Cyclotron emission

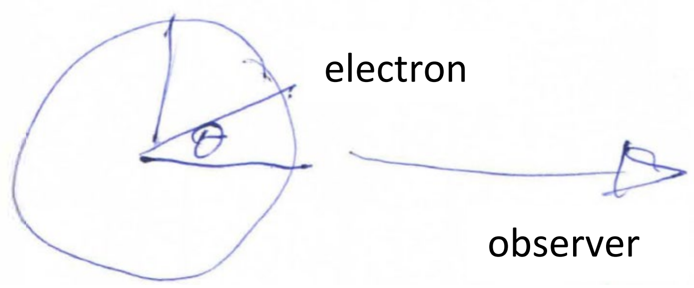
Electron spiraling with the angle α around a magnetic field line moves with acceleration *a*

$$\vec{F} = -e\,\vec{v}\times\vec{B}_0 \qquad v_\perp = v\,\sin\alpha \qquad |\vec{a}| = \left|\frac{\mathrm{d}\,\vec{v}}{\mathrm{d}\,t}\right| = \frac{e\,v_\perp B_0}{m_e} = \omega_c\,v\sin\alpha$$

Power of emitted radiation is given by the Larmor formula

$$P = \frac{2}{3}\frac{e^2\,a^2}{4\pi\varepsilon_0\,c^3} = \frac{2}{3\,c}\frac{e^2}{4\pi\varepsilon_0}\frac{e^2\,B_0^2}{m_e^2}\frac{v^2\sin^2\alpha}{c^2}$$

Observation in the plane $\perp B_0$ (for simplicity)



angular distribution of emission $P_\Omega \sim \sin^2\theta$

time dependence of incident radiation at observer position $P(t) \sim \sin^2\theta = \sin^2\omega_c t$

Other observation angles – harmonics $n\omega_c$ are observed in addition to the frequency ω_c

Emitted power integrated over full solid angle (isotropic electron distribution assumed)

$$\overline{P} = \int_0^\pi P\, 2\pi \sin\alpha \,\mathrm{d}\alpha = \frac{2}{3\,c}\,\frac{e^2}{4\pi\varepsilon_0}\,\omega_c^2 \left(\frac{v}{c}\right)^2 \int_0^\pi \; 2\pi \sin^3\alpha \,\mathrm{d}\alpha$$

angular integral = 8π/3

$$\overline{P} = \frac{e^2}{4\pi\varepsilon_0}\,\frac{4\,\omega_c^2}{3\,m_e c^3}\,\frac{8\pi}{3}\,\frac{m_e v^2}{2} = \frac{e^2}{4\pi\varepsilon_0}\,\frac{4\,\omega_c^2}{3\,m_e c^3}\,\frac{8\pi}{3}\,E_{kin}$$

Evolution of electron kinetic energy

$$\frac{\mathrm{d}\,E_{kin}}{\mathrm{d}\,t} = -\overline{P} = -\frac{E_{kin}}{t_0} \qquad\qquad t_0 = \frac{9\,\varepsilon_0}{8}\,\frac{m_e^3\,c^3}{e^4\,B_0^2} \simeq \frac{0.3}{B_0^2}\;\;[\mathrm{s}]$$

Electron loses its energy due to cyclotron emission relatively fast in a very dilute plasma, where reabsorption is negligible.

We shall derive now the absorption coefficient K_c to estimate reabsorption.

Absorption coefficient

The absorption coefficient will be calculated via principle of detailed balancing
In equilibrium – emission = absorption

$$n_e \overline{P} = K_c \overline{I}_{p\nu_c} \Delta\nu$$

Radiation intensity in equilibrium

$$\overline{I}_{p\nu_c} = 4\pi B_{\nu_c} = \frac{8\pi h \nu_c^3 / c^2}{\exp(\frac{h\nu_c}{k_B T_e}) - 1}$$

As ν_c is of order 10^{11} Hz, $h\nu_c \ll k_B T_e$, intensity obeys Rayleigh-Jeans law

$$\overline{I}_{p\nu_c} \simeq \frac{8\pi \nu_c^2 k_B T_e}{c^2} = \frac{2\omega_c^2 k_B T_e}{\pi c^2}$$

Line width of cyclotron emission is due to Doppler broadening

$$\Delta\nu = \nu_c \frac{v}{c} = \frac{\omega_c}{2\pi} \frac{v}{c} \simeq \frac{\omega_c}{2\pi} \sqrt{\frac{2k_B T_e}{m_e c^2}}$$

Absorption coefficient is then

$$K_c = \frac{n_e e^2}{4\pi\varepsilon_0} \frac{2\omega_c^2}{3c} \left(\frac{v}{c}\right) \frac{8\pi}{3} \frac{\pi c^2}{2\omega_c^2 k_B T_e} \frac{2\pi c}{\omega_c v} = \frac{n_e e^2}{4\pi\varepsilon_0} \frac{16\pi^3}{9B_0} \sqrt{\frac{2m_e}{k_B T_e}}$$

Numerically $$K_c \simeq 8.46 \times 10^4 \frac{1}{B_0} \frac{1}{\sqrt{T_e}} \frac{n_e}{10^{19}}$$ $[B]$=T, $[T_e]$=keV, $[n_e]$=m^{-3}, $[K_c]$=m^{-1}

For $n_e = 10^{19}$ m^{-3}, $B = 1$ T, $T_e = 1$ keV $\quad l_f = K_c^{-1} = 1.18\times10^{-5}$ m

and the energy confinement time is estimated $\tau_{energy} \cong t_0 R/ l_f$,

for R = 0.1 m $\quad \tau_{energy} \cong 0.3\times0.1/1.18\times10^{-5} = 2540$ s

Thus, energy losses due to cyclotron emission are negligible in magnetic confinement