

Applications of ALE Method to Laser Plasma Studies

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Overview

- **motivation example for Lagrangian formulation**
- **hydrodynamical model with heat conductivity and laser absorption**
- **numerical methods used in our PALE (Prague ALE) code**
 - **hyperbolic part – Arbitrary Lagrangian Eulerian (ALE) method**
 - **parabolic part – heat conductivity**
 - **laser absorption – source term in internal energy equation**
- **laser plasma application, which cannot be treated by pure Lagrangian method**
 - **high velocity impact problem**
 - **double foil target**
 - **foam target**

Motivation for Lagrangian Formulation

- laser plasma is created by laser interaction with targets
- target is $0.8\mu m$ thin Aluminum foil; Prague Asterix Laser System (PALS) laser at 3-rd harmonics, pulse duration $250ps$, focus $40\mu m$, energy $200J$; [animation](#)
- computational mesh is fixed to the fluid and moves with the fluid
- no mass flux between cells through edges
- computation domain changes with time
- problems with large changes of computational domain volume and/or shape (compression or [expansion](#))
- naturally treated moving boundaries
- typically used in laser plasma simulations

Euler Equations in Lagrangian Coordinates

- density ρ , velocity \mathbf{u} , pressure p , internal energy $\epsilon = E/\rho - \mathbf{u}^2/2$, temperature T , heat conductivity κ , laser intensity I

$$\begin{aligned}\frac{d\rho}{dt} + \rho \operatorname{div} \mathbf{u} &= 0, & \frac{d\mathbf{x}}{dt} &= \mathbf{u} \\ \rho \frac{d\mathbf{u}}{dt} + \operatorname{grad} p &= 0 \\ \rho \frac{d\epsilon}{dt} + p \operatorname{div} \mathbf{u} &= -\operatorname{div}(\mathbf{I}) + \operatorname{div}(\kappa \operatorname{grad} T)\end{aligned}$$

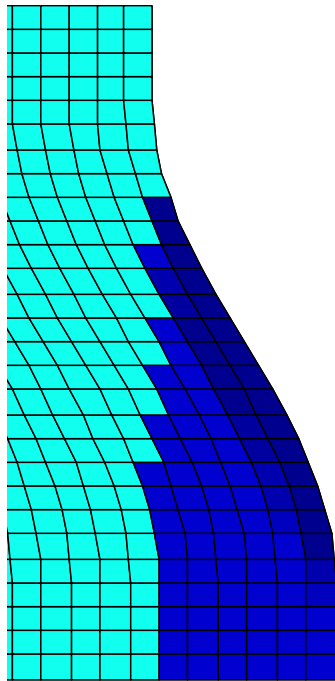
- total Lagrangian time derivatives include convective terms

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \operatorname{grad}$$

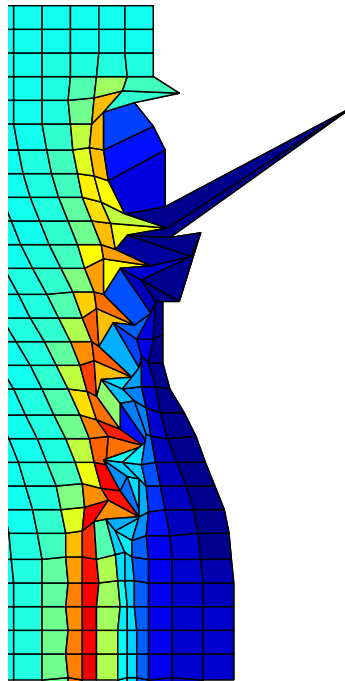
- equation of state – ideal gas and QEOS for plasma
- splitting – hyperbolic and parabolic part
- heat conductivity essential – faster shock wave

