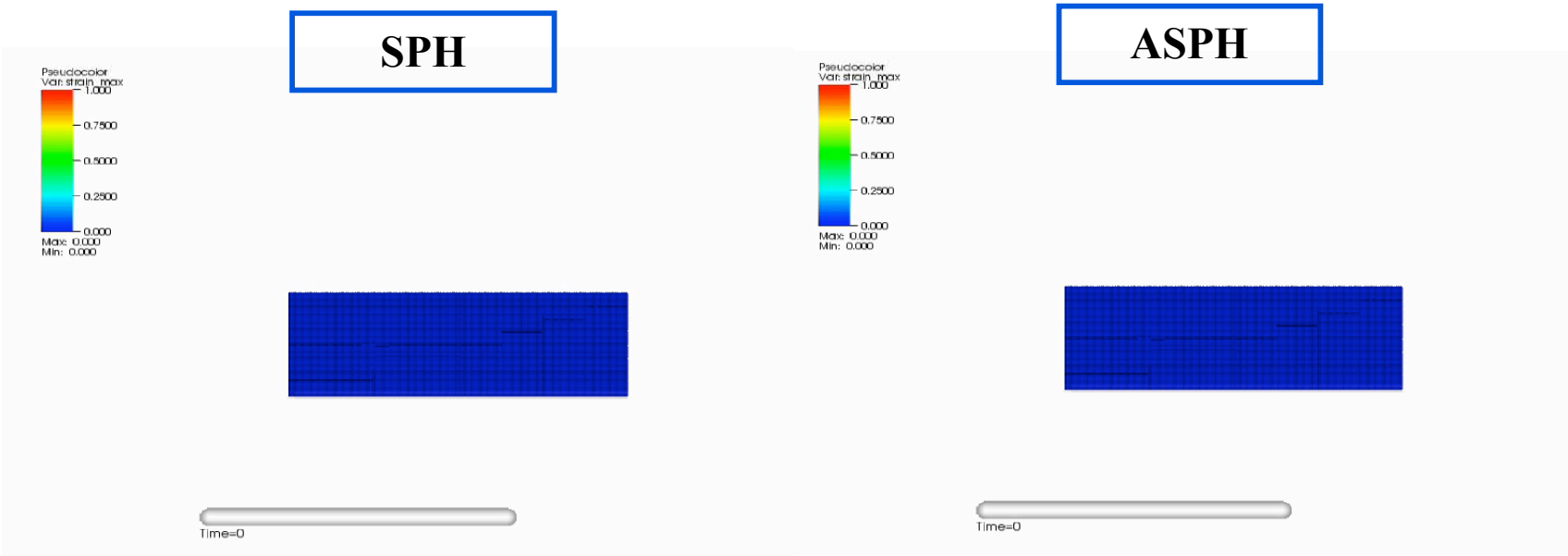
- The improved adaptability of ASPH allows us to solve elastic/plastic problems more accurately.
- However in fragmentation scenarios the $H_i^{\alpha\beta}$ tensor will incorrectly stretch over opening cracks, keeping nodes in contact that we would like to cleanly decouple.
- On possible correction is to boost the weight of damaged nodes when measuring the moments with which we set $H^{\alpha\beta}$,

$$H_i^{\alpha\beta} \propto \sum_j \left(1 + D_j^{\max}\right)^{2\nu} \frac{(W_i^H)^2}{r_{ij}^{\nu-1}} \, \hat{x}_{ij}^{\alpha} \, \hat{x}_{ij}^{\beta}$$

  where $D_j^{\max}$ is the maximum eigenvalue of $D_j^{\alpha\beta}$.
  - This causes the measurement of $H_i^{\alpha\beta}$ to pull back from these enhanced weight nodes.
- This is still an issue under development…

2-D Tensile Rod

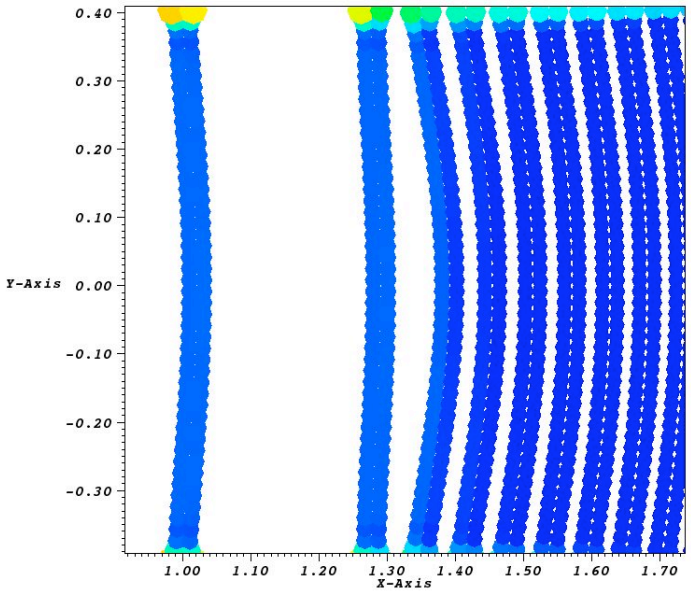
- Consider a rod being stretched by a constant velocity on the ends:
  - 3 cm long, 1 cm high
  - $v_o = 10^4$ cm/sec on ends
- Plotted here are the maximum eigenvalues of the strain, run without the damage model (should simply stretch).



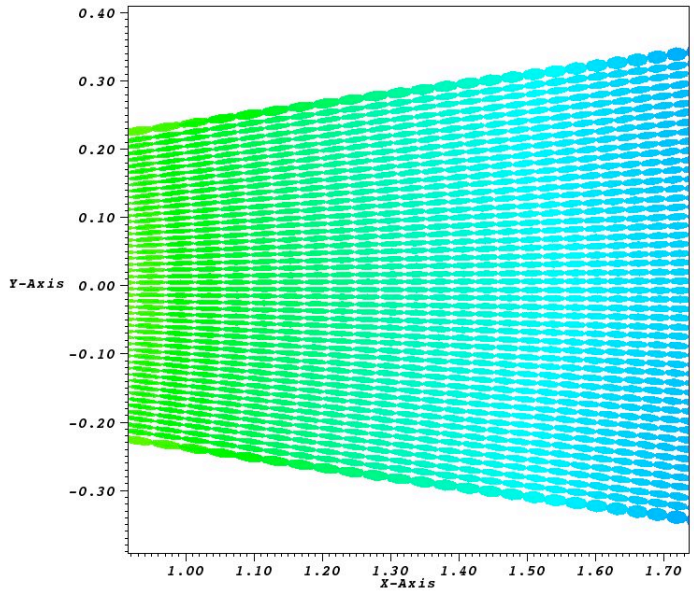
2-D Tensile Rod: H Tensors

- **The SPH run clearly suffers from numerical fragmentation, where ASPH does not.**
  - **This is due to ASPH's ability to adapt to the anisotropic evolution of the node spacing.**

