ALE simulations of high-velocity impact problem

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We describe here our newly developed ALE code on 2D quadrilaterals. The code is employed here in the numerical simulations of the crater formation during impact of small accelerated object on bulk target. The simulation of the high-velocity impact problem by the Lagrangian hydrodynamical codes leads in later stages to severe distortion of the Lagrangian grid which prevents continuation of computation. Such situations can however be treated by the Arbitrary Lagrangian-Eulerian (ALE) method. In order to maintain the grid quality, the ALE method applies grid smoothing regularly after several time steps of Lagrangian computation. After changing the grid, the conservative quantities have to be conservatively remapped from the old grid to the new, better one. After remapping, Lagrangian computation can continue.

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1 Introduction

In the nature many phenomena are understood as conservation laws. Fluid dynamics and plasma physics problems are modeled by a set of hydrodynamical conservation laws for mass, total energy, and momentum in each direction. Mathematically, these conservation laws are expressed by a set of hyperbolic partial differential equations. Generally, solving these equations must be done numerically.

One can use two different approaches for solving hydrodynamical equations, Eulerian and Lagrangian methods. In the case of Eulerian method, the computational grid is static, and the fluid flows through it. In Lagrangian methods the computational grid is moving with the fluid, it can resolve the solution on the regions of big changes of fluid volume and shape. Unfortunately, it may happen that the moving grid becomes tangled and loses its regularity. Now the mathematical assumption is violated and the Lagrangian method can not continue the computation.

Arbitrary Lagrangian-Eulerian (ALE) method [1] is a way, how to deal with this problem. After several time steps, we smooth the computational grid by rezoning and recompute conservative variables to the new, better grid. This algorithm adds additional diffusion to the numerical solution, the computational grid remains reasonable, and the Lagrangian numerical computation can continue.

In this paper we present the description of the 2D ALE code on logically orthogonal grids, and its application on the high-velocity impact problem. Previously, crater formation during interactions of intense pulses of laser PALS with targets has been simulated by high-velocity impact problem [2]. Intense laser beam can be also used for ablative acceleration of a macroparticle against solid targets. Here, we simulate the impact of an accelerated solid Aluminum disc onto a massive target by our hydrodynamical ALE code. The simulations parameters used here are nearly similar to the conditions in the experiment on PALS laser [3].

The hydrodynamical equations describing the conservation of the mass, total energy, and momenta in both directions, can be written in the form

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\nabla\cdot\vec{v}, \qquad \rho\frac{de}{dt} = -p\,\nabla\cdot\vec{v}, \qquad \rho\frac{d\vec{v}}{dt} = -\nabla\cdot p, \tag{1}$$

where t is time, ρ mass density, $\vec{v}$ vector of speeds, p pressure, and e the specific internal energy. To close this system, we must add the equation of state (EOS), which relates the density, internal energy, and pressure for the particular fluid. The simplest possibility of the EOS is the equation of state of the ideal gas. It can easily be written as $p = (\gamma - 1)\,\epsilon\,\rho$, where ϵ is the density of the internal energy $e = \rho\,\epsilon$. This form of the equation of state can be used for many testing computations as its evaluation is very fast, but its usage for plasma simulations is limited as plasma differs a lot from the ideal gas. To approximate real plasma behavior, we employ the quotidian equation of state (QEOS) [4], which is valid in a broad range of plasma conditions. It calculates the thermodynamic properties for the given conditions by applying Thomas-Fermi theory for electrons, and Cowan model for ions.

2 ALE Code

Our 2D Cartesian geometry ALE code on logically orthogonal quadrilateral grids was tested on a wide variety of problems from fluid dynamics, as Noh problem, Sedov problem, Rayleigh-Taylor instability. All numerical results are reasonable.