Moving Lagrangian Mesh

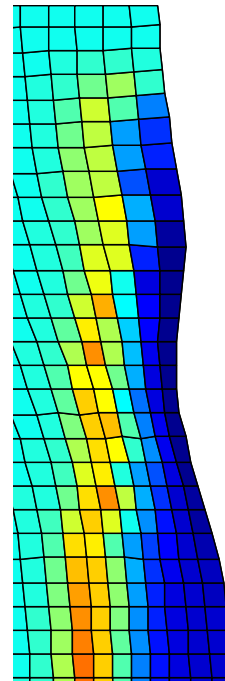
- moving mesh can degenerate



initial



Lagr



ALE

- degenerate typically for shear flow like high velocity impact
- can be treated by ALE method

ALE Method

- ALE – Arbitrary Lagrangian Eulerian method. Combination of Lagrangian and Eulerian methods [Hirt, Amsden, Cook (JCP 1974, 1997)]
 - I. Lagrangian computation several time steps
 - II. Rezoning – mesh untangling and smoothing
 - III. Remapping – conservative interpolation of the conservative quantities from old to new, better mesh. Then, back to Lagrangian computation.
- remapping corresponds to Eulerian part of ALE method, allows mass flux between cells
- ALE method combines positives of both approaches – grid moves with fluid (as Lagrangian), but Eulerian part keeps it smooth

I. Lagrangian Step

- staggered discretization – scalar quantities (density, pressure, internal energy, temperature) defined inside grid cells, vector quantities (positions, velocities) defined on grid nodes
- compatible method [Caramana, Burton, Shashkov, Whalen (JCP, 1998)]
- based on zonal, subzonal, and viscosity forces in each grid node
 - **zonal pressure force** – force from all neighboring grid cells to the node due to the pressure inside cells
 - **subzonal pressure force** – depends on difference between pressure in cell, and the pressure in cell corners, reduces artificial grid distortions
 - **viscosity force** – adds artificial viscosity in compression regions; edge [Caramana, Shashkov, Whalen (JCP, 1998)] or tensor [Campbell, Shashkov (2000)] viscosity

II. Rezoning

- rezoning – mesh untangling and smoothing.
- for remapping we need to move only those vertices which are necessary and as little as possible
- simple smoothing [**Winslow (1963)**]

$$\mathbf{x}_{i,j}^{k+1} = \frac{1}{2(\alpha^k + \gamma^k)} \left(\alpha^k (\mathbf{x}_{i,j+1}^k + \mathbf{x}_{i,j-1}^k) + \gamma^k (\mathbf{x}_{i+1,j}^k + \mathbf{x}_{i-1,j}^k) - \frac{1}{2} \beta^k (\mathbf{x}_{i+1,j+1}^k - \mathbf{x}_{i-1,j+1}^k + \mathbf{x}_{i-1,j-1}^k - \mathbf{x}_{i+1,j-1}^k) \right),$$

where coefficients $\alpha^k = x_\xi^2 + y_\xi^2$, $\beta^k = x_\xi x_\eta + y_\xi y_\eta$, $\gamma^k = x_\eta^2 + y_\eta^2$, and where (ξ, η) are logical coordinates.

- Reference Jacobian method [**Knupp, Margolin, Shashkov (JCP, 2002)**]
- combination of feasible set method and numerical optimization [**Váchal, Garimella, Shashkov (JCP, 2004)**].

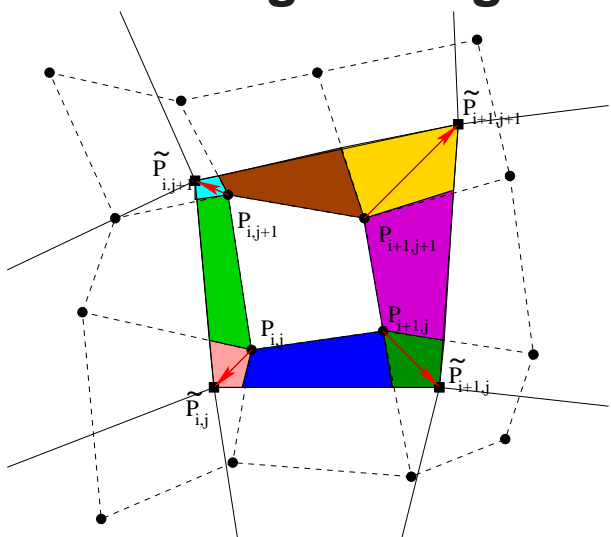
III. Remapping/1

- conservative interpolation of conservative quantities from the old Lagrangian mesh to the new smoothed mesh

