Ale with Mixed Elements

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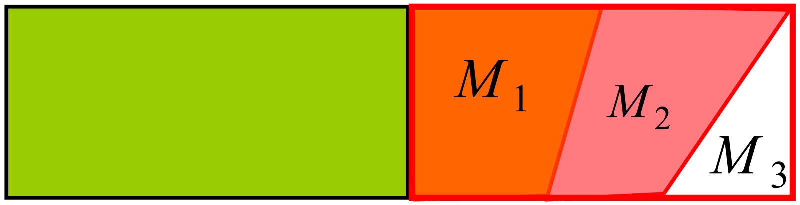
Contents

Introduction

- Because hydrocodes with lagrangian scheme lack of robustness specially on thin meshes, an ALE formulation with multi-material elements has been implemented to perform simulations with small sizes cells :
  - We introduce ALE blocks with several materials in the same mesh (mixed and pure cells) which can interact with each other or with pure Lagrangian by slide lines.
  - In a same simulation, we can track accurately material boundaries (sliding, void opening and closure) and high material deformation.

Ω_1^L **= Lagrangian** Ω_2^A **= Ale block with M1/M2/M3 materials**



Governing equations

Notations

u(x, t) : material velocity
v(x, t) : mesh velocity

Time derivative :

Lagrangian : $\frac{d}{dt} = \frac{\partial}{\partial t} + u \cdot \nabla$

Ale Reference system : $\frac{D}{Dt} = \frac{\partial}{\partial t} + v \cdot \nabla$

$$\begin{cases} \frac{D\rho}{Dt} + \rho \nabla . u = -(u - v) \cdot \nabla \rho \\ \frac{D\rho u}{Dt} + \nabla p = -(u - v) \cdot \nabla \rho u \\ \frac{D\rho e}{Dt} + p \nabla . u = -(u - v) \cdot \nabla \rho e \end{cases}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} - (u - v) \cdot \nabla$$

In Practice : Splitting method

Lagrangian phase : u = v

$$\begin{cases} \rho V = Cste \\ \rho \frac{du}{dt} = -\nabla p \\ \frac{de}{dt} + p \frac{dV}{dt} = 0 \\ p = \mathrm{EOS}(\mathrm{e}, \rho) \end{cases}$$

Advection phase :

1 – Mesh smoothing under criteria
2 – Remapping : nodal or cell values (ρ, ρu, ρe) from the old mesh onto the new mesh

Splitting method

- **Lagrangian step : with multi-material cells**
  - Use the classical Wilkins second order scheme
  - Special treatment in multi-material cells :
    - ⇨ Assumption of equal material volumetric strain for all materials
    - ⇨ Interface reconstruction

- **Advection step :**
  - Mesh smoothing under criteria : a new grid is defined.
  - Remapping phase : the material quantities are interpolated from the old configuration onto the new one.
    - ⇨ Material by material, after material mesh reconstruction
    - ⇨ Dukowicz method (intersection old mesh / new mesh), second order accuracy

Lagrangian step with multi-material cells

- **Treatment of the multi-material cell :** Note Volume fraction $f_\alpha = \frac{V_\alpha}{V}$
  - Assumption : equal volumetric strain for all materials

    Only one velocity for all material : $u = u_\alpha$

    → imply the relation closure for volume fraction in a cell :

$$\frac{dV_\alpha}{dt} = V_\alpha \nabla \cdot u_\alpha = V_\alpha \nabla \cdot u = f_\alpha V \, \nabla \cdot u = f_\alpha \frac{dV}{dt}$$

    → equal material volumetric strain rate for all materials $\frac{df_\alpha}{dt} = 0$

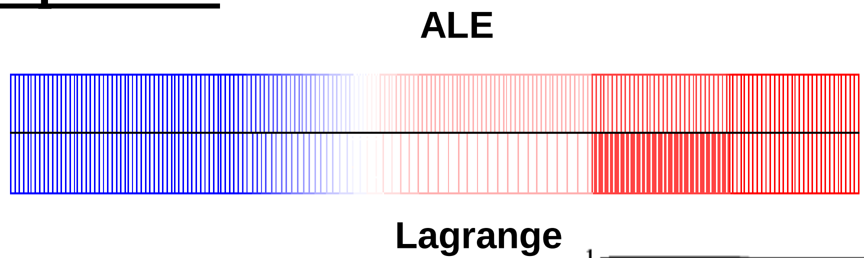
    → And so with the internal energy balance equation, we have the average pressure definition $p = \sum_\alpha p_\alpha f_\alpha$

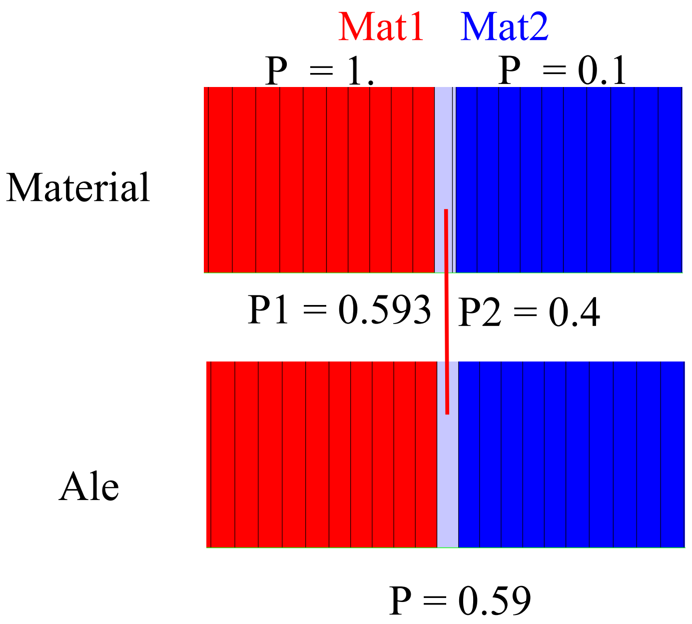
- **More robust multi-material cell treatment : Relaxation pressure**

we perform an iterative method after Lagrangian step (without volume variation) to balance material pressure in a cell.