The Lagrangian part of the ALE code is based on the zonal, subzonal, and viscosity forces [5, 6] computed in each grid node, which define new nodal speeds in the next time step. The zonal part of the force depends on the cell geometry, and on the pressure inside the cell. It represents the force from the grid cell to each its node due to the pressure inside. The subzonal pressure force depends on the difference between the pressure in the cell, and the pressure in the cell corner (quarter of the cell adjacent to a particular node). It reduces the artificial grid distortions, prevents hourglass grid motion [6], and its value is controlled by one of the input parameters, the merit factor. Thus, we can regulate vorticity of the solution. The last part of the nodal force, the viscosity force, adds artificial viscosity to the solution. Without this component there is not enough dissipation and the solution fails soon. We use a bulk viscosity force based on classical Kuropatenko formula for the viscosity pressure correction.

Smoothing the Lagrangian grid is called rezoning and it has to be followed by remapping of conservative quantities from the old grid to the new, smoothed one. For rezoning, many different techniques can be used. One possibility is the combination of the feasible set and the global optimization techniques described in [7]. This method is very powerful for tangled or very distorted grids, but it is also very time consuming. When doing regular rezoning-remapping during the computation,

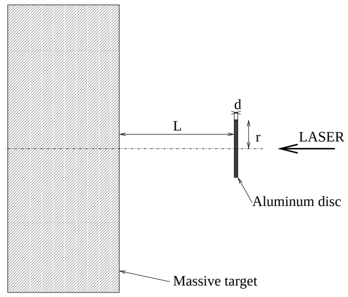

Fig. 1. Set-up of the problem, parameters $L = 300$ μm, $d = 11$ μm, and $r = 150$ μm.

a more efficient method is suitable. The grid is not distorted so strongly, and an easier method can be used. One of the most common methods is based on Winslow's work [8] which is discussed in details in paper [9].

For remapping, method based on the exact integration is the most natural one. Unfortunately, due to old and new cells intersections computation this method is very slow. A new method based on approximate integration was developed [10]. In the first stage, the conservative quantity is piecewise-linearly reconstructed on the original grid. We use Barth-Jasperson limiter for slopes computation to ensure local monotonicity of the reconstructed function. In the second stage, approximate integration over swept regions is used. By swept region, we mean the movement of the edge of the original grid to its new position. This method does not require intersections computation, but it may create new local extrema. Thus, the third stage is necessary - the repair. Local mass redistribution enforces local bound preservation and keeps the conservativity. This complete remapping algorithm is than local bound and linearity preserving, conservative, and much more efficient than the exact integration. This remapping method is used not only for mass recomputation, but also for the other conservative variables - energy and momenta. As conserved quantities are not mutually independent, a special procedure must be applied for remapping all conservative quantities.

3 Numerical results

Problem solved by our 2D ALE code simulates a part of the experiment performed on the PALS laser facility [3]. In this experiment, Aluminum disc is accelerated by ablation pressure induced by intense laser pulse against thick massive solid target. A scheme of the experimental set-up is presented in Fig. 1.

The density of the flying disc is estimated from 1D Lagrangian simulation of the disc ablatively accelerated by laser pulse to about a half of solid state Aluminum density. Our simulation starts in the moment of the disc impact onto the solid target. We chose the impact speed 54 km/s, that is near to the value measured in paper [3] for the 250 J laser beam. The disc starts to sink into the massive target.

Pure Lagrangian computation fails very soon due to the computational grid

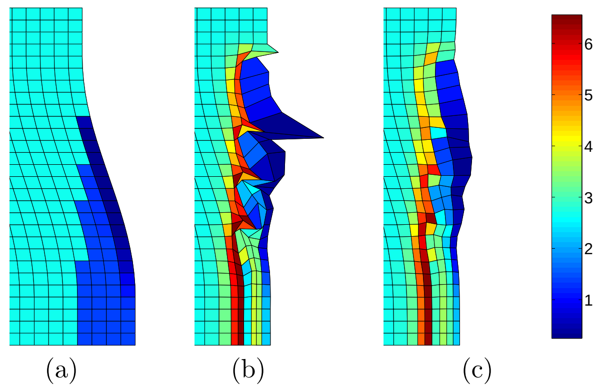

Fig. 2. Density colormap at critical part of the computational grid. (a) shows the initial grid before computation. After very short 0.5 ns computation with the pure Lagrangian method, the grid is very distorted (b) and includes non-convex cells. On the other hand, the grid during the complete ALE method (c) is still very nice.

