Mixed Finite Element Methods for Lagrangian Hydrodynamics

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Algorithms for the numerical simulation of hydrodynamics sometimes give rise to spurious unphysical modes which can lead to artificial grid distortion and symmetry breaking. Such spurious behaviour can be attributed to the way in which the acceleration of grid nodes is computed. Typically a control volume is used to define a finite difference approximation for the gradient operator. This approach is combined with an "hourglass-filter" which is used to damp or project out the so called hourglass modes. We present results concerning the development and use of mixed finite element methods for the numerical solution of the hydrodynamic equations in a Lagrangian frame. Mixed finite element methods are a promising alternative to traditional finite difference methods for discretizing spatial differential operators such as the gradient and divergence, and can subsequently be used to define an acceleration operator which is inherently free of spurious modes. We utilize the Brezzi-Douglas-Marini (BDM) elements on quadrilaterals for discretizing the pressure and velocity. In this approach, the pressure is piece-wise constant in a zone (as is typically the case) while the velocity is discretized on mesh faces (edges in 2D) using a divergence conforming basis set where the degrees of freedom are the normal projections of the nodal velocity on mesh faces. The BDM basis functions maintain coordinate system invariance by transforming covariantly. We present early results that suggest such an approach is much better at controlling (or altogether eliminating) spurious grid distortion on a set of canonical test problems. We also point out that the main drawback to this approach is the need to assemble and solve a global sparse linear system at each Lagrange time step.