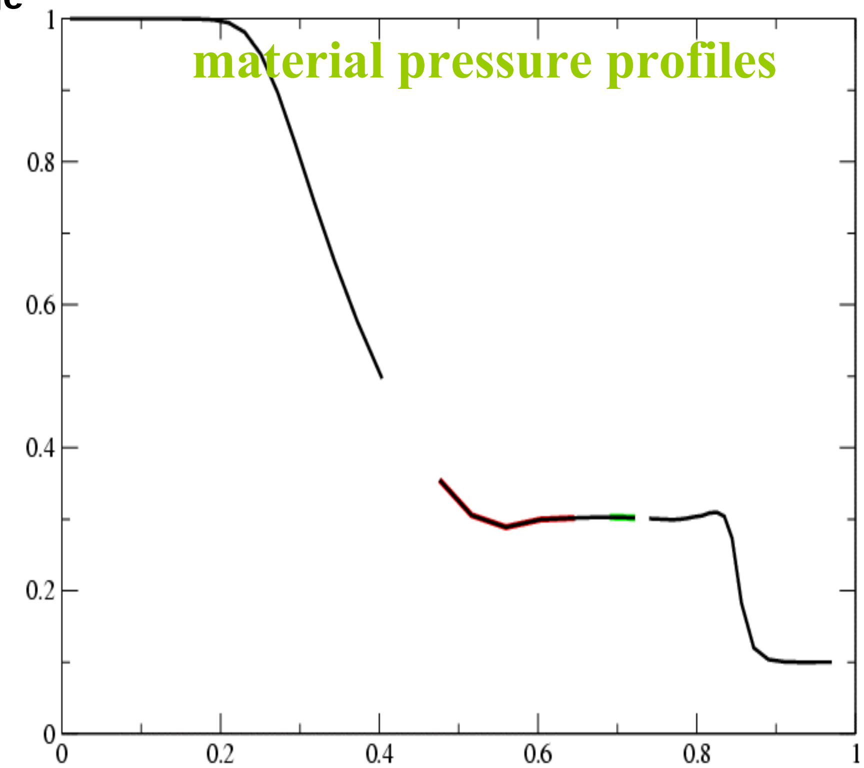
Lagrangian step with multi-material cells

Sod shock tube problem





t = 0. s



t = 0.2 s

Lagrangian step with multi-material cells

- **Elastic-plastic treatment**
  - For each elastic material α, characterized by mechanical characteristics (μ_α, Y_α), we compute the deviatoric stress tensor:

Same deformation Assumption

Strain rate $\overline{\overline{D}}_\alpha^{n+1/2} = \overline{\overline{D}}^{n+1/2}$; $D_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$

Spin tensor $\overline{\overline{W}}_\alpha^{n+1/2} = \overline{\overline{W}}^{n+1/2}$; $W_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$

$$\begin{cases} \overline{\overline{S}}_{el\,\alpha}^{n+1} = \overline{\overline{S}}_\alpha^{n} + \overline{\overline{R}}_\alpha^{n} + 2\mu_\alpha \cdot \left[\overline{\overline{D}}^{n+1/2} - \frac{1}{3} \cdot tr(\overline{\overline{D}}^{n+1/2}) \cdot \overline{\overline{I}}\right] \\ \overline{\overline{S}}_\alpha^{n+1} = f(Y_\alpha)\, \overline{\overline{S}}_{el\,\alpha}^{n+1} \end{cases}$$

  - On the Ale Block : For the momentum conservative equation

Deviatoric stress tensor

Stress tensor

$$\begin{cases} \overline{\overline{S}}^{n+1} = \sum_\alpha f_\alpha \cdot \overline{\overline{S}}_\alpha^{n+1} \\ \overline{\overline{\sigma}}^{n+1} = -P \cdot \overline{\overline{I}} + \overline{\overline{S}}^{n+1} \end{cases}$$

Average quantities

The average stress is the average of the individual material stresses weighted by the volume fraction.