SPH

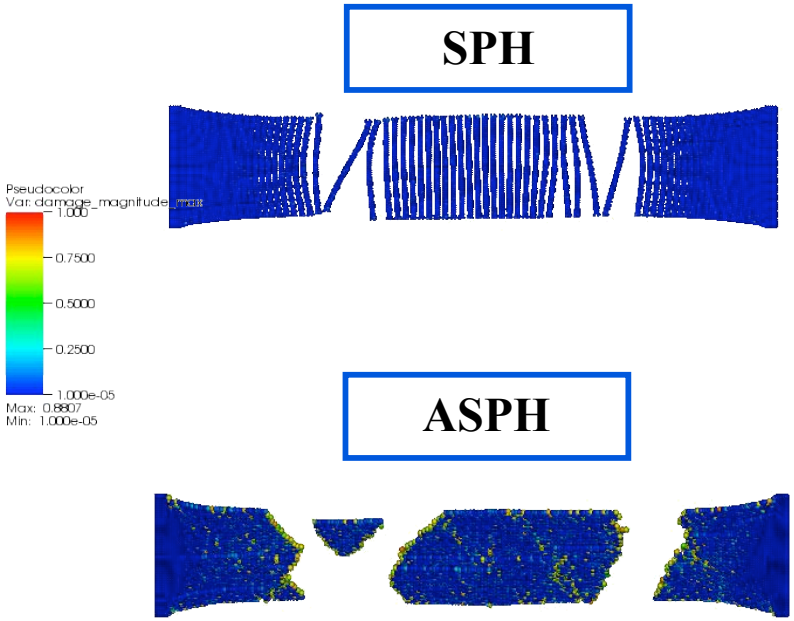


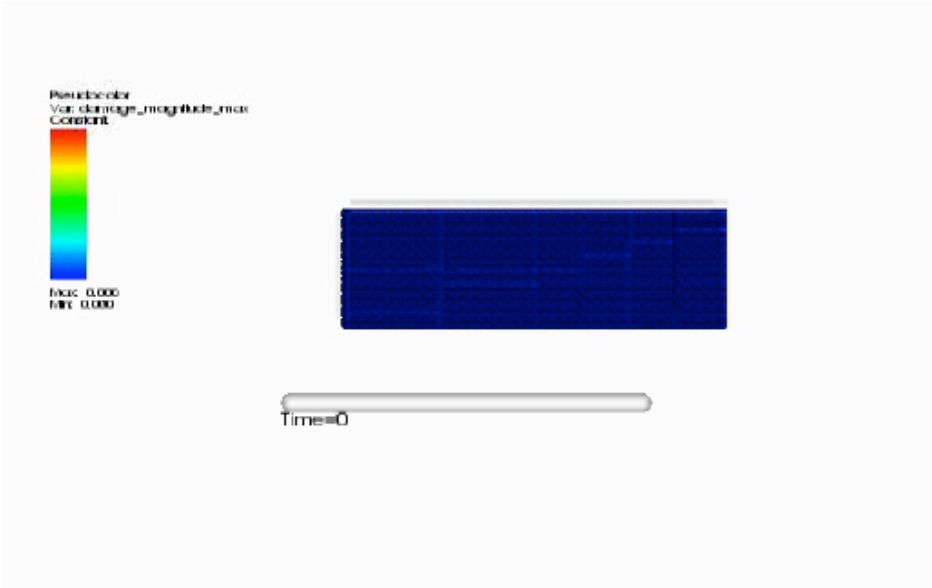
ASPH



2-D Tensile Rod: Damage and Fragmentation

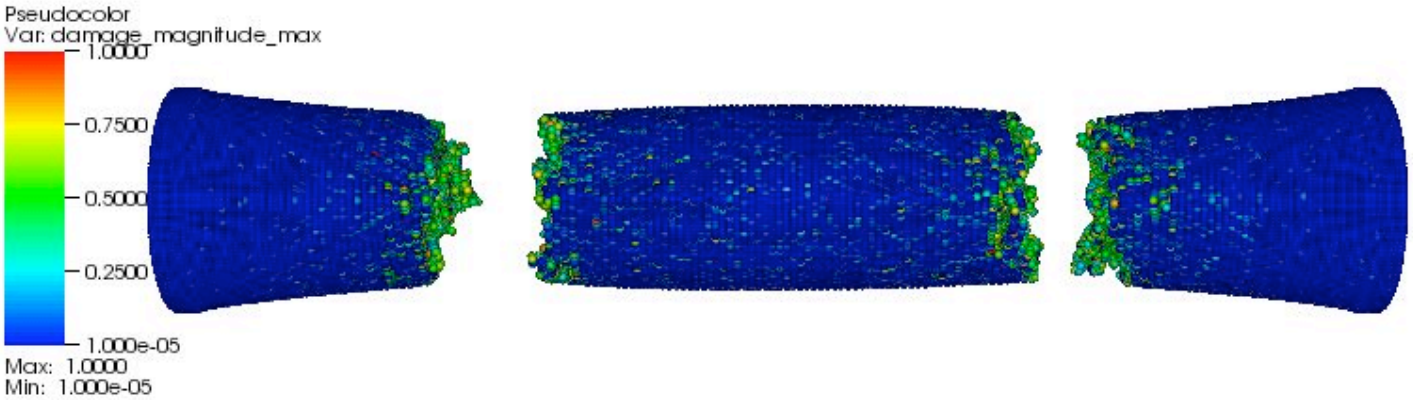
- As we expect, when run with the damage model the numerical fragmentation dominates the SPH calculation.
- ASPH breaks up due to the evolution of the damage model.
- Plotted here is the maximum eigenvalue of the damage.