1. piecewise linear **reconstruction** with Barth-Jasperson limiter [Barth (1997)]

$$g(x, y) = g_c + \left(\frac{\partial g}{\partial x} \right)_c (x - x_c) + \left(\frac{\partial g}{\partial y} \right)_c (y - y_c)$$

2. **quadrature** of reconstruction over cells of new mesh
 - **exact quadrature** – intersection of new cell with all neighboring old cells

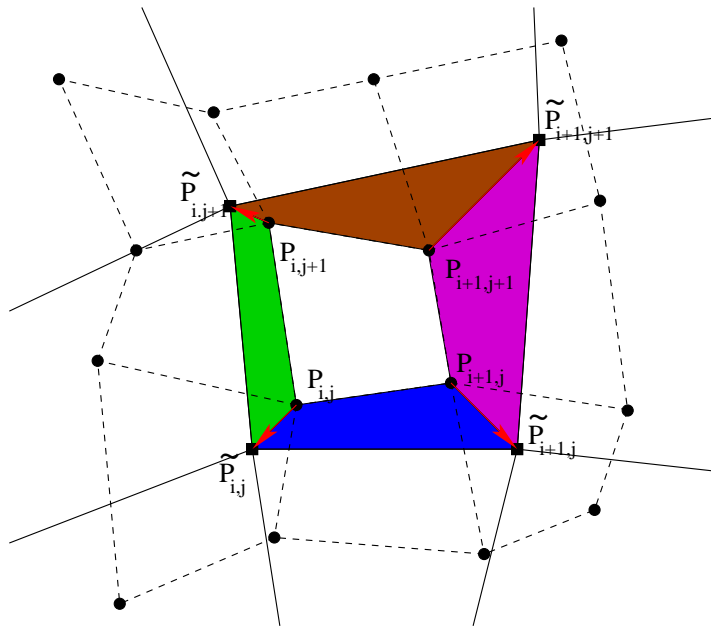


* old mesh dashed, new mesh solid

* integration of linear function over each intersection polygon – Green theorem transforms into integration over polygon edges

III. Remapping/2

- **approximate quadrature** over regions swept by edges moving from old to new position [Kuchařík, Shashkov, Wendroff (JCP, 2003)]



- exact integration is very expensive, requires finding intersections.
- integral over new cell can be decomposed as sum of integrals over swept regions.

3. repair

- Barth-Jasperson limiter guarantees monotonicity in 1D
 - in 2D new local local extrema might appear – repair [Shashkov, Wendroff (JCP, 2004)]
- remapping of staggered quantities more complicated

ALE in cylindrical geometry

- generalized to cylindrical r, z geometry [Kuchařík, Liska, Loubere, Shashkov (HYP2006)]

- additional factor r in integrals

$$\int f(x, y) dx dy \rightarrow \int f(r, z) r dr dz$$

- Lagrangian step
 - control volume method
 - cell center moved to center of cell mass instead of original average of vertexes – so that ALE remapping can be conservative
- rezoning – mesh nodes move on the z axis
- remapping – additional factor r in integrals

Heat Conductivity

- heat conductivity represented as parabolic term in the energy equation. By splitting, we solve $aT_t + \text{div}\mathbf{w} = 0$, $\mathbf{w} = -\kappa \text{grad } T = 0$ using mimetic operators method [Shashkov, Steinberg (1996)]