Advection step : **Mesh smoothing under criteria**

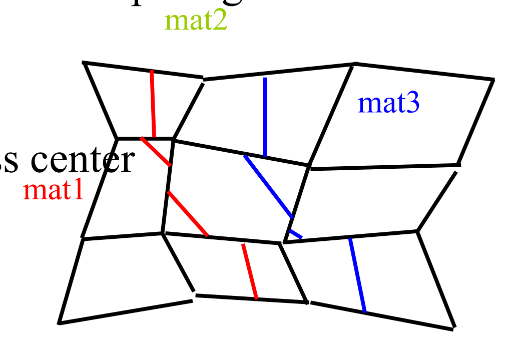
- Object : improve the mesh quality at the end of the lagrangian step, by finding some ideal position for the nodes, ideal in the sense of minimizing element distortions.
- We use zones definitions which allow user to specify and to adjust locally the mesh smoothing.
  - We define nodal remapping criteria to identify the nodes which need to be relaxed :
    - For boundary nodes : distance criterion
    - For internal nodes : volume criterion, angular criterion …
  - The boundary nodes are moved by specific algorithms (specially for the sliding material boundaries) ;
    - To apply an equidistant node distribution along the boundary,
    - To preserve the shape of the boundary.
  - The new position of internal nodes is calculated from classical iterative mesh smoothing algorithms, with a possibility of remapping constraint :
    - Winslow's equipotential relaxation method
    - Simple average method

Multi-material cell advection step

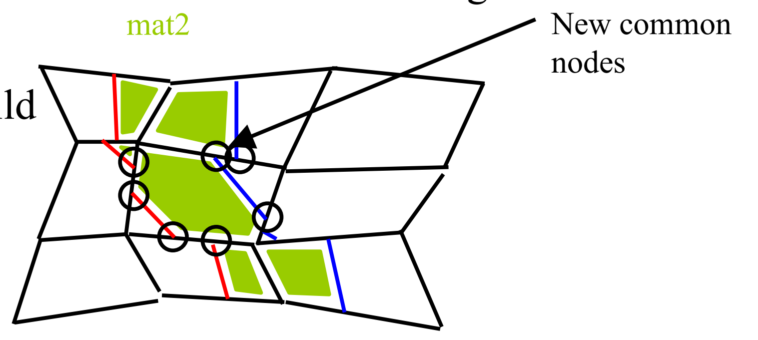
- Remapping phase :
  - Interface Material Reconstruction in mixed cells
    - Multi-material cells are characterized by their volume fractions.
    - To track the material interface, we use the Young's method : the slope is given by the gradient of the volume fractions field :

$$\vec{N} = -\vec{\nabla} f_{\beta} \text{ where } \beta \text{ realizes } \max_{\alpha=1,..\text{nbmat}} \left\| \vec{\nabla} f_{\alpha} \right\|$$

    - Materials are ordered in a cell by locating each material mass center along the normal direction.
    - Only **one** normal for all interfaces : onion skin method

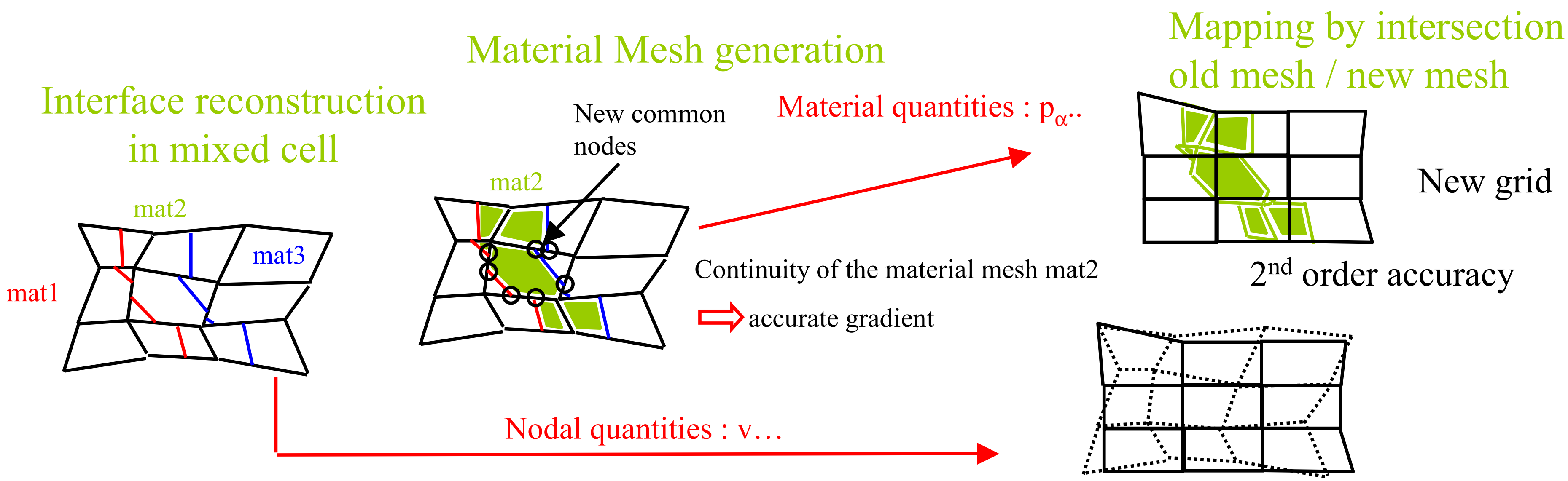


  - Material mesh reconstruction
    - From the old grid, we build nodal-cell connectivity for each material :
      - ⇨We create the nodes between the material interface and the mesh edge.
      - ⇨The common nodes are merged.
      - ⇨The new material lagrangian mesh is build with non structured connectivity



