

## Introduction to plasma physics

**Plasma definition** ([Ich1] S. Ichimaru, Statistical Plasma Physics, Vol I)

Plasma is any statistical system containing mobile charged particles.

*Note* *Statistical means macroscopic. For the scale length  $L$  and the free charge density  $n$ , the relation  $L \gg n^{-1/3}$  holds.*

*Note* *Nearly any system contains some mobile charged particles, but if their impact on the system behavior is negligible, it has no sense to describe it using methods of plasma physics.*

Other **Plasma definition** ([Chen] F.F. Chen, Introduction to Plasma Physics)

Plasma is **quasi-neutral** system of mobile charged (and possibly also neutral) particles that exhibits **collective** behavior.

*Note* *This definition is narrower. Systems are not classified as plasmas if they are not macroscopically neutral even if they behave collectively. So-called **non-neutral plasmas** (e.g. charged particle beams) do not meet this definition.*

*Note.* *Here 2 basic plasma properties are mentioned.*

*Note.* *Collective behavior is thus essential, but need not be dominant. Collective behavior is dominant for **ideal plasmas**.*

## Plasma formation – ionization processes

### 1. Ionization by cosmic rays

*E.g. Ionosphere* – a plasma layer is around the Earth from the height of about 45 km up to the height ca 500 km that is formed due to ionization by cosmic rays (both electromagnetic, and corpuscular), the main source of which is the Sun. Electron density maximum of about  $10^6 \text{ cm}^{-3}$  is in the upper F-layer (above 200 km), where electron temperature is  $T_e \cong 1500 \text{ K}$ .

*Note.* Certain amount of mobile charge particles is in any matter around us due to the action of ionizing radiation. Even at a small height above the sea level, 10 free electrons and positive ions are formed per second in air in nature. A fraction of electrons attach to molecules and **negative ions** are created. Typically, there are  $\sim 10^3$  positive ions in  $\text{cm}^3$  in the outside air and the ratio of the positive to the negative ion number is about 1.15.

Ionization is due to the photon action is called **photoionization**. Photon energy must be higher than the ionization energy  $U_i$  ( $\hbar\omega > U_i > 3.9 \text{ eV}$ ). Multiphoton ionization possible only for radiation intensities  $> 10^{11} \text{ W/cm}^2$

## 2. Ionization in electric field (collisional)

If gas is placed in a strong electric field, existing free electrons are accelerated and when they gain a sufficient energy to cause ionization during collision by separation of outer orbital electrons of neutral atoms or molecules. Electrons released by **impact ionization (collisional ionization)** are again accelerated by the electric field and **ionization avalanche** is formed. Thus, **electric discharge** occurs.

E.g. in a glow discharge at the pressure of 1 Torr, the electron density is  $n_e = 10^9 - 10^{11} \text{ cm}^{-3}$  and electron temperature is  $T_e \cong 10^4 \text{ K}$ .

## 3. Ionization by heating

Without using electric field, a plasma state may be attained by raising the temperature of a neutral gas. Electron thermal energy is 1 eV at temperature 11600 K. Binding energy of outer electrons in an atom/molecule is a few eV, and thus the thermal energy of free electrons is sufficient for impact ionization when electron temperature is  $10^4 - 10^5 \text{ K}$ . Moreover, the energy of emitted photons is then sufficient for **photoionization**. In the thermodynamic equilibrium ionization is given by **ionization equilibrium**.

## Ionization equilibrium - Saha equation [SI units]

$$\frac{n_i n_e}{n_n} = 2.4 \times 10^{21} T^{3/2} \exp\left(-\frac{U_i}{k_B T}\right) \quad (1)$$

Boltzmann constant is  $k_B = R/N_A = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}$ , and thus  $k_B T = 1 \text{ eV}$  at the temperature  $T = 11600 \text{ K}$ , ionization potential is for instance for nitrogen atom  $U_i = 14.5 \text{ eV}$  (15.58 for molecule  $\text{N}_2$ ), for Argon it is  $U_i = 15.76 \text{ eV}$ .

The atom density in pure argon at the standard atmospheric pressure and temperature  $0^\circ\text{C}$  (Loschmidt constant) is  $n_0 = 2.6868 \times 10^{25} \text{ m}^{-3} = n_n + n_i \approx n_n$  and according to (1) equilibrium ionization is  $n_i/n_n = 2.9 \times 10^{-146}$ . Even at temperature of  $1 \text{ eV}$ , the argon ionization is  $n_i/n_n \approx 0.004$ .

*Note. Plasma temperature is mostly high, and thus is usually given in eV or in keV. It is also practical for the comparison of temperature with ionization energies. At higher temperatures plasma is often multiply ionized.*

## 4. Pressure ionization

At higher densities, the orbital radius of valence electrons may be  $\geq$  inter-atomic distances and then valence electrons may be freed at room temperature.

