Two-layer flows with free surface

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We obtain a dispersive model for the description of large amplitude waves propagating in a two-layer system with free surface. The model is a ``two-layer" generalization of the Green-Naghdi (GN) model. The novelty of the derived model in comparison with the work by Liska, Margolin and Wendroff (1995) is using the Lagrangian approach in the spirit of the work by Miles and Salmon (1985) done for the derivation of the GN model. The Lagrangian approach gives the background for application of general theoretical methods. In particular, this concerns the generalization of the notion of vortex motions, which was proposed in our earlier paper (Gavrilyuk and Teshukov, 2001) for general class of Lagrangian models, and which was developed here for a two-layer model.
As in the case of the full problem, the present model captures the resonance between short waves and long waves. In this framework it is shown, by using numerical computations, the existence of homoclinic trajectories embedded into the continuous spectrum. They correspond to true solitary waves having the same velocities at infinity in each layer. Their study reduces to the analysis of a Hamiltonian system with two degrees of freedom. The traveling-wave solutions depend on three parameters: the *density ratio*, the *depth ratio* and the *Froude number* based on the bottom layer. Two wave regimes, characterized by the elevation or depression of the interface between the layers are presented. A critical depth ratio separates these two regimes and it will be shown how it relates to a change of the structure of the potential for the Hamiltonian system. The analysis of the number and nature of critical points turned out to be decisive in this work. It was found that the number of critical points can be four or two, depending on the value of the Froude number (for fixed density and depth ratios). For sets of parameters corresponding to oceanic conditions we have perceived the existence of true solitary waves and their broadening whenever the speed wave increases towards a limit value. Finally, other sets of parameters are considered for which multi-humped solitons exist, highlighting the richness and complexity of the system considered.