Multi-material cell advection step

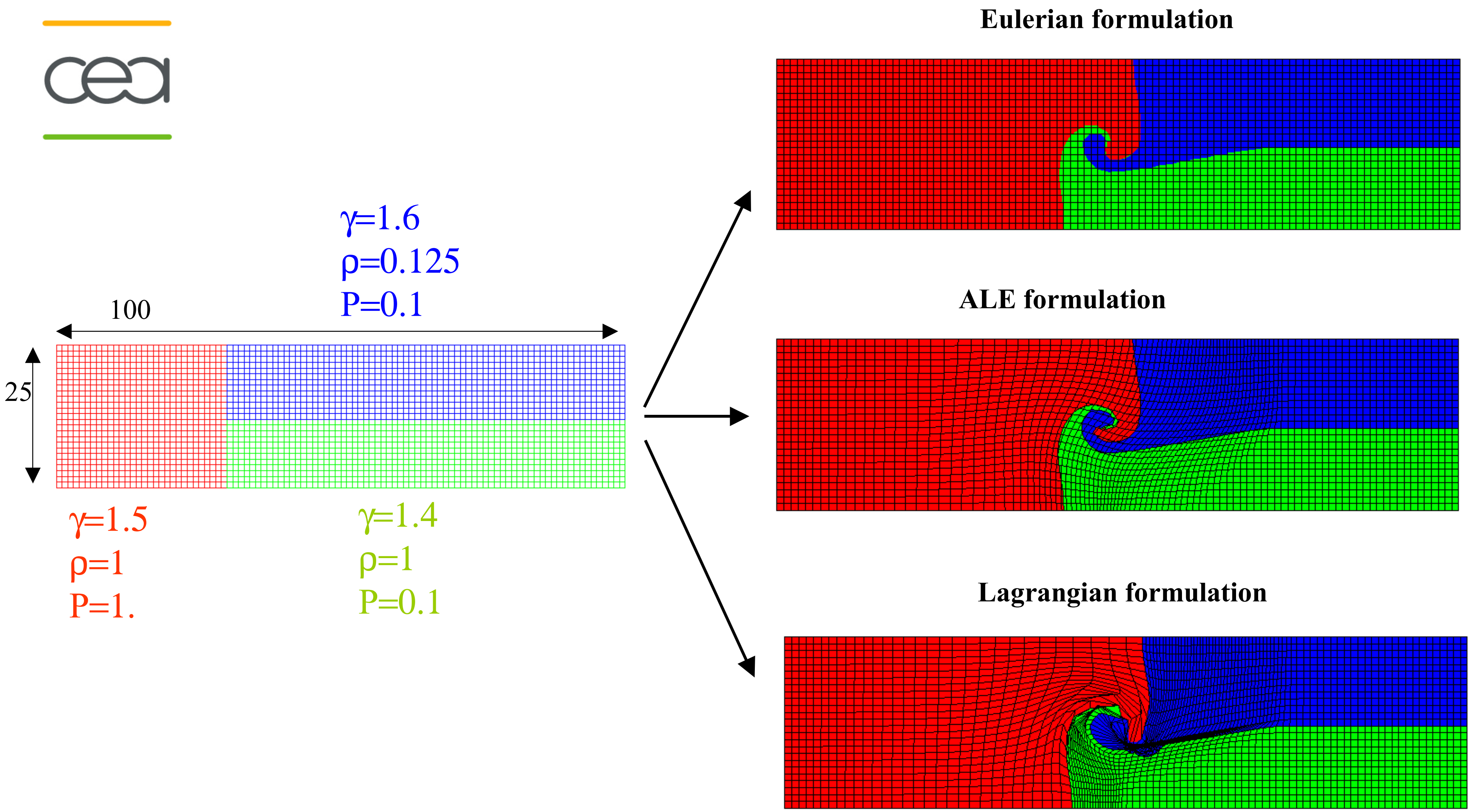
- **Remapping step :** we perform the second order Dukowicz method (intersection old mesh / new mesh) :
  - Interfaces Reconstruction
  - For each material, we
    - generate an old material mesh,
    - map material cell quantities from this old material mesh onto the new grid.
  - Nodal quantities velocities are mapped from the old dual grid (global mesh) onto the new dual one.



Some Results

- Instability with 3 materials :
- Taylor bar impact in both Lagrangian and ALE formulation
- Calculation of a dynamic friction experiment