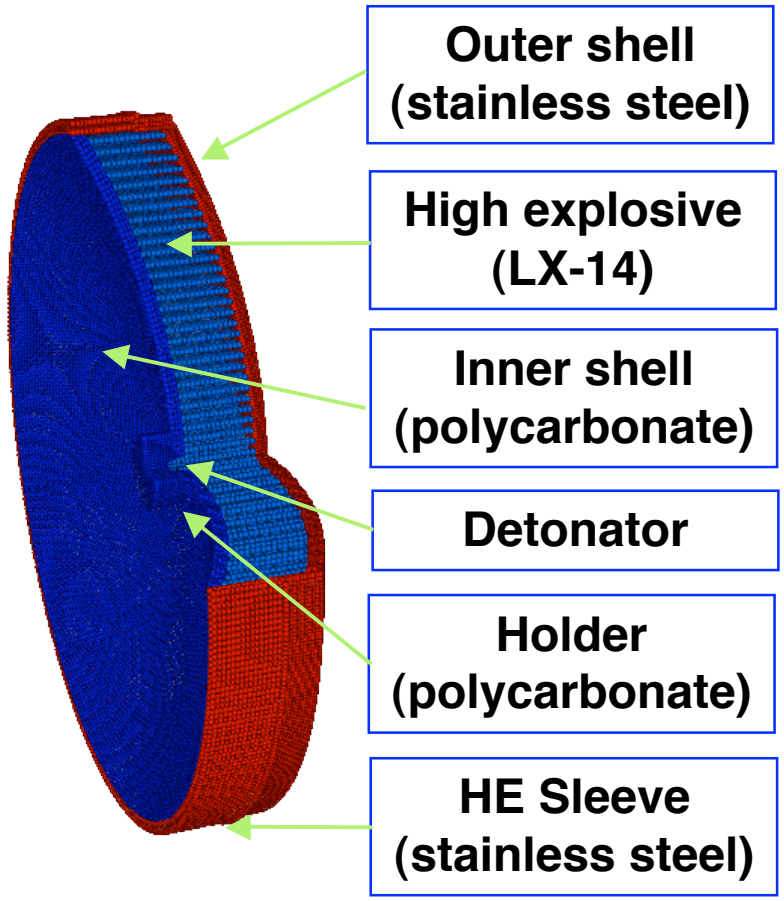
3-D Tensile Rod

- The results we see in 2-D are reflected in 3-D as well.
- Here we show an example of a 3-D cylindrical rod:
  - Initially 3 cm long, 1/2 cm radius.
  - $v_o = 10^4$ cm/sec on ends.
- Plotted here is the maximum eigenvalue of the damage.

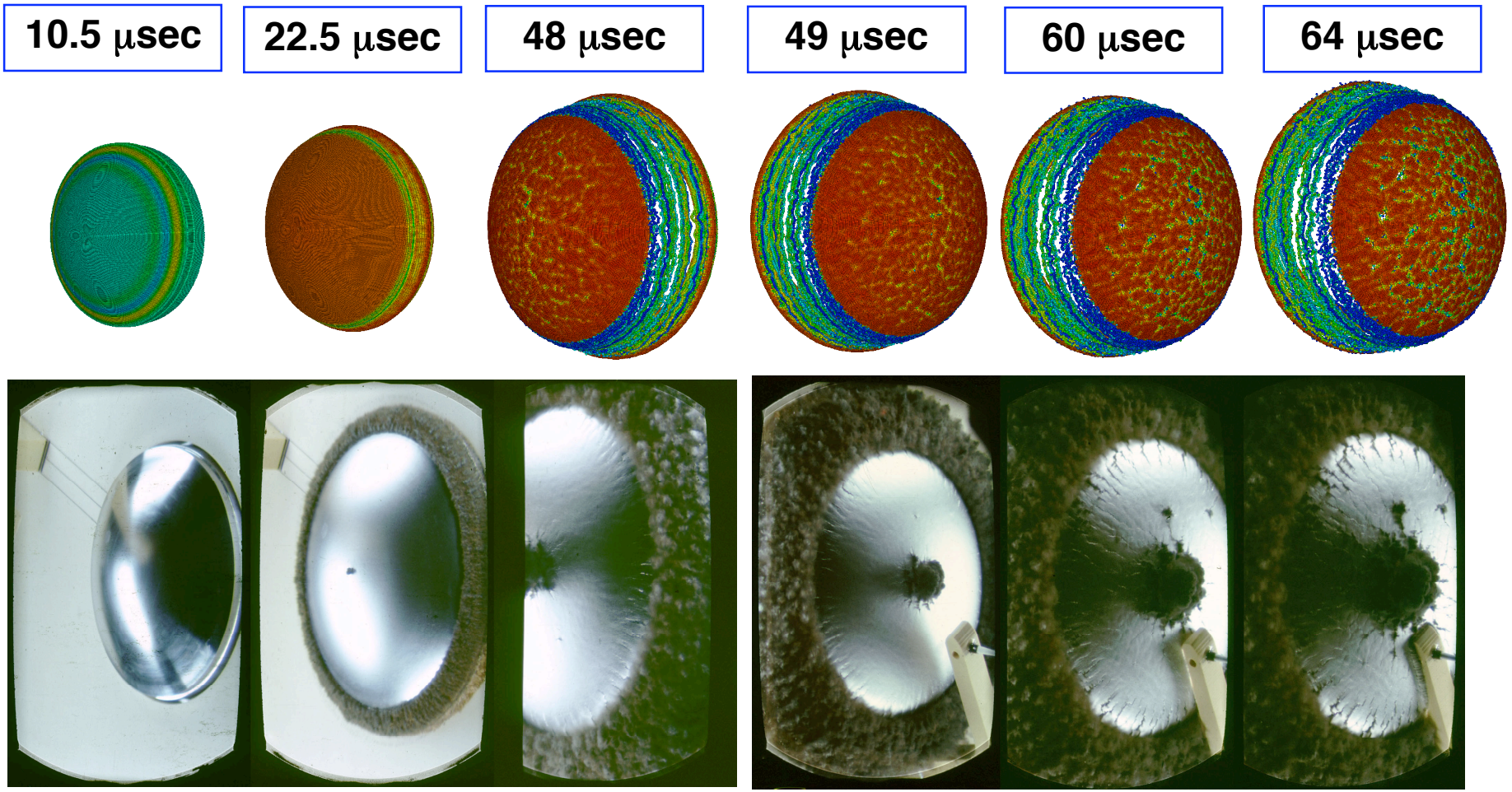


Steel hubcap setup.

- HE driven fragmentation of hemispherical shell of steel.
- This is a model of an experiment that was shot, so we have the data for comparison:
  - Velocimetry
  - Soft capture fragmentation statistics