In metals at room temperature, the density of free electrons is of order  $10^{23} \text{ cm}^{-3}$ . Electron Fermi energy  $E_F$  is at such density

$$E_F = \frac{\pi^2 \hbar^2}{2 m_e} \left( \frac{3 n_e}{\pi} \right)^{2/3} \cong 7.9 \text{ eV} \gg T_e \quad , \quad (2)$$

and thus electron gas in metals is degenerate. The ratio  $\Theta = T/E_F$  is called the **degeneration parameter**. For degenerate electron gas ( $\Theta \ll 1$ )  $E_F$  is a good estimate of electron kinetic energy.

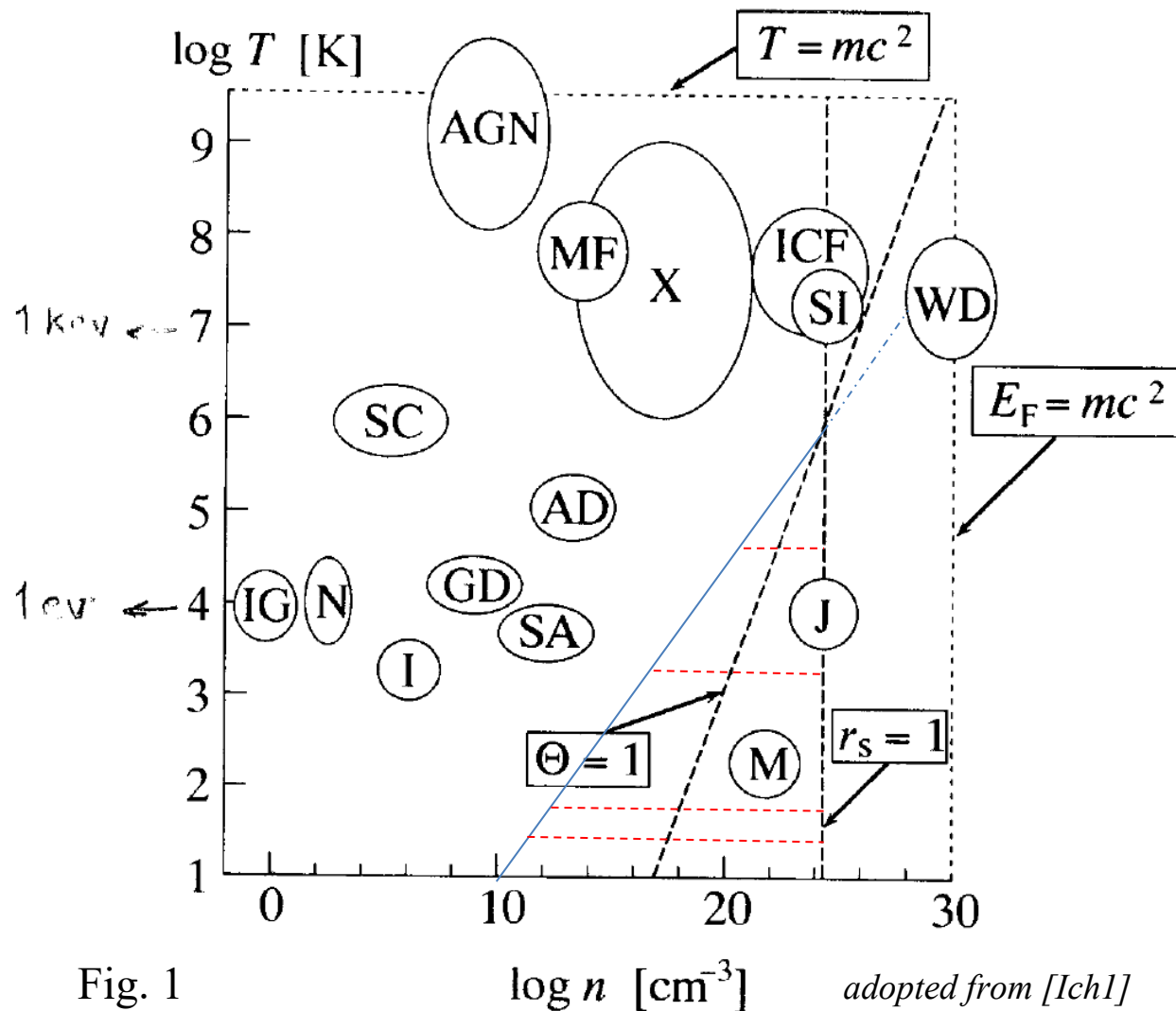
In semiconductors, the density of free electrons and holes is much lower.

Typical example of plasma originated by pressure ionization is the interior of burnt-out star. It is compressed to such high density that electron Fermi energy is  $\gg$  binding energy of all electrons in an atom, and consequently all atoms are fully ionized.

**One-component plasma (OCP)** approximation – system of single species of charged particles embedded in a uniform background of neutralizing charges.