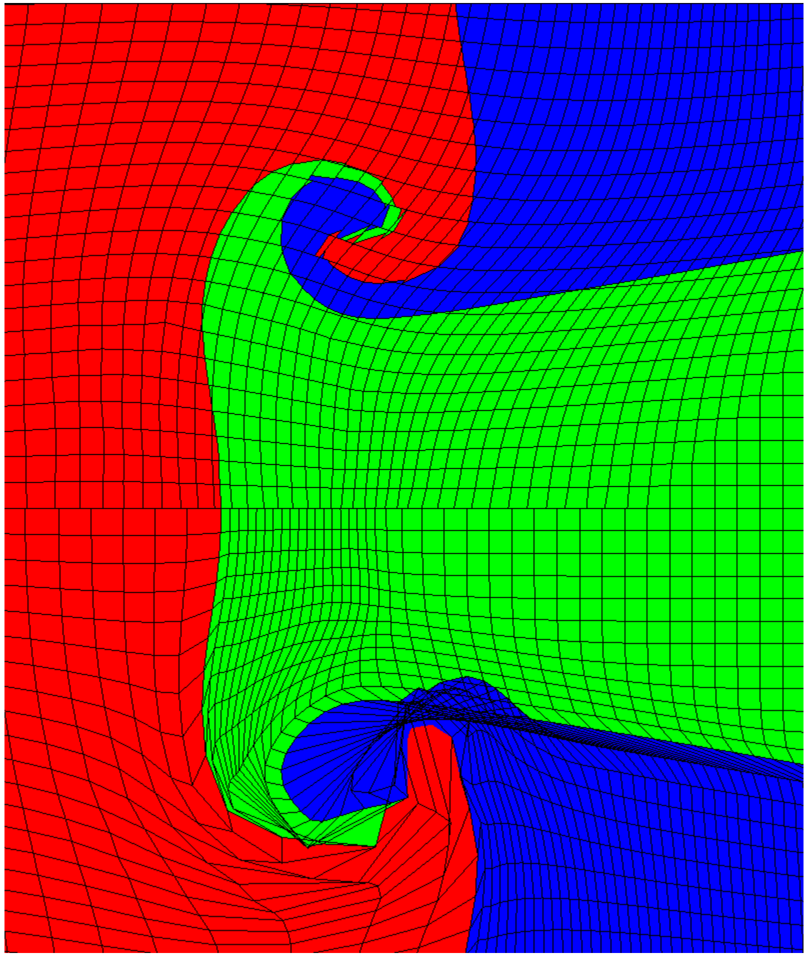
Instability with 3 Materials



Instability with 3 materials

Ale

Lagrangian



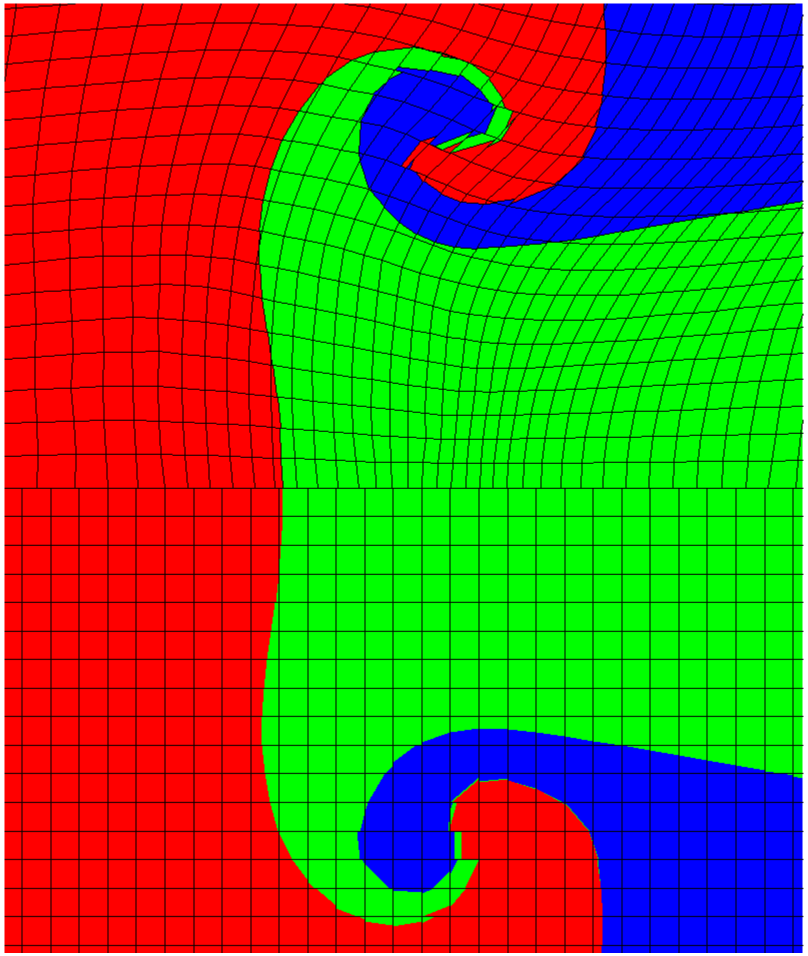
Lagrangian Interface very distorted.

The calculation stops.