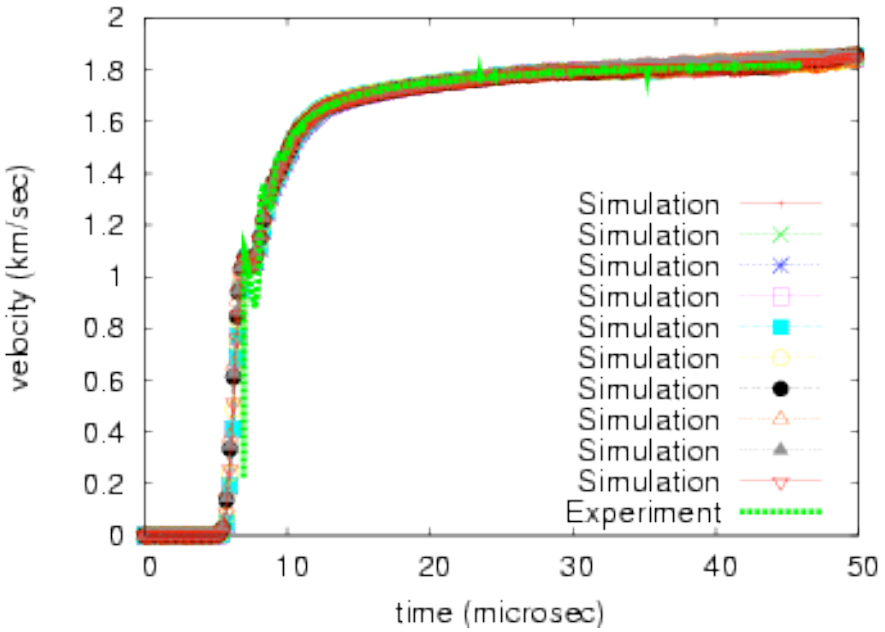


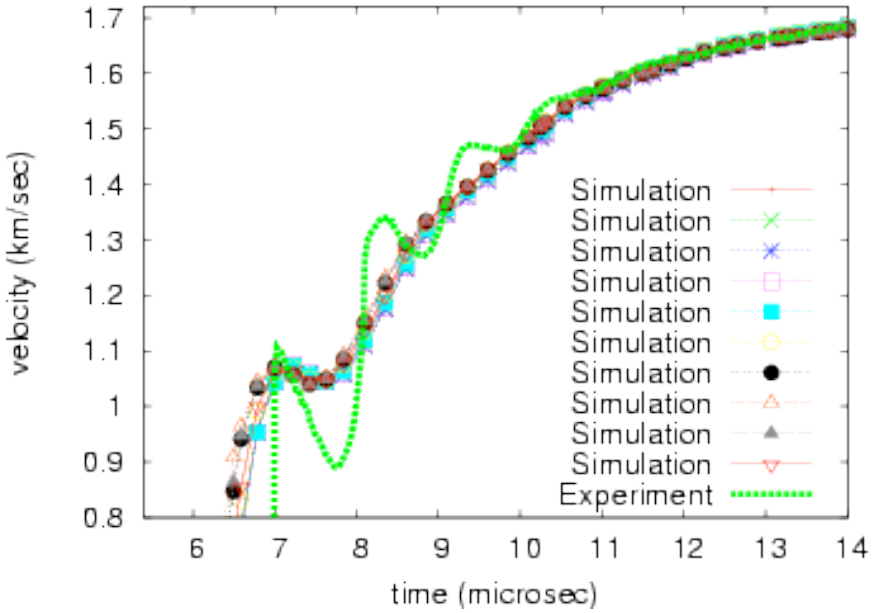
Steel hubcap: experimental photos.



Steel hubcap: velocimetry

- **The experiment included a velocimetry trace of the surface velocity of the outer shell, which we reproduced in the simulation (in 10 bins around the shell).**
- **The bulk motion matches the experiment well, but the pullback of the surface is only roughly captured due to the lack of resolution across the shell thickness (we only have 5 nodes).**

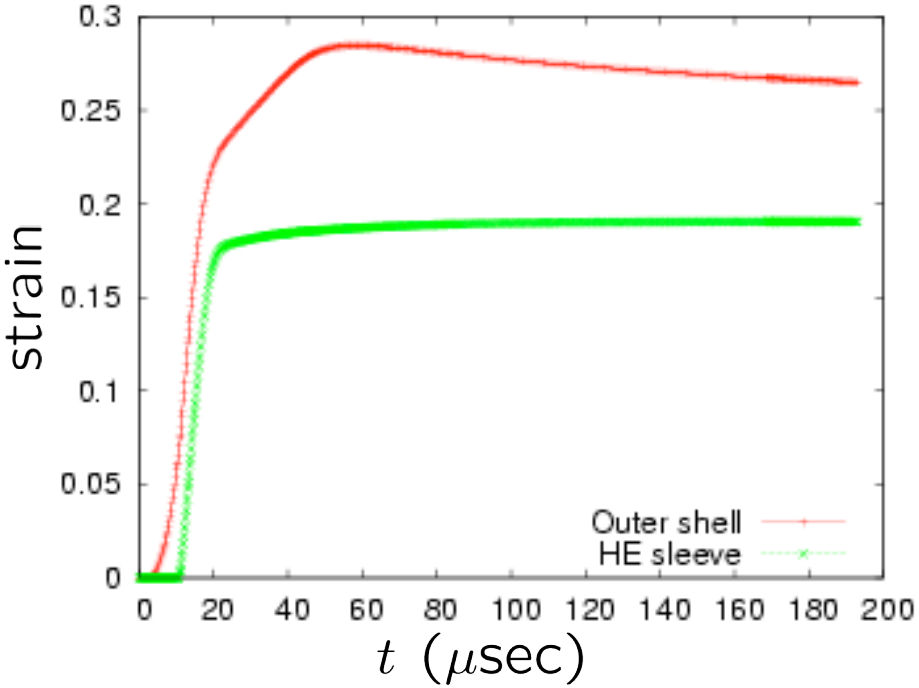




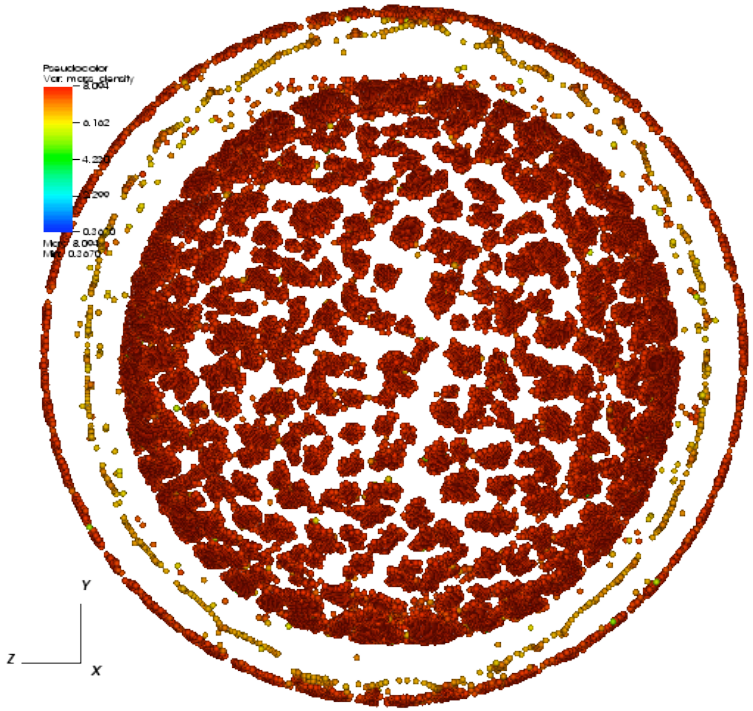
Steel hubcap: strain history and damage

- The HE sleeve and hubcap have different strain rates, and therefore fail at different strains.
- The fragmentation pattern follows the expected uniaxial strain failure on the edges, and biaxial strain failure in the middle.