- mimetic discrete operators G, D have the same discrete integral properties, namely gradient is adjoint of divergence $G = D^*$

$$a \frac{T^{n+1} - T^n}{\Delta t} + D\mathbf{W}^{n+1} = 0$$

- implicit scheme in flux form

$$\mathbf{W}^{n+1} - GT^{n+1} = 0$$

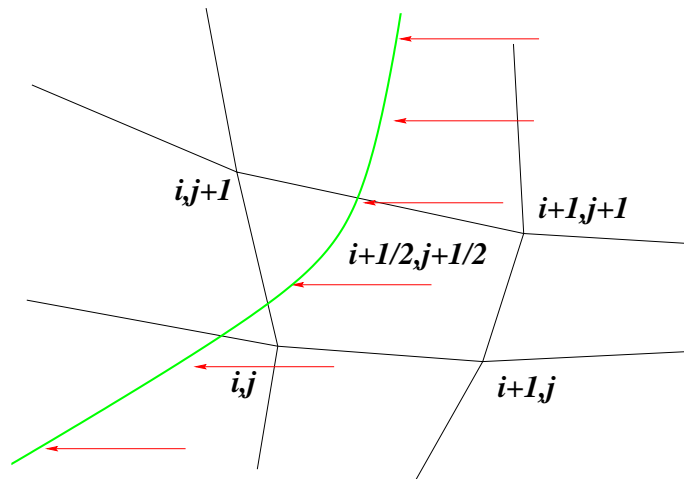
- temperature T^{n+1} is eliminated and the system is solved for heat flux \mathbf{W}^{n+1} ; the matrix of the system is symmetric and positive definite; same time step as in hyperbolic Lagrangian/ALE step

- computed fluxes have to be smaller than physical heat flux limit $\mathbf{W}^{n+1} = \text{sign}\mathbf{W}^{n+1} \min(|\mathbf{W}^{n+1}|, W_{limit})$

- generalization to cylindrical geometry.

Laser absorption

- simplest model – laser penetrates till critical density ρ_c and is absorbed on the critical surface

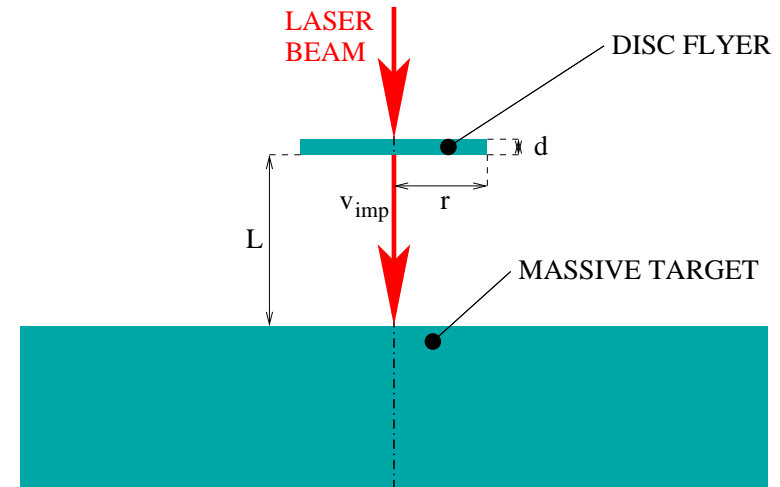


- source in internal energy equation $\rho \frac{d\epsilon}{dt} + p \operatorname{div} \mathbf{u} = -\operatorname{div}(\mathbf{I})$
- ray tracing model – curved rays with refraction, no reflected wave
[N. Demchenko]

High velocity impact

- disc flyer impact problem
- high power laser-irradiated Aluminum disc ablatively accelerates up to very high velocity (40-190 km/s) and strikes to massive Aluminum target