Instability with 3 materials

Ale

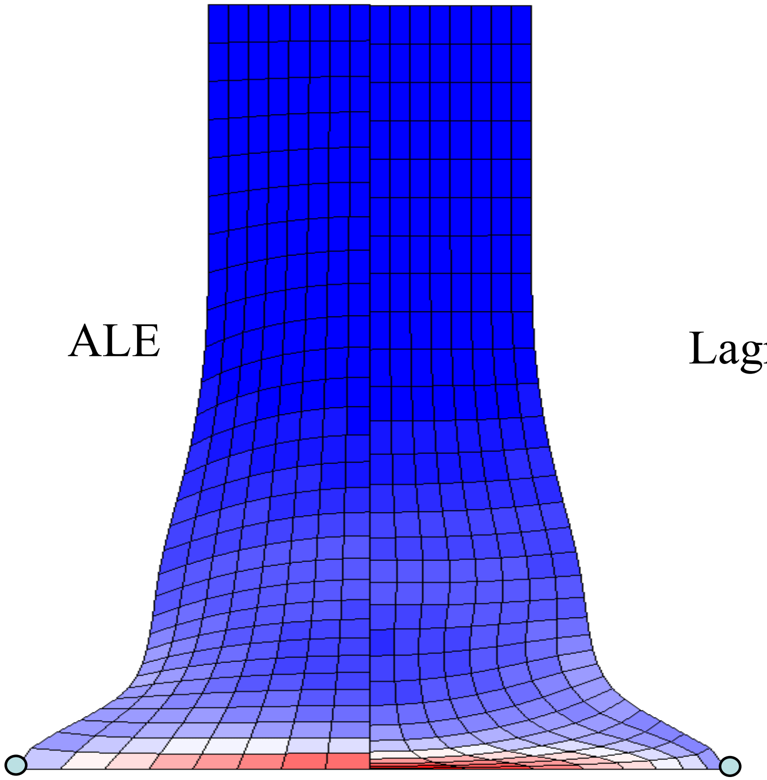
Eulerian

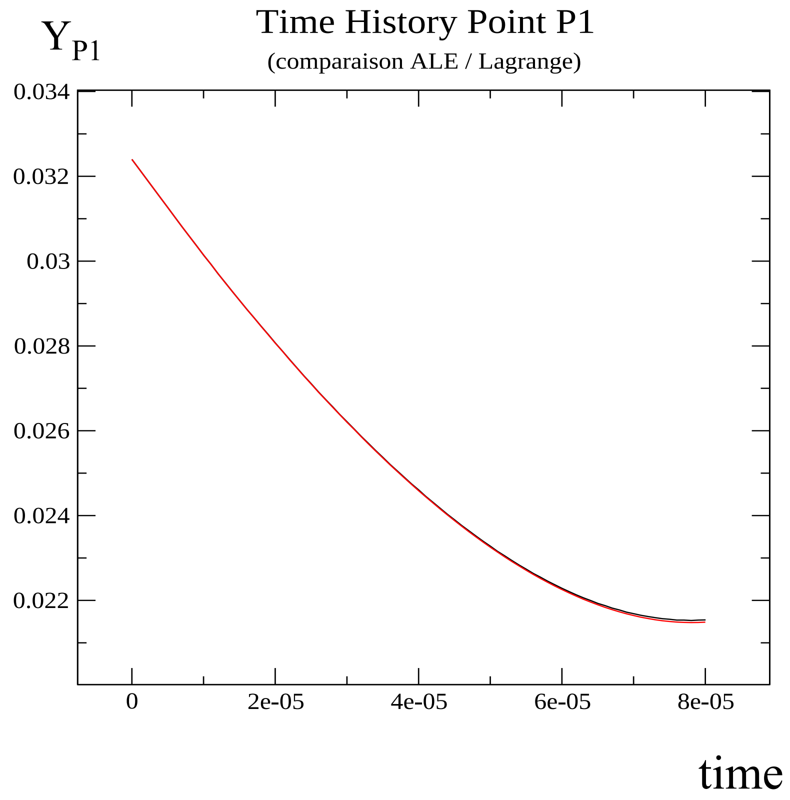


Same robustness of eulerian approach

ALE Taylor bar impact

Plastic strain



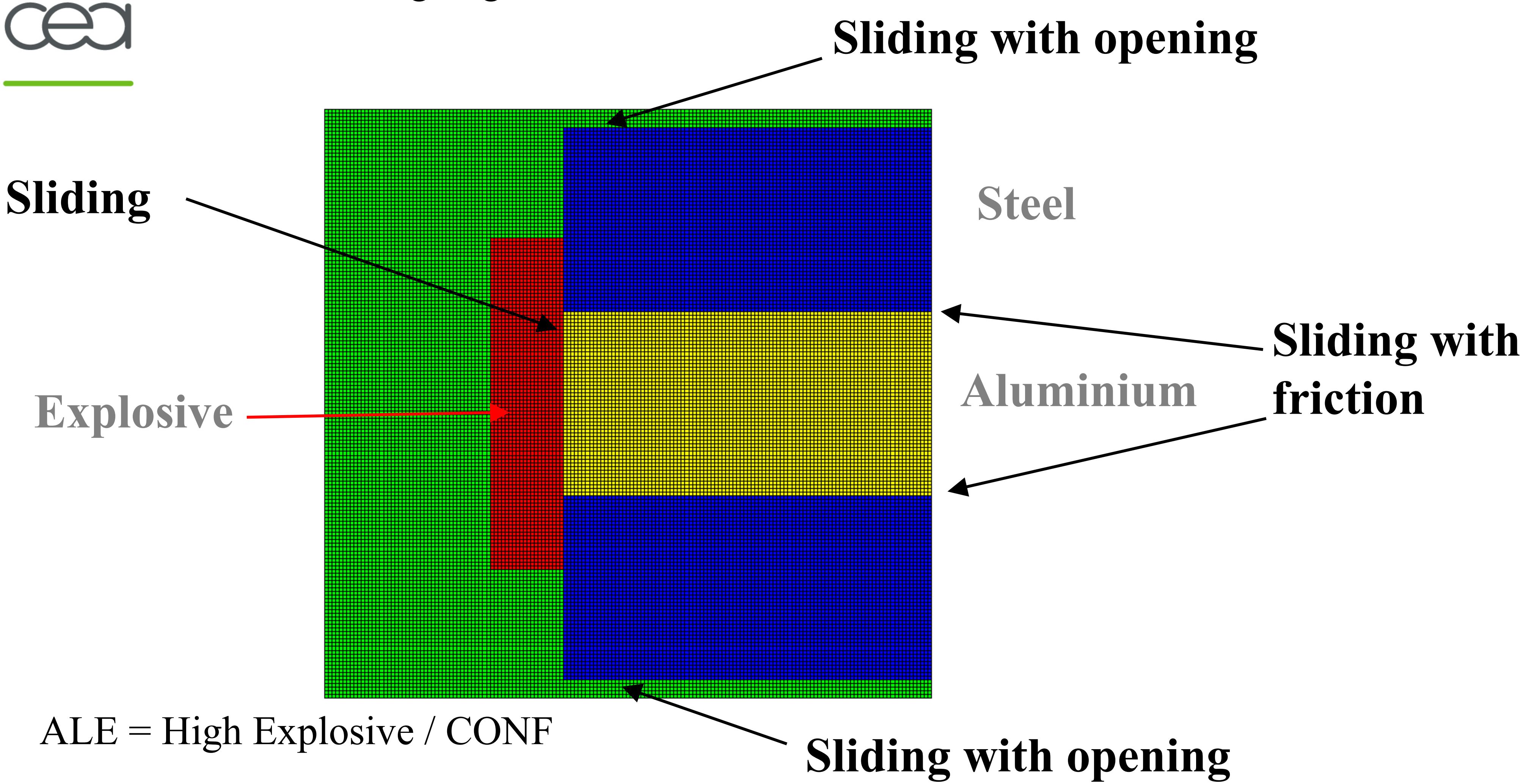


Good deformation

Elements severely distorted ➔ Loss of accuracy and reducing the time step.
(unphysical mesh distortion and hourglassing)