Strain history



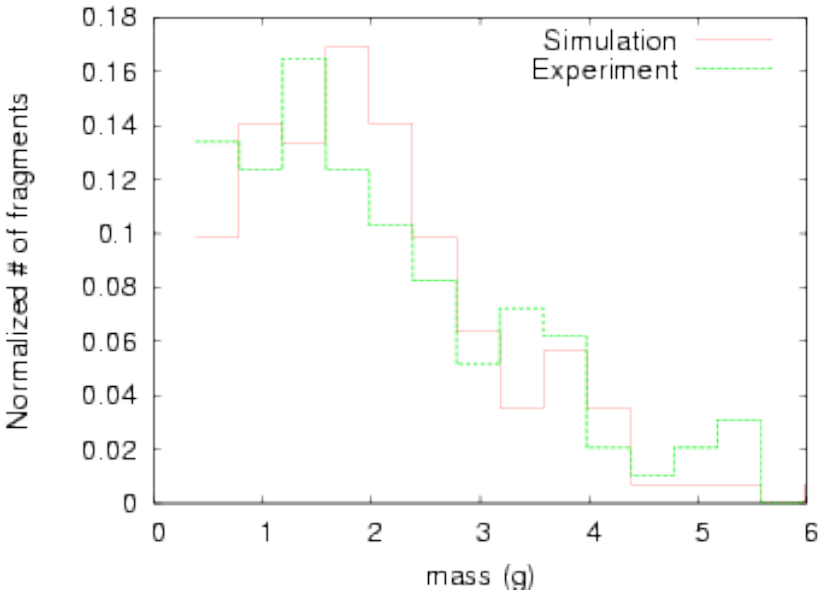
Mass density @ 100 μsec



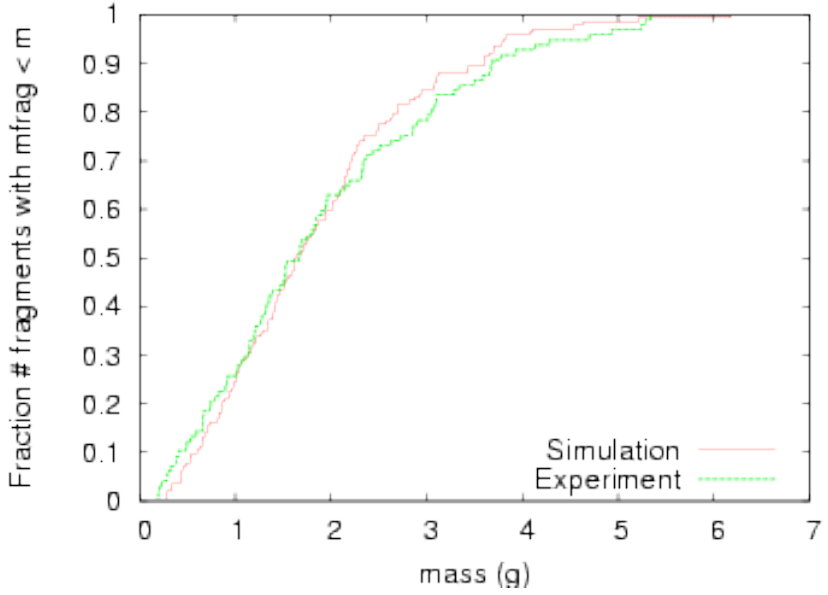
Steel hubcap: fragment properties.

- We employ a simple algorithm to identify distinct fragments in the simulation.
  - Any nodes nodes with damage $D^{trace} < 1$ still linked within a radius $r < 1$ h are considered part of the same fragment.
  - Results @ 193 μsec

