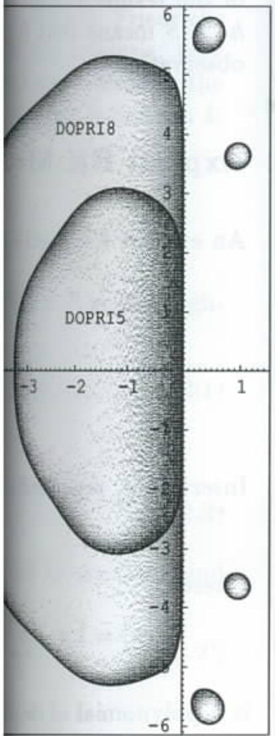

The method of Dormand & Prince DOPRI5 (Section II.4, Table 4.6) is of order 5 with $s=6$ (the 7th stage is for error estimation only). Here $R(z)$ is obtained by direct computation. The result is

$$R(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24} + \frac{z^5}{120} + \frac{z^6}{600} \,. \tag{2.13}$$

For DOPRI8 (Section II.6, Table 6.4), $R(z)$ becomes

$$R(z) = \sum_{j=0}^{8} \frac{z^j}{j!} + 0.27521279901 \cdot 10^{-5} z^9 + 0.24231996586959 \cdot 10^{-6} z^{10} + 0.24389718205443 \cdot 10^{-7} z^{11} - 0.2034615289686 \cdot 10^{-9} z^{12} \,. \tag{2.14}$$

The stability domains for these two methods are given in Fig. 2.2.

Extrapolation Methods

The GBS-algorithm (see Section II.9, Formulas (9.12), (9.13)) applied to $y' = \lambda y$, $y(0)=1$ leads with $z = H\lambda$ to

$$\begin{aligned} y_0 &= 1, \qquad y_1 = 1 + \frac{z}{n_j} \\ y_{i+1} &= y_{i-1} + 2\,\frac{z}{n_j}\,y_i \qquad\qquad i = 1,2,\ldots,n_j \\ T_{j1} &= \frac{1}{4}(y_{n_j-1} + 2y_{n_j} + y_{n_j+1}) \\ T_{j,k+1} &= T_{j,k} + \frac{T_{j,k} - T_{j-1,k}}{(n_j/n_{j-k})^2 - 1} \,. \end{aligned} \tag{2.15}$$

The stability domains for the diagonal terms T_{22}, T_{33}, T_{44}, and T_{55} for the harmonic sequence

$$\{n_j\} = \{2, 4, 6, 8, 10, \ldots\}$$

(the one which is used in ODEX) are displayed in Fig. 2.3. We have also added those for the methods *without* the smoothing step (II.9.13c), which shows some difference for negative real eigenvalues.