Rozpad plazmatu difuzí R(1)

$$\frac{\partial n}{\partial t} = \mathrm{div}(D \nabla n)$$

if $D =$ constant

$$\frac{\partial n}{\partial t} = D \nabla^2 n$$

we search for solution

$$n(\vec{r}, t) = T(t) S(\vec{r})$$

$$S \frac{dT}{dt} = D T \nabla^2 S$$

$$\frac{1}{T} \frac{dT}{dt} = \frac{D}{S} \nabla^2 S = -\frac{1}{\tau}$$

τ konst

$$T = T_0 e^{-t/\tau}$$

$$\nabla^2 S = -\frac{1}{D\tau} S$$

planar geometry wall $\pm L$

$$S(\pm L) = 0$$

$$S = A \cos \frac{x}{\sqrt{D\tau}} + B \sin \frac{x}{\sqrt{D\tau}}$$

lowest mode

$$\frac{x}{\sqrt{D\tau}} = \frac{\pi}{2} \qquad B = 0$$

$$\tau = \left(\frac{2L}{\pi}\right)^2 \frac{1}{D}$$

$R(\ell)$

$$n = n_0 e^{-t/\tau} \cos\frac{\pi x}{2L}$$

all mode

$$n(t=0) = n_0\left(\sum_\ell a_\ell \cos\frac{(\ell+\frac{1}{2})\pi x}{L} + \right.$$

$$\left. + \sum_m b_m \sin\frac{m\pi x}{L}\right)$$

$$\tau_\ell = \left[\frac{L}{(\ell+1/2)\pi}\right]^2 \frac{1}{D} \qquad \tau_m = \frac{1}{D}\left[\frac{L}{m\pi}\right]^2$$

Cylindric geometry (radius of wall = a)

$$\nabla^2 S = \frac{d^2S}{dr^2} + \frac{1}{r}\frac{dS}{dr}$$

$$\frac{d^2S}{dr^2} + \frac{1}{r}\frac{dS}{dr} + \frac{1}{D\tau}S = 0$$

solution $J_0(kr)$ ($Y(kr)$ excluded as $Y(0) \to -\infty$)

modes correspond to ~~the~~
k for which $J_0(ka) = 0$

lowest mode $ka \cong 2.4 \Rightarrow S = J_0\left(\frac{2.4r}{a}\right)$

$$\sqrt{D\tau} = \frac{a}{2.4} \Rightarrow \tau = \frac{1}{D}\left(\frac{a}{2.4}\right)^2$$

higher modes - other 0 of J_0

2nd $\simeq 5.5$

decay via recombination R ③

photorecombination $\sim \alpha n_e n_i = \alpha n^2$

3-body recomb $\sim \beta n_e^2 n_i$ small omitted

$$\frac{\partial n}{\partial t} = -\alpha n^2$$

$$\frac{1}{n(\vec{r},t)} = \frac{1}{n_0(\vec{r})} + \alpha t$$

$$n(\vec{r},t) = \frac{n_0(\vec{r})}{1 + n_0(\vec{r})\alpha t}$$

decay curve Chen p. 167

Steady state with source in center

$$\underbrace{\frac{\partial n}{\partial t}}_{=0} - D\nabla^2 n = Q(\vec{r}) \quad (= I n)$$

$$\nabla^2 n = -\frac{I}{D} n \qquad I n = Q$$

$$\frac{d^2 n}{dx^2} = -\frac{Q}{D}\delta(0) \qquad n = n_0\left(1 - \frac{|x|}{L}\right)$$

cylindric $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) = 0 \qquad r > 0$

$$n = n_0 \ln\left(\frac{a}{r}\right)$$