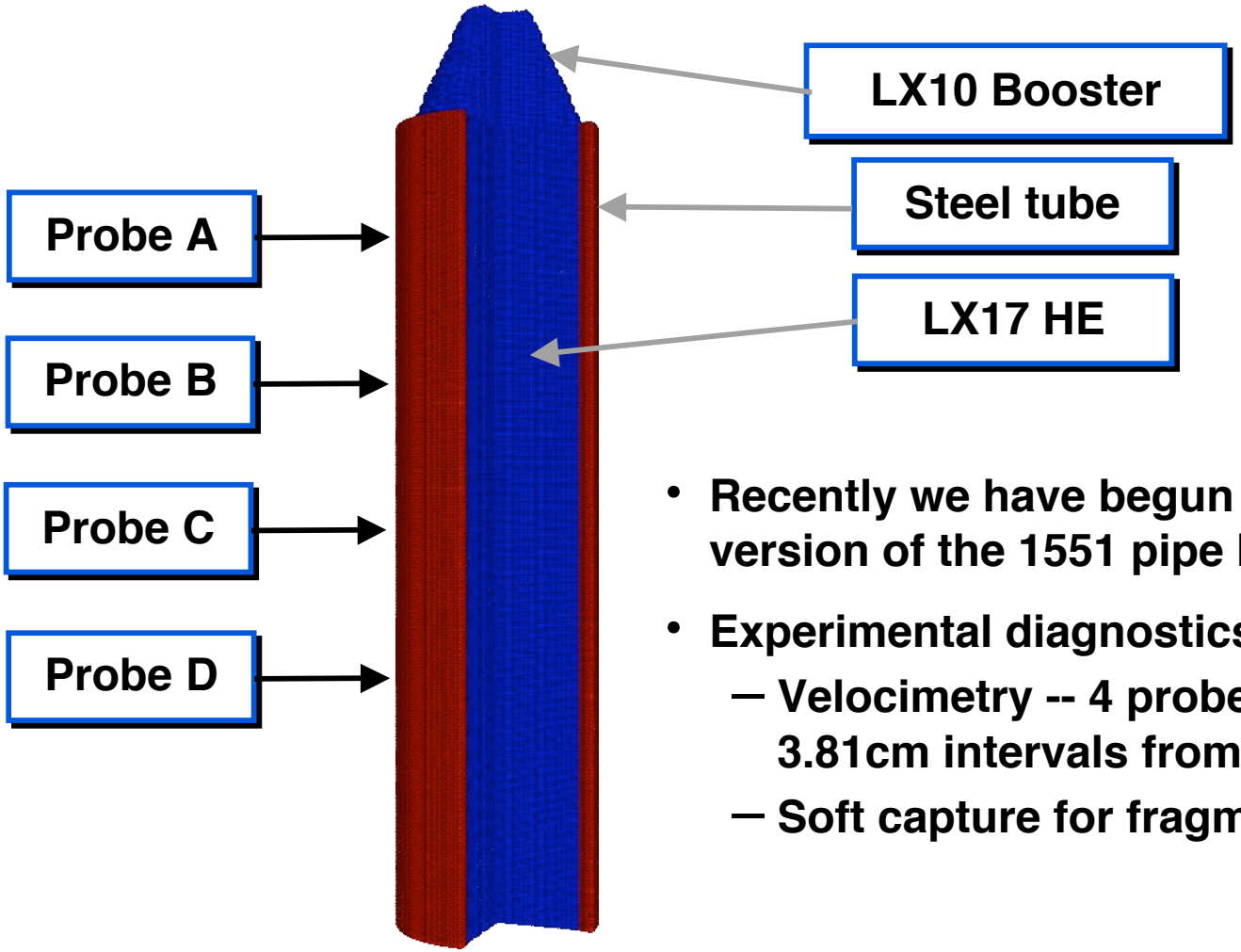
Fragment mass distribution



Cumulative fragment mass distribution



Modeling the 1551 Pipe bomb experiments

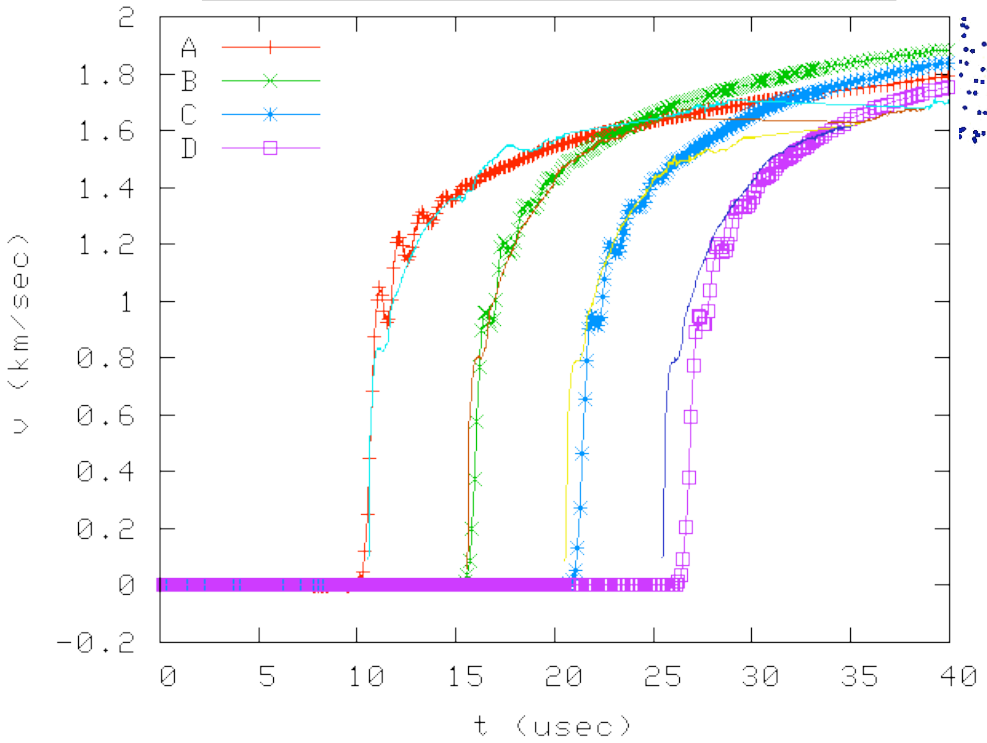


- Recently we have begun modeling the steel version of the 1551 pipe bomb experiments.
- Experimental diagnostics:
  - Velocimetry -- 4 probes, positioned at 3.81cm intervals from top along tube.
  - Soft capture for fragments.

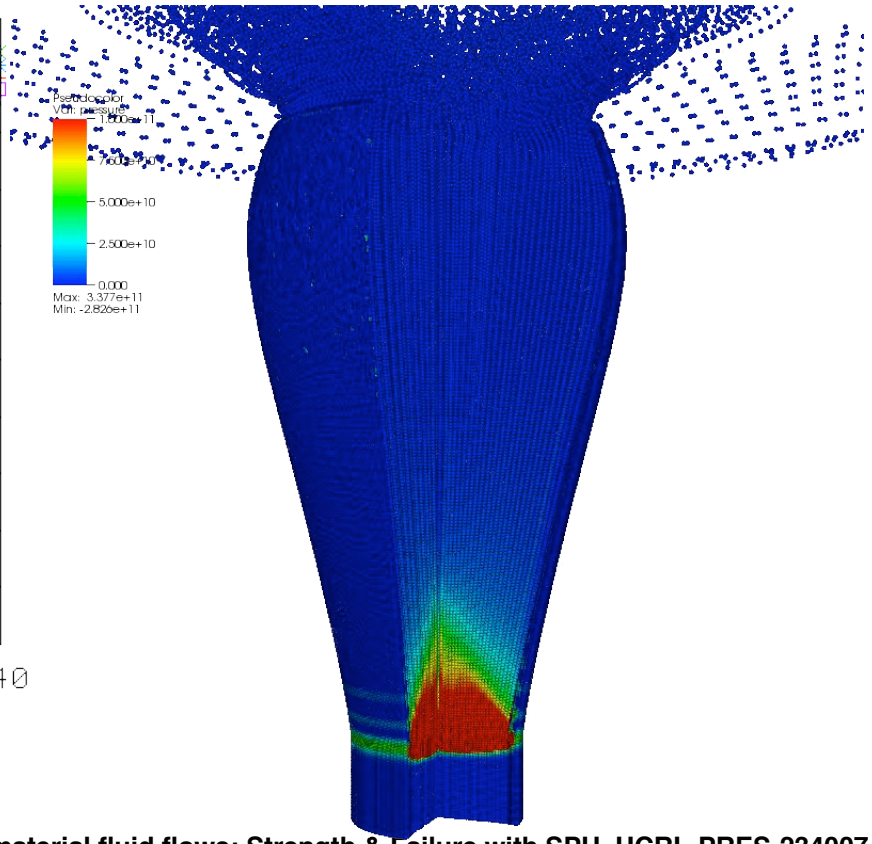
Pipe bomb simulation -- HE burn phase.

- **Dynamics look reasonable, though in this case we actually see more pullback in the simulated steel, and the HE burn velocity is slow.**

Velocimetry



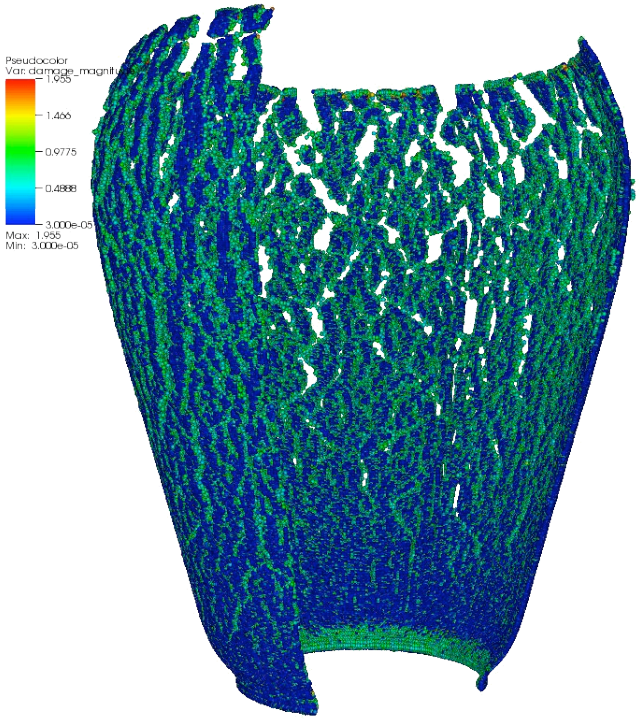
Pressure @ 50μsec



Pipe bomb simulation -- fragmentation.