*Note. Properties of electrons and ions may differ considerably; therefore focus to one species of charged particles is sometimes useful.*

# Typical values of electron density and temperature of some plasmas



IG – interstellar gas  
 N – gaseous nebula  
 I – ionosphere  
 GD – glow discharge  
 SA – sun atmosphere  
 AD – arc discharge  
 SC – sun corona  
 AGN – active galactic nucleus  
 MF – magnetic fusion  
 X – X-ray star  
 ICF – inertial confinement fusion  
 SI – sun interior  
 degenerate M – metal  
 J – Jovian interior  
 WD – white dwarf

$r_s$  is the ratio of mean inter-electronic distance to Bohr radius  $a_B = 5.29 \times 10^{-9}$  cm

## Coupling parameter, weakly and strongly coupled plasma

Coupling parameter for OCP is the ratio of Coulomb energy at the average interparticle distance to their average kinetic energy  $\max(3/2 k_B T, E_F)$ .

The average distance  $R$  of particles of the density  $n$  is

$$R = \left( \frac{3}{4\pi n} \right)^{1/3} . \quad (3)$$

For ions  $R_i$  is usually called ion sphere radius or also Wigner-Seitz radius. Ion sphere contains all bound and free electrons belonging to particular ion  $\Rightarrow$  atomic physics description for dense plasmas

For degenerate electrons coupling parameter  $\Gamma_e$  is expressed, as follows

$$\Gamma_e = \frac{e^2}{4\pi\epsilon_0 R_e E_F} = \frac{2^{7/3}}{3^{4/3} \pi^{2/3}} \left( \frac{3}{4\pi n_e} \right)^{1/3} \frac{m_e e^2}{4\pi\epsilon_0 \hbar^2} = 0.543 \frac{R_e}{a_B} = 0.543 r_S , \quad (4)$$

where  $a_B$  is the Bohr radius. Average electron distance is equal to the Bohr radius for density  $n_e = 1.6 \times 10^{24} \text{ cm}^{-3}$ . Coupling parameter of degenerate electrons decreases with density!!

For classical plasmas (particles with charge  $Ze$ ) the expression is

$$\Gamma = \frac{(Ze)^2}{4\pi\epsilon_0 RT} = 0.0027 Z^2 \left( \frac{n}{10^{18} \text{ cm}^{-3}} \right)^{1/3} \left( \frac{10^6 \text{ K}}{T} \right), \quad (5)$$

and thus coupling parameter rises with density and decreases with temperature. For electrons and hydrogen ions  $\Gamma = 0.543$  at the blue line in the previous figure. Electrons are thus strongly coupled only in red hatched triangle. If ions are fermions, Fermi energy is defined, but it is very small. So ions are not degenerate. Hydrogen ions are strongly coupled everywhere below the blue line.

We will mostly study classical weakly coupled plasmas. Especially for multiply ionized plasmas, it is more probable that ions are strongly coupled. Consequently, **ion coupling parameter  $\Gamma_i$**  is used as coupling indicator.

In weakly coupled plasmas, mutual potential energy of particles is small compared to their kinetic energy, and thus their thermodynamic properties are close to a gas and the equation of state can often be approximated by **equation of state of ideal gas**.



## Plasma properties

*(suggested reading [Nich] D.R. Nicholson, Introduction to Plasma Theory chap. 1; alternatively [Chen] chap. 1)*

### Quasi-neutrality

System is quasi-neutral, if the total charge in volumes comparable with the cube of its scale length  $L$  is much less than total charge of all positive charges (and absolute value of total negative charge).

*Note. The scale length  $L$  of the system must be much greater than the distance, to which negative charges may be separated from positive charges (usually electrons from ions).*

Certain energy is needed for the separation of charges of the opposite signs. Macroscopic charge clouds may separate only to the distance, where their thermal energy is fully converted to the potential energy.

Simple physical model – what is the maximum thickness  $\Delta$  of an infinite planar electron layer that can move against static ions by its full thickness? (classical statistics is assumed)

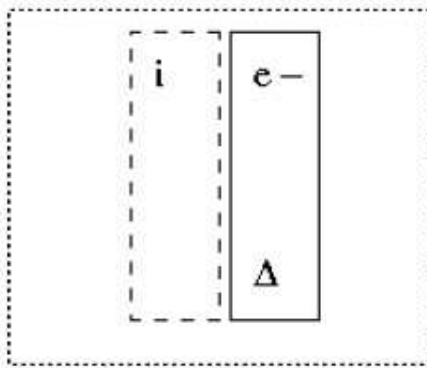


Fig. 2 Shift of  $e$  layer

Planar capacitor emerges with surface charge density  $\sigma$  and electric field  $E$  is inside

$$\sigma = -e n_e \Delta \quad E = \sigma / \epsilon_0$$

Electron potential energy is equal to its thermal energy

$$U_{pot} = -e E \Delta = \frac{e^2 n_e \Delta^2}{\epsilon_0} = k_B T_e$$

This  $\Delta$  is called **electron Debye length**  $\lambda_{De}$

$$\lambda_{De} = \Delta = \left( \frac{\epsilon_0 k_B T_e}{n_e e^2} \right)^{1/2} \quad (6)$$

Electron Debye length grows with the square root of electron temperature  $T_e$  and decreases with the square root of electron number density  $n_e$ .

Thus, plasma is quasi-neutral at the distances that are significantly larger than the Debye length; the quasi-neutrality condition is a scale length  $L \gg \lambda_{De}$ .

