Conservative Formulation and Numerical Methods for Multiphase Compressible Media

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Although a substantial progress has been made in last few decades in the theoretical and numerical modelling of multiphase media, there is no widely accepted mathematical model even for two-phase compressible flows. The main challenge in the development of high-accuracy numerical methods for multiphase compressible flows is associated with the formulation of a mathematical model that satisfies important properties such as hyperbolicity, symmetric hyperbolic system in particular, fully conservative form of the governing equations and compatibility and consistency of the mathematical model with the thermodynamic laws. These properties provide a solid mathematical framework for the theory of different initial-boundary value problems and allow developing highly accurate numerical methods.

According to classical theories, multiphase mulifluid flows are considered as interacting continua governed by mass, momentum and energy balance laws for each phase. We propose a new approach for the modelling of multiphase flows based on the theory of thermodynamically compatible systems and irreversible phenomenological thermodynamics.
This approach allows us to formulate classes of hyperbolic conservation-form equations using generalized potentials and variables. Using phenomenological thermodynamic laws a structure of governing balance laws is derived according to which the mixture is assumed as a continuum in which multiphase character of flow is taken into account by the appropriate choice of parameters of state. Thus the multiphase flow is governed by the additional differential equations in addition to the mass, momentum and energy conservation laws for the mixture. The full set of conservation-form hyperbolic equations can be derived by using the formalism of thermodynamically compatible systems. The most general model governs a multiphase compressible flow with different phase pressures and temperatures. Constitutive relationships such as equation of state (EOS) for the mixture and source terms responsible for phase interaction are required to close the system. The EOS for the mixture can be derived using the equations of state for each phase.

The conservation form of the governing equations provides a straightforward basis for the development of high-order accurate numerical method. A second-order finite volume method based on the solution of the Riemann problem as obtained by the GFORCE method has been developed. A number of numerical examples for one- and two-dimensional problem for two-phase flow are presented.