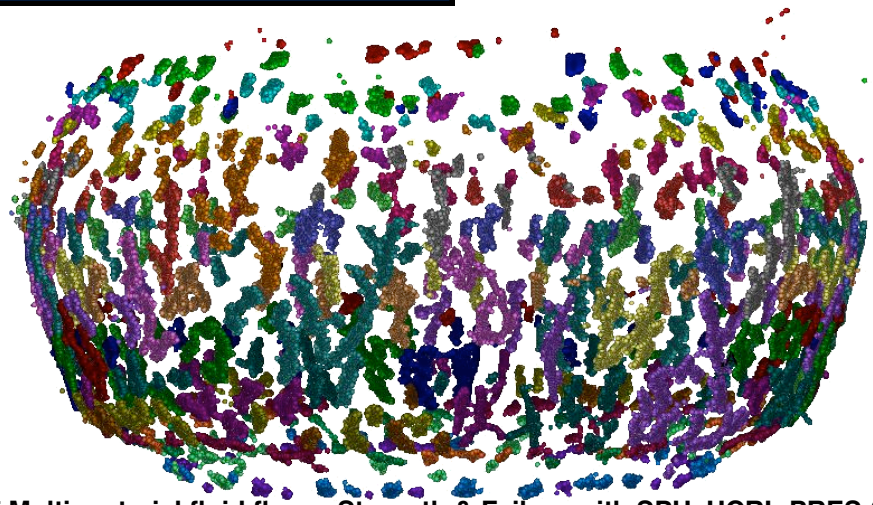
- **Fragmentation characteristics are qualitatively similar to the experiment.**

Damage @ 50 μsec



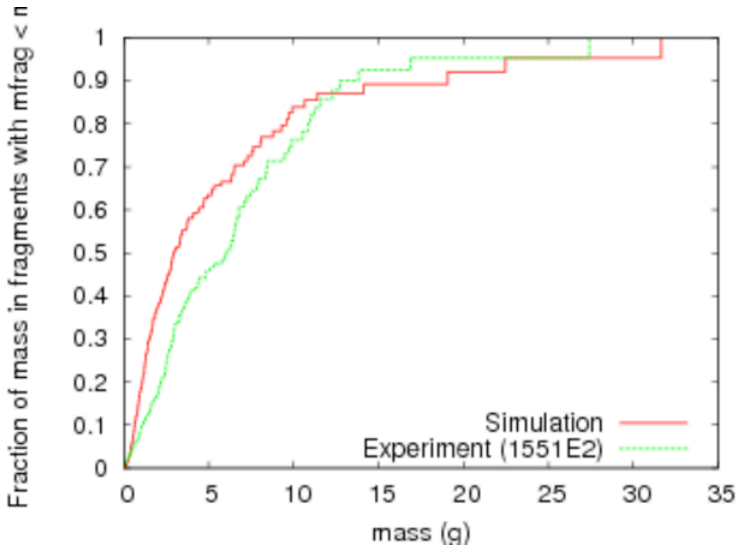
1551A recovered fragments

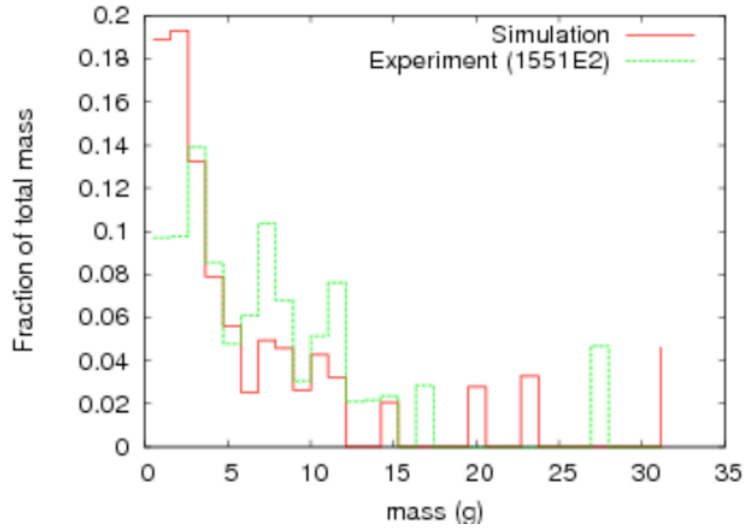
Fragments @ 150 μsec

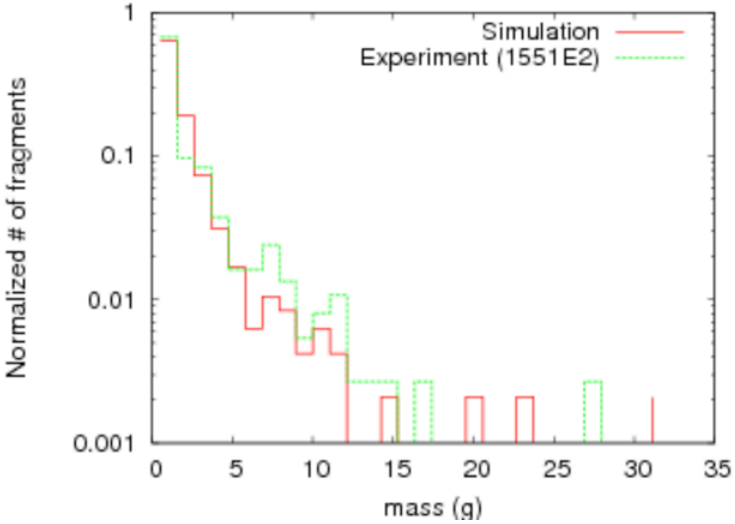


Pipe bomb fragment properties.