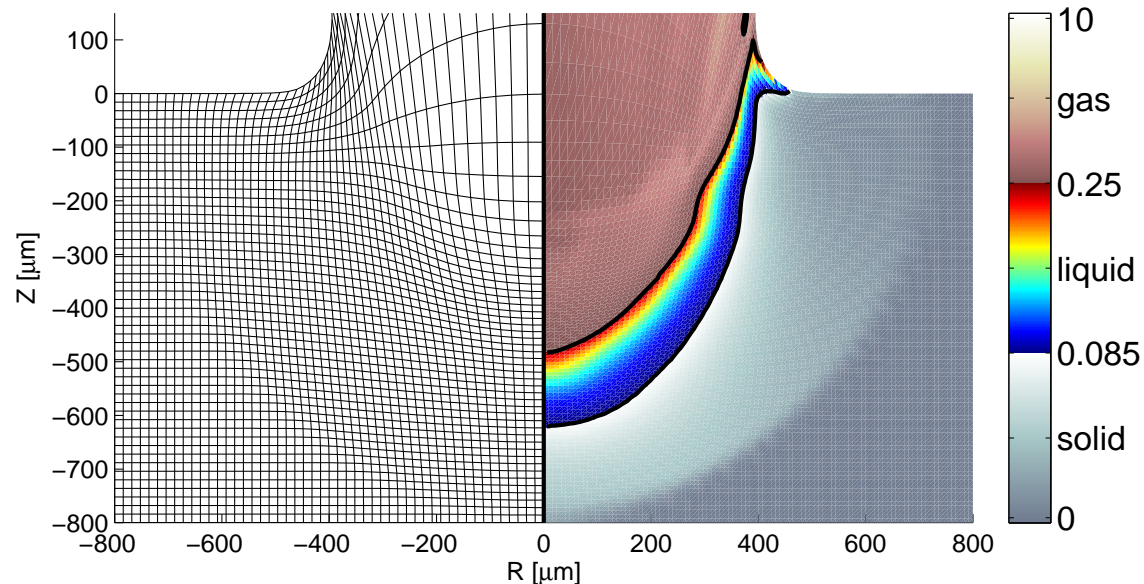
- $d = 6;11\mu\text{m}$, $r = 150\mu\text{m}$, $L = 200\mu\text{m}$, laser energy 120 – 390 J, 1-st or 3-rd harmonics, pulse length 400 ps, focus $r_f = 125\mu\text{m}$.



- problem split into two parts for simulations:
 - ablative disc flyer acceleration by laser beam; [animation](#)
 - impact of disc flyer into massive target
- problem parameters similar to the experiment performed on the PALS laser facility in Prague [Borodziuk et al. CJP (2003), Kálal et al. ECLIM (2004)].

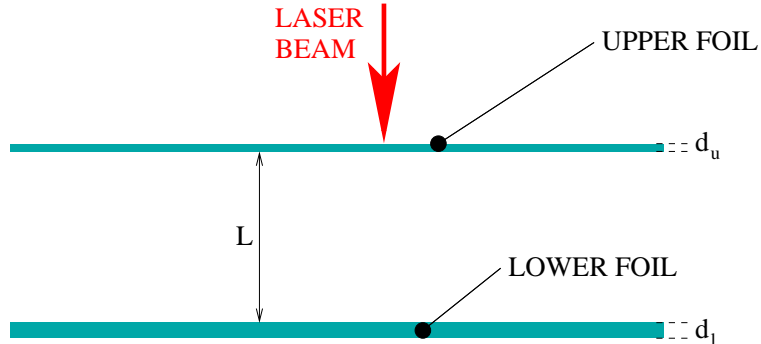
Crater creation

- after impact – increase of temperature, melting and evaporating material, circular shock wave
- crater (gas - liquid interface) formed inside the target



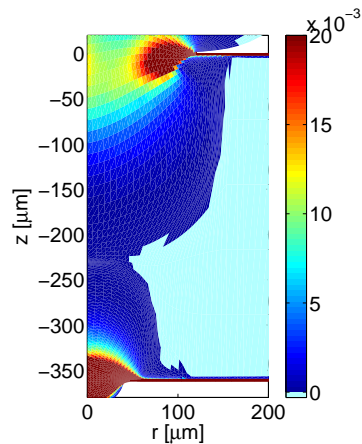
- **temperature animation**
- simulated craters size and shape correspond reasonably well to experimental data [Kuchařík, Liska, Limpouch (2006)]