distortions and the complete ALE algorithm with its rezoning-remapping stage is required. One can see the example on the Fig. 2. The right blue part (with density $1.35\mathrm{g/cm^2}$) of the initial grid with lower density represents the disk moving very fast (54 km/s) to the left. In a very early moment of the computation (0.5 ns), the material just started to compress, which is visible also from the grid. The pure Lagrangian computation leads to a very distorted grid in the critical area of the disc edge, and after several next steps the computation fails. The complete ALE algorithm continues computing and resolves the problems with the distorted Lagrangian grid very nicely.

After the impact, both the massive target and the disc start to raise their temperature. A shock wave is formed that propagates into the target and causes heating, melting and evaporation of the target material. QEOS equation of state is used here for Aluminum. This equation of state is fairly universal and can model matter from solid up to the plasma state. However, it does not include phase transitions explicitly, and thus material melting and evaporation cannot be satisfactorily determined from the target temperature. We use here internal energy of the shocked material instead. The crater boundary is found from condition that energy delivered by the shock wave must be at least equal to the sum of energy needed for heating material up to boiling point, and of latent heat of vaporization.

The increase of the internal energy measured in erg/g is plotted in Fig. 3 in time 30 ns after disk impact which is the final moment of crater formation. Different colormaps are used for the different states of the matter - evaporated gas in shades of pink on the right, solid metal in shades of gray on the left and melted liquid in colors in between. The different phase states are separated by the isolines at $9.72 \cdot 10^9$ erg/g and $1.35 \cdot 10^{11}$ erg/g levels of internal energy, which are energies needed to heat and melt and heat, melt and evaporate of Aluminum. By the crater we understand the interface between the liquid and gas phase. In fact, some amount

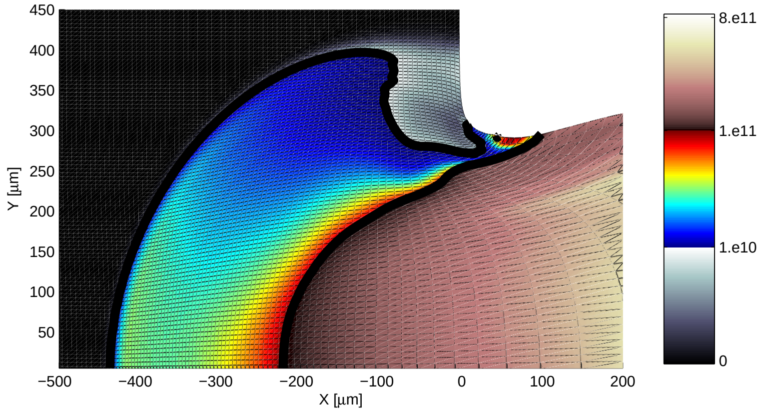

Fig. 3. Increase of internal energy in erg/g in time 30 ns after disk impact. Different energy colormaps for solid material, melted Aluminum, and evaporated gas. Two sick isolines divide three phase states.

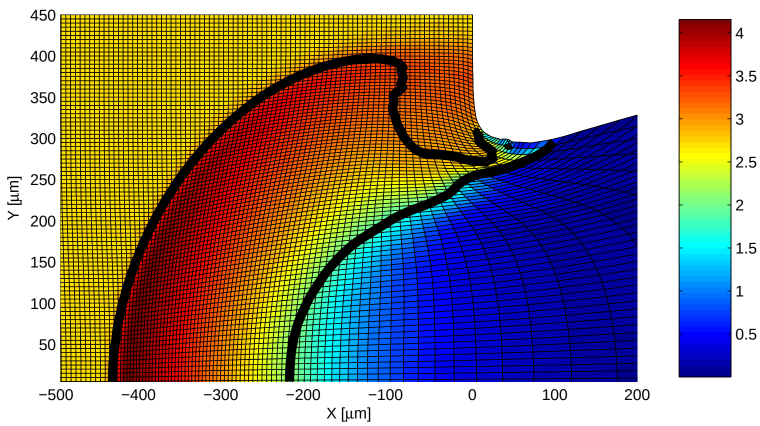

Fig. 4. Density colormap in g/cm^2 in time 30 ns after disk impact with computational grid and two internal energy isolines dividing three phase states.

