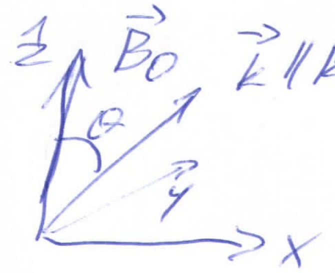
# Electrostatic oscillations in cold plasma in $B_0$ field

High frequency oscillations – ions homogeneous neutralizing background

We solve equations for electron fluid




$\vec{k} \parallel \vec{E}_1$

$$\vec{k} = (k\sin\theta, 0, k\cos\theta)$$

$$\vec{E}_1 = (E_1\sin\theta, 0, E_1\cos\theta)$$

$$\vec{B}_0 = (0, 0, B_0)$$

$$n_e = n_0 + n_1$$

$$\vec{v}_1 = (v_x, v_y, v_z)$$

Continuity equation

$$\frac{\partial n_1}{\partial t} + n_0 \operatorname{div} \vec{v}_1 = 0$$

Equation of motion

$$m_e \frac{\partial \vec{v}_1}{\partial t} = -e(\vec{E}_1 + \vec{v}_1 \times \vec{B}_0)$$

Poisson's equation

$$\operatorname{div} \vec{E}_1 = -\frac{e n_1}{\varepsilon_0}$$

Solution $\sim e^{i(\vec{k}\vec{r} - \omega t)}$

Continuity

$$-i\omega n_1 + i n_0 (k_x v_x + k_z v_z) = 0$$

Momentum

$$-i\omega m_e v_x = -eE_1\sin\theta - e v_y B_0$$

$$-i\omega m_e v_y = e v_x B_0$$

$$-i\omega m_e v_z = -eE_1\cos\theta \qquad v_z = -\frac{eE_1}{m_e\omega}\cos\theta$$

$$\Rightarrow v_x = -\frac{i\omega e E_1\sin\theta}{m_e(\omega^2-\omega_c^2)}$$

$$\Rightarrow n_1 = -i\frac{k n_0 e E_1}{m_e}\left(\frac{\sin^2\theta}{\omega^2-\omega_c^2}+\frac{\cos^2\theta}{\omega^2}\right)$$

Poisson's equation

$$i\vec{k}\vec{E}_1 = ikE_1 = -\frac{en_1}{\varepsilon_0} = ik\frac{e^2 n_0}{m_e\varepsilon_0}\left(\frac{\sin^2\theta}{\omega^2-\omega_c^2}+\frac{\cos^2\theta}{\omega^2}\right)E_1$$

Dispersion relation

$$1 = \omega_p^2\left(\frac{\sin^2\theta}{\omega^2-\omega_c^2}+\frac{\cos^2\theta}{\omega^2}\right)$$

$$\omega^4 - \underbrace{(\omega_p^2+\omega_c^2)}_{\omega_h^2}\omega^2 + \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\omega_{1/2}^2 = \frac{\omega_h^2 \pm \sqrt{\omega_h^4 - 4\omega_p^2\omega_c^2\cos^2\theta}}{2}$$

When $\omega_p > \omega_c$

$\theta = 0$

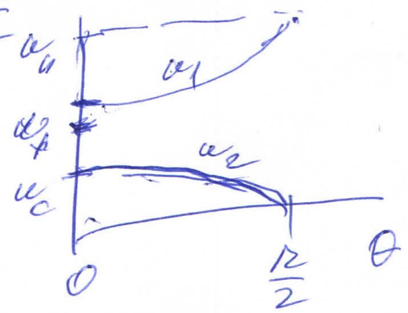
$\omega_1^2 = \omega_p^2$

$\omega_2^2 = \omega_c^2$

$\theta = \frac{\pi}{2}$

$\omega_1^2 = \omega_h^2$

$\omega_2^2 = 0$



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