Double foil target

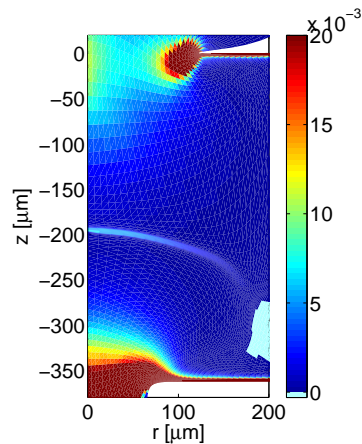
- foils thickness $d_u = 0.8\mu m$, $d_l = 2\mu m$
 - foils distance $L = 360\mu m$
 - laser energy 78 J, 3-rd harmonics, pulse length 250 ps, focus $r_f = 40\mu m$
 - almost vacuum between foils
- 
- mass of neighboring vacuum and foils cells should not differ much
 - vacuum cells are big while foils cells small
 - initially e.g. one foil rectangular cell has r/z edges lengths ratio 10^4 and neighbors the vacuum cell with r/z ratio 0.2
 - pure Lagrangian simulation fails due to mesh degeneration soon after laser burns through the upper foil

Double foil target – ALE results

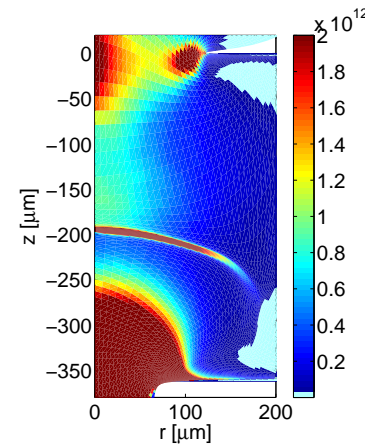
- laser maximum is at 0 ps



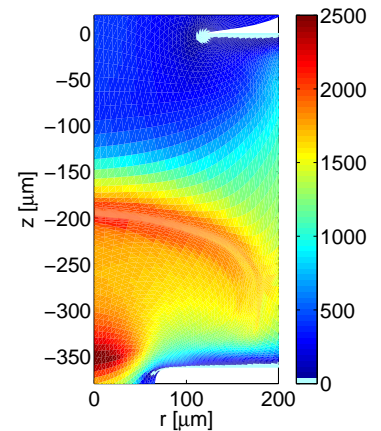
$\rho[\text{g/cm}^3]$ at 0 ps



$\rho[\text{g/cm}^3]$ at 100 ps



$p[\text{g/(s}^2\text{cm)}]$ at 100 ps

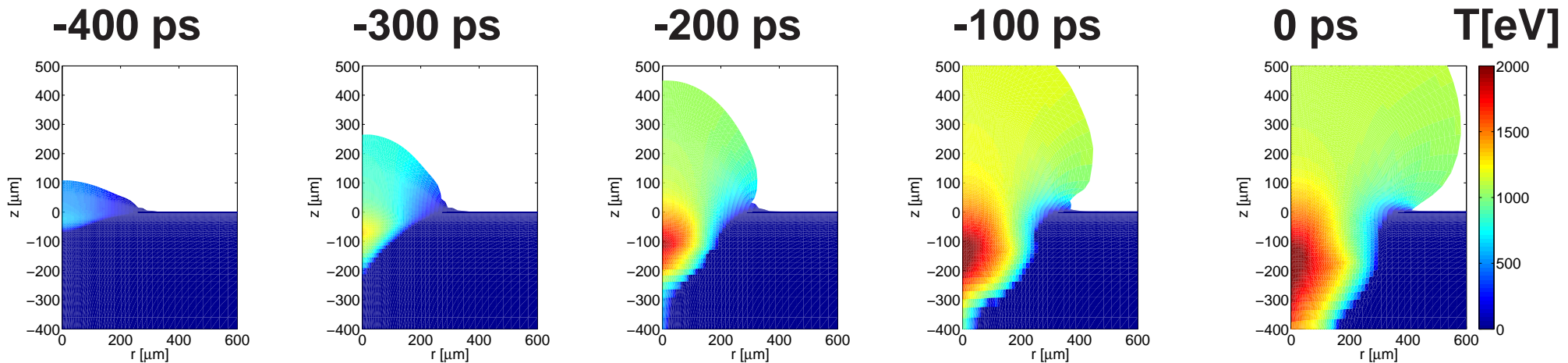


$T[\text{eV}]$ at 100 ps

- double foil target density and pressure [animation](#)