- **Quantitative comparison of the fragment mass properties shows we are close, but not an exact match.**
- **All results at 150 μsec.**

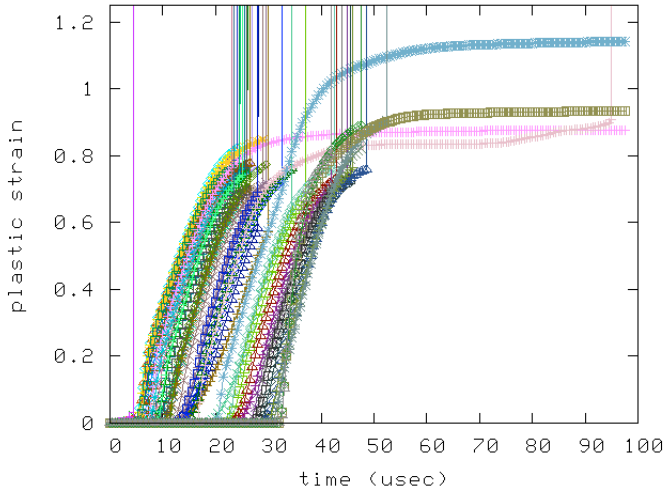


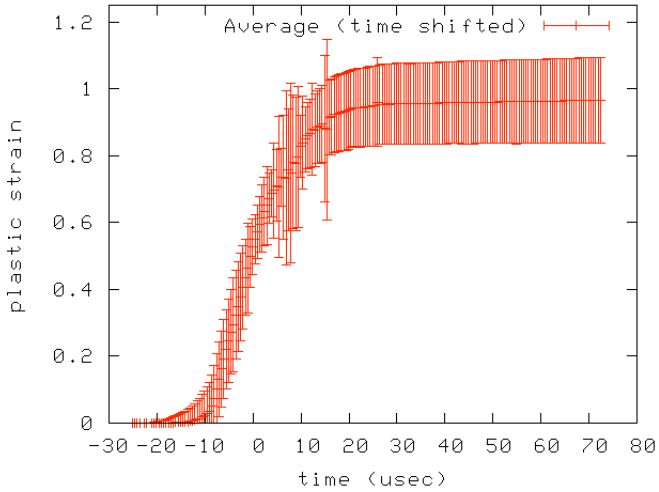




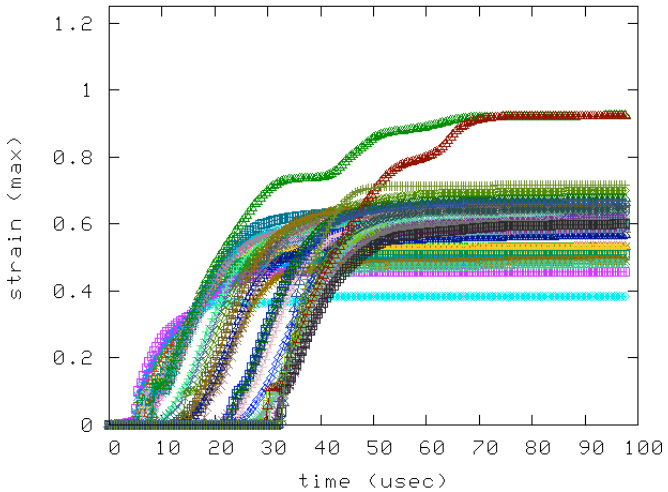
Pipe bomb fragment strain histories.

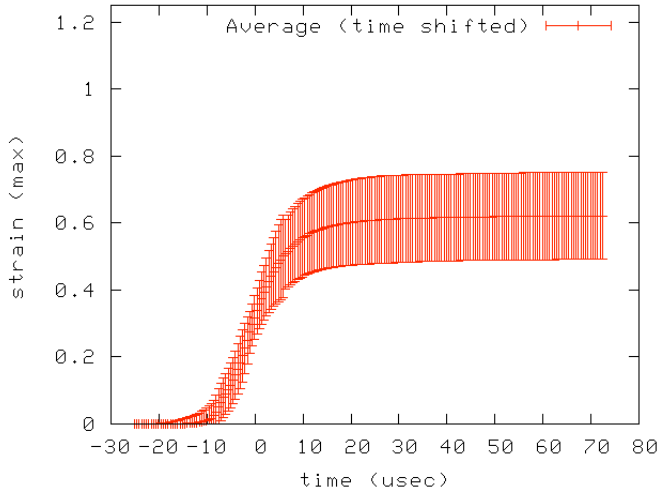
Plastic strains





Strain maximum eigenvalue





Conclusions and Future Work.

- We have found that ASPH is important for realistic modeling of the process of fragmentation.
  - Necessary to model anisotropic distortions in the material.
  - Allows more flexibility in how the initial resolution is distributed in the problem.
- Even using simple damage models seems to yield reasonable results on the experiments we have considered thus far.
  - We are in process of adding a wider variety of damage models (to represent $DD_S(\varepsilon)/Dt$).
    - Johnson-Cook
    - Hancock-McKenzie
    - …
  - We hope this will allow us to make quantitative comparisons of these different models within the same framework.
- We should look at the impact of incorporating some of the correction terms for SPH gradients (CSPH).

SPH evolution of the deviatoric stress.

- **The stress tensor is given by the deviatoric stress and pressure**

$$\sigma^{\alpha\beta} = S^{\alpha\beta} - P\delta^{\alpha\beta}$$

- **The traceless deviatoric stress $S^{\alpha\beta}$ is evolved assuming Hooke's law ($S^{\alpha\beta} \propto$ strain rate).**

$$\frac{DS^{\alpha\beta}}{Dt} = 2\mu\left(\dot{\epsilon}^{\alpha\beta} - \frac{1}{3}\dot{\epsilon}^{\gamma\gamma}\delta^{\alpha\beta}\right) + A^{\alpha\beta}$$

$$\dot{\epsilon}^{\alpha\beta} = \frac{1}{2}\left(\partial_\beta v^\alpha + \partial_\alpha v^\beta\right)$$

- **Where the rotation terms $A^{\alpha\beta}$ are given by**

$$A^{\alpha\beta} = -R^{\alpha\gamma}\sigma^{\gamma\beta} + R^{\alpha\psi}\sigma^{\psi\beta}$$

$$R^{\alpha\beta} = \frac{1}{2}\left(\partial_\beta v^\alpha - \partial_\alpha v^\beta\right)$$

- **All based on the SPH estimate of the velocity gradient**

$$\partial_\beta v^\alpha = -\rho_i^{-1}\sum_j m_j\left(v_i^\alpha - v_j^\alpha\right)\partial_\beta W_{ij}$$

Plastic yielding of the deviatoric stress.

- To account for the plastic behavior of materials pushed beyond their yield stress, we apply the von Mises yielding criterion as a simple limit on the deviatoric stress:

$$S_i^{\alpha\beta} \rightarrow f_i S_i^{\alpha\beta}$$

$$f_i = \min\left(\frac{Y_i}{\sqrt{3J_2}}, 1\right), J_2 = \frac{1}{2} S_i^{\alpha\beta} S_i^{\alpha\beta}$$

- And the equivalent plastic strain is updated accordingly

$$\Delta\dot{\epsilon}_p = \frac{1 - f_i}{3\mu}\sqrt{3J_2}$$

- To close the equations, we use either the Gruneisen or Linear polynomial equations of state to estimate ***P*** & ***K***, and the Steinberg-Guinan rate independent strength model to to estimate ***Y*** & μ.