of the liquid material probably flows also away, as the experimental crater depth is actually somewhat greater, about 300 μm. The discrepancy may be also caused by the difference in geometry used in simulations (Cartesian) and experiment (cylindrical). The shape of crater cross-section is close to circular, which corresponds well to the experimental results. Density is presented in Fig. 4 together with actual computational mesh and internal energy isolines dividing three phase states of Aluminium. At later times the liquid-gas interface does not move further inside the solid target, while the solid-liquid interface continues to move inside together with the circular shock wave.

4 Conclusion

We have developed a 2D ALE code on logically rectangular grids, which is robust enough to be applicable to complex simulations from plasma physics. As an example of the code application, we have presented here simulation of an impact of the thin metallic disc onto a massive target for parameters similar to experiment [3]. Our code derived the shape and volume of the resulting crater that may be compared with the experimental data.

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References

[1] C. W. Hirt, A. A. Amsden, J. L. Cook: *An Arbitrary Lagrangian-Eulerian Computing Method for All Flow Speeds*. Journal of Computational Physics **14** (1974) 227-253.

[2] S. Borodziuk, A. Kasperczuk, T. Pisarczyk, K. Rohlena, J. Ullschmied, M. Kalal, J. Limpouch, P. Pisarczyk: *Application of Laser Simulation Method for the Analysis of Crater Formation Experiment on PALS Laser*. Cz. J. Phys. **53** (2003) 799-810.

[3] S. Borodziuk, A. Kasperczuk, T. Pisarczyk, M. Kalal, J. Ullschmied, J. Limpouch, K. Rohlena, N. N. Demchenko, S. Yu. Gus'kov, V. B. Rozanov, V. N. Kondrashov, P. Pisarczyk: *Application of the Three Frame Interferometric and Crater Replica Methods for Investigation of a Laser Accelerated Macroparticles - Massive Target Interaction in the PALS Experiment*. Optica Applicata **34** (2004) (in press).

[4] R. M. More, K. Warren, D. Young, G. Zimmerman: *A New Quotidian Equation of State (QEOS) for Hot Dense Matter*. Phys. Fluids **31** (1988) 3059-3078.

[5] E. J. Caramana, D. E. Burton, M. J. Shashkov, P. P. Whalen: *The Construction of Compatible Hydrodynamics Algorithms Utilizing Conservation of Total Energy*. Journal of Computational Physics **146** (1998) 227-262.

[6] E. J. Caramana and M. J. Shashkov: *Elimination of Artificial Grid Distortion and Hourglass-Type Motions by Means of Lagrangian Subzonal Masses and Pressures*. Journal of Computational Physics **142** (1998) 521-561.

[7] P. Váchal, R. Garimella, M. Shashkov: *Untangling of 2D Meshes in ALE Simulations*. Journal of Computational Physics **196** (2004) 627-644.

[8] A. M. Winslow: *Equipotential Zoning of Two-Dimensional Meshes*. Technical report, Lawrence Livermore National Laboratory (1963) UCRL-7312.

[9] J. S. Peery, D. E. Carroll: *Multi-Material ALE Methods in Unstructured Grids*. Comput. Methods Appl. Mech. Engrg. **187** (2000) 591-619.

[10] M. Kuchařík, M. Shashkov, B. Wendroff: *An Efficient Linearity-and-Bound-Preserving Remapping Method*. Journal of Computational Physics **188** (2003) 462-471.