Foam target

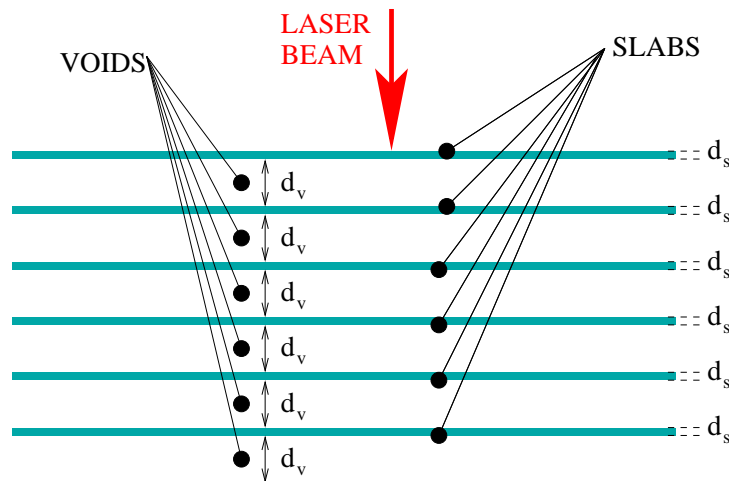
- $400\mu\text{ m}$ thick TAC foam with density $9.1\text{mg}/\text{cm}^3$ with $2\mu\text{m}$ pores.
- Gaussian laser pulse on the third harmonics with wavelength $0.438\mu\text{m}$, total energy 170 J , the radius of laser spot on target $300\mu\text{m}$ and FWHM length 320 ps .
- foam modelled by uniform density $9.1\text{mg}/\text{cm}^3$ material



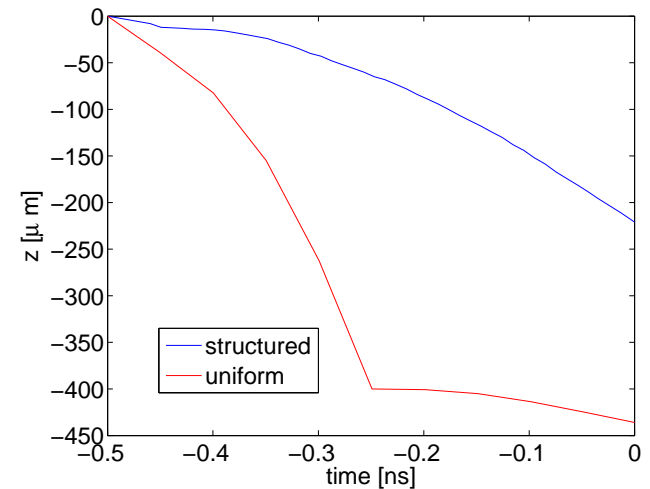
evolution of temperature; timing relates to the laser pulse maximum at 0 ps.

Foam target - structured model

- foam modelled by the sequence of $d_s = 0.018\mu\text{m}$ thick dense slabs with density $\rho_s = 1\text{ g/cm}^3$ separated by $d_v = 1.982\mu\text{m}$ thick voids with density $\rho_v = 1\text{ mg/cm}^3$



