# Numerial Calculations for Hydrodynamics Based on Multi-dimensional Riemann Solvers

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A Numerical scheme for multi-dimensional hydrodynamics will be reported based on an approximate multi-dimensional Riemann solver at grid points. The scheme is one of the first attempts to use multi-dimensional Riemann solvers in numerical simulations for hydrodynamics. The scheme is truly multi-dimensional. It is second order accurate in both space and time. It satisfies conservation laws for mass, momentum, and total energy exactly.

The set of the two-dimensional (2D) Euler equations may be written as

$$\rho \frac{dU}{dt} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y}.$$

Here, $U$, contains conserved quantities of mass, mmentum, and energy, and $F_x$ and $F_y$ are fluxes in the x- and y- directions. A 2D Riemann problem is the equation above with a set of constant states for each region surrounding a point, for example, four constant states in the four quadrants in a structured mesh. If the equation is integrated over a cell and one time step, $0 < t < \Delta t$, the following equation will be obtained.

$$\langle U \rangle = \langle U \rangle_0 + \frac{\Delta t}{\Delta m} \{ \overline{\oint F_x dy} + \overline{\oint F_y dx} \}.$$

Here, $\langle U \rangle$ is a space-averaged value of $U$ over the cell at $t = \Delta t$, $\langle U \rangle_0$ is its initial value, $\Delta m$ is the mass in the cell, the integral is count-clockwise along the perimeter of the cell, and the bar over the integral stands for the time-average during the time step. In our scheme, the time-averaged integral is approximately calculated through the time-averaged values obtained from an approximate multi-dimensional Riemann solver. To get the second order accuracy of the scheme, the states surrounding a grid point will not be the states of the cells, but are the states on domains of dependence.

The scheme has been tested in ALE calculations. The image below shows the pressure in an ALE calculation at $t = 0.0002$ for a 2D Riemann problem. The initial pressures on the four quadrants are $10^6$, 1.0, $10^6$, 10 respectively. The initial velocity is zero, and initial density is unity everywhere. No artificial viscosity has been used in the calculation.

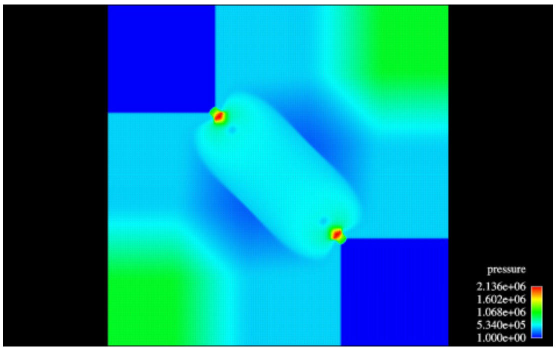


Figure 1: Pressure of a two-dimensional Riemann problem.