Progress toward an Improved Staggered-Grid Hydrodynamics Method

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The Lagrangian formulation of the equations of hydrodynamics has a very old and venerable history going back over 60 years as a practical tool for large-scale numerical simulations. Problems associated with mesh tangling have been largely addressed through adaptivity in the forms of Arbitrary Lagrange Euler (ALE), free-Lagrange reconnection, and more recently Adaptive Mesh Refinement (AMR). This has led to the development of formulations extended for unstructured polyhedral cells.

Most Lagrange formulations have employed a spatial discretization in which the evolution equations for stress and velocity are solved on staggered control volumes arranged such that the logical center of each lies on the boundary of the other. This overlap avoids the interpolation to obtain boundary fluxes characteristic of cell-centered schemes. For uniform grids, this formulation is second-order away from discontinuities and first-order near them.

The basic numerics have evolved from simple finite difference approximations to multi-dimensional, fully conservative, compatible, finite-volume formulations that mimic the fundamental hydrodynamics equations. In recent years, significant progress has been made in addressing major historical issues associated with energy conservation, hourglass instabilities, and shock-induced oscillations.

In spite of successes, the staggered formulation has flaws that bear investigation. Monotonic viscosity formulations have not been adapted to unstructured grids. The stress divergence operator is only first order for non-uniform grids, in effect transferring the burden for accuracy from the algorithm to grid generation tools. The conventional nodal definition of kinetic energy cannot be conserved simultaneously with momentum during advection, and alternatives may conflict with energy compatibility requirements. Volumes calculated from coordinates and from velocity fluxes are not identical, leading to either energy or entropy error.

The use of a higher-order differencing scheme was key to resolving hourglass problems. This suggests that similar techniques might be helpful in improving some of the aforementioned flaws. This paper will address work in this area.