structured foam model

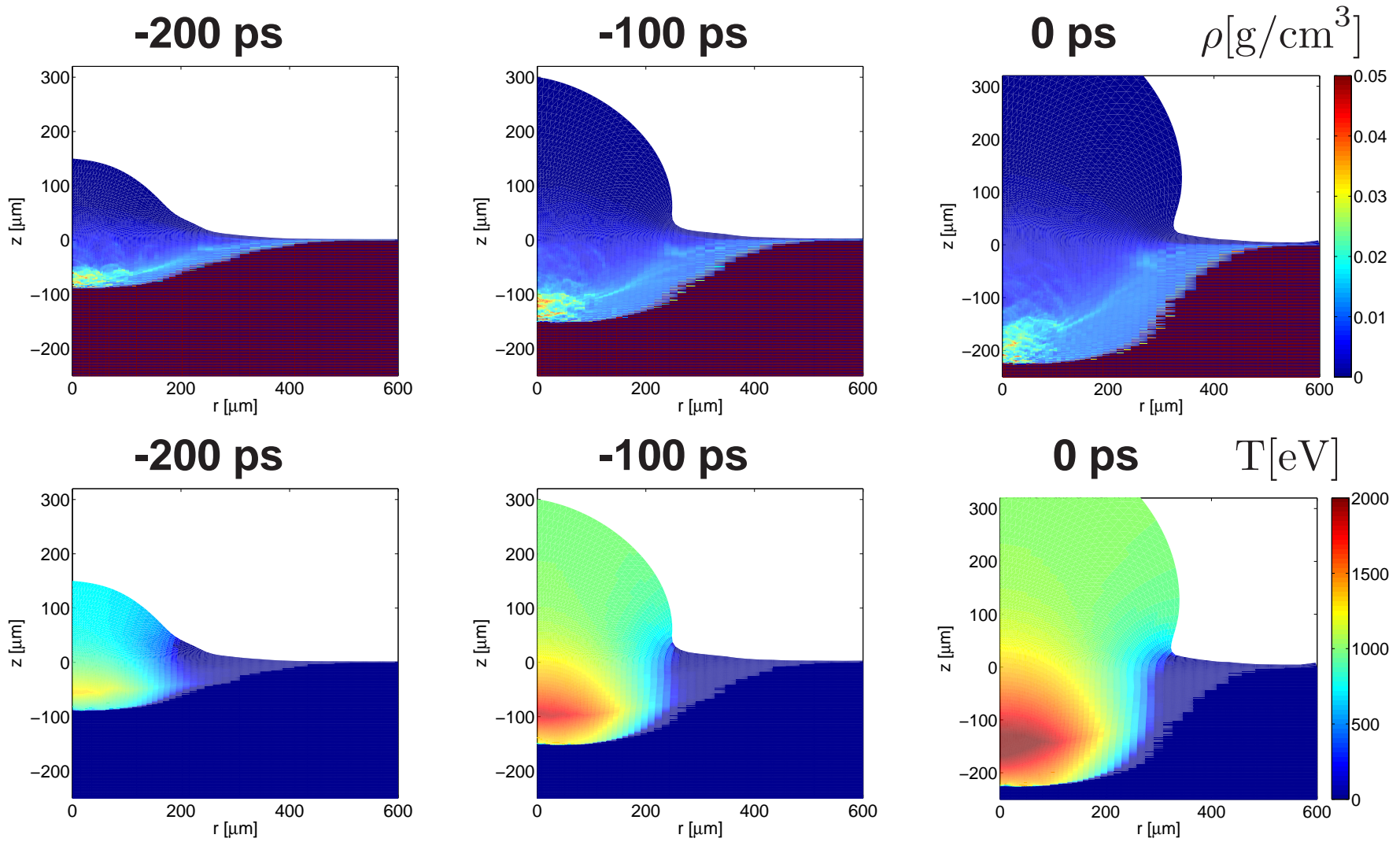


burning of laser through the target

- experimental speed of laser penetration into the foam is about $600 \sim 700\mu\text{m/ns}$, speed from structured simulation is about $500\mu\text{m/ns}$ (average in time interval (0.1, 0.5) ns) and from uniform simulation about $1600\mu\text{m/ns}$ (average in time interval (0.0, 0.25) ns).
- structured model approximates experimental data much better.

Foam target - structured model/2

- evolution of density and temperature



- density **animation** , **zoomed animation** ,

Conclusion

- **ALE method in Cartesian and cylindrical geometry**
- **heat conductivity, laser absorption**
- **applications – simulations of disc flyer, double foil and foam targets from PALS experiments**
- **pure Lagrangian simulation fails while ALE gives reasonable results**
- **perspectives – multimaterial, two temperature model, radiation transport, AMR.....**