Problem - degenerate electron gas in ICF

Problem

Inertial confinement fusion typically includes compression of DT ice from solid density $0.25\ \mathrm{g/cm^3}$ to $1000\times$ higher density. Assuming isentropic compression at $T = 0$, determine energy per unit mass needed for the compression. Compare with energy needed for heating of plasma with ideal gas equation of state (EoS) to ignition temperature 5 keV.

Solution

number of electrons per unit mass

$$N_m = \frac{1}{m_p + 3/2 m_n + m_e} \simeq \frac{1}{2.5\ m_p}$$

(m_p, m_n, m_e are masses of protons, neutron and electron, respectively)
electron density

$$n_e = \rho\ N_m = \frac{\rho}{2.5 m_p}$$

energy of fuel with density ρ (ion energy is negligible)

$$E^d\left(\rho\right) = N_m\ \frac{3}{5}\ \varepsilon_F = \frac{1}{2.5\, m_p}\ \frac{3}{5}\ \frac{h^2}{2\, m_e}\left(\frac{3\ \rho}{8\pi\ 2.5\, m_p}\right)^{2/3}$$

energy needed for compression

$$E_{dk} = E_d\left(\rho = 250\ \mathrm{g/cm^3}\right) - E_d\left(\rho = 0.25\ \mathrm{g/cm^3}\right) = 1.3\times 10^{10}\ \mathrm{J/kg}\ \ (13\ \mathrm{kJ\ for\ 1\ mg})$$

Heating of plasma with ideal gas EoS

$$E^{id} = E_e^{id} + E_i^{id} = (3/2 + 3/2)\ N_m\, k_B\, T = 5.8\times 10^{11}\ \mathrm{J/kg}\ \ (580\ \mathrm{kJ\ for\ 1\ mg})$$

Energy consumption for heating is thus about $40\times$ greater than for compression at $T = 0$. Therefore, ICF research is trying to compress fuel with temperature as low as possible and to heat only small part of fuel to ignition temperature. The rest fuel is heated due to braking of α particles generated in DT fusion.