The Comoving-frame and Laboratory-frame Nonequilibrium Grey Radiation Diffusion Approximations in the Nonrelativistic Limit

Jarrod D. Edwards and Jim E. Morel
Texas A&M University
College Station, Texas, USA

We contrast the comoving-frame and laboratory-frame non-equilibrium grey radiation diffusion approximations in the nonrelativistic limit. This limit corresponds to nonrelativistic material motion, which we define as $v \leq 0.01c$, where v is the material speed and c is the speed of light. All of the non-relativistic equations we consider are correct to $O(v/c)$ unless otherwise stated. Our main results are as follows.

One need only neglect the time derivative of the flux in the laboratory-frame grey P_1 equations to obtain the laboratory-frame diffusion approximation, but one must neglect several additional terms in the comoving-frame grey P_1 equations to obtain the comoving-frame diffusion approximation.

The comoving-frame grey diffusion equation does not rigorously conserve laboratory-frame radiation energy. Conservation is only meaningful with respect to laboratory-frame quantities because the comoving frame is an accelerated reference frame. Thus the comoving-frame grey diffusion approximation is not conservative. However, the error is small. Furthermore, if one neglects the difference between the comoving-frame and laboratory-frame radiation energy densities (a reasonable nonrelativistic approximation), the equation becomes conservative.

The comoving-frame P_1 equations conserve the laboratory-frame radiation energy. Thus the lack of conservation in the diffusion approximation arises from the terms that are dropped from the P_1 equations to obtain the diffusion approximation.

In static media the equilibrium diffusion approximation is known to be asymptotically correct through $\mathrm{O}(\epsilon)$. We show that both the laboratory-frame and comoving-frame grey diffusion approximations preserve the asymptotic equilibrium diffusion limit through $\mathrm{O}(\epsilon)$. This means that both approximations are fully valid in this limit.

The comoving-frame grey diffusion equation is considerably simpler than the laboratory-frame diffusion equation. A simplification to the laboratory-frame radiation energy and momentum source terms results in an laboratory-frame grey diffusion equation that has exactly the same form as the comoving-frame equation. The simplified equation is not correct to $\mathrm{O}(v/c)$, but it nonetheless preserves equlibrium solutions, preserves the equilibrium-diffusion limit, and is always conservative. Thus we believe that this equation is a viable alternative to the comoving-frame equation.