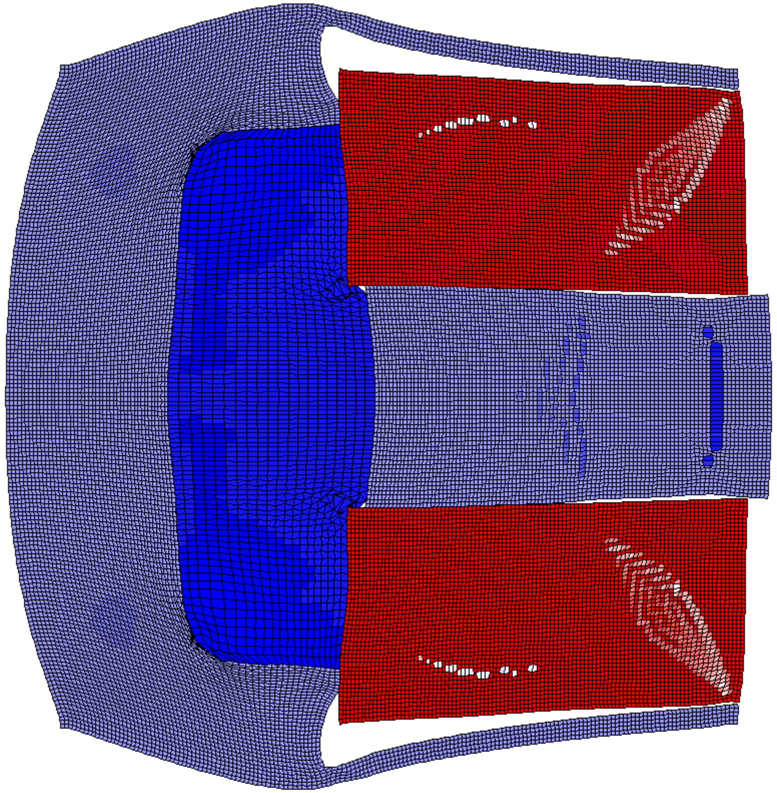
Dynamic friction experiment

Ale and Lagrangian mesh motion in the same framework

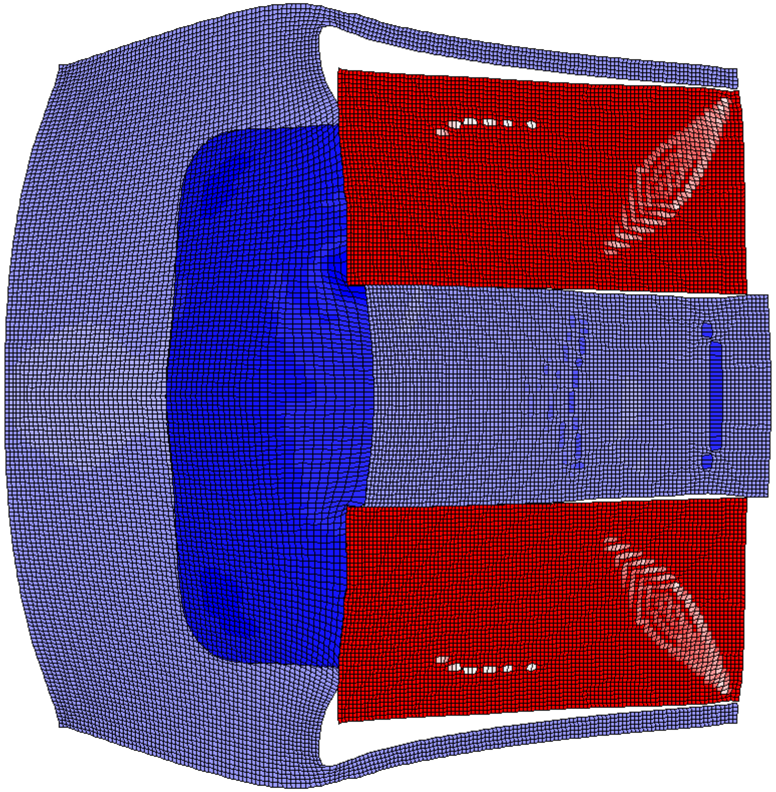


ALE = High Explosive / CONF

Comparison at 30 μs



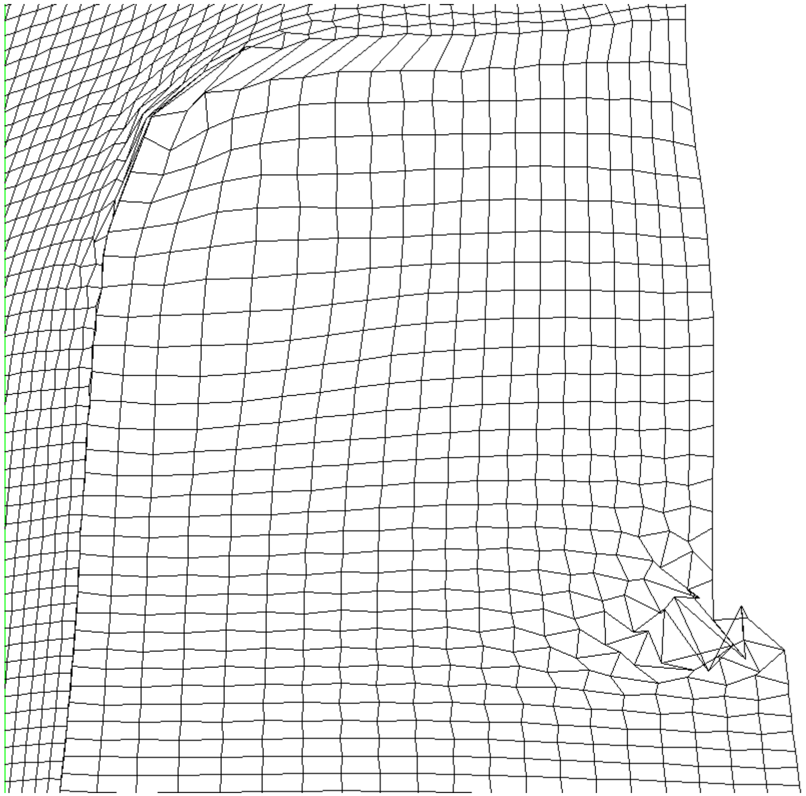
Pure Lagrangian



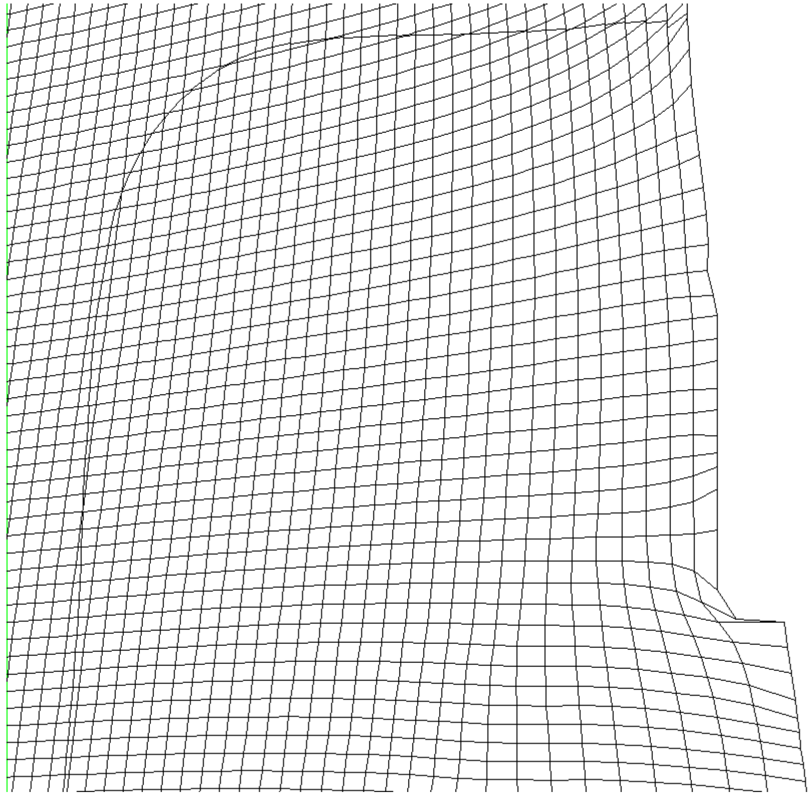
Lagrangian and ALE

➔Good agreement with the Lagrangian deformation

Zoom



Lagrangian materials



ALE block

➔ALE reduces mesh distortion in the mixed cells but the Ale boundary remains sharp.

Conclusion

- **Ale method with mixed cells :**
  - Lagrangian step with treatment of the multi-material cells (equal volumetric strain assumption + relaxation pressure)
  - Full 2D advection step with interface reconstruction in multi-material cells
- **2 different treatments for materials interfaces :**
  - Lagrangian interfaces with slidelines
  - Multi materials cells.

⇨ Provides the accuracy of Lagrangian formulation and the robustness of the Eulerian Multi materials cells

- **Future works :**
  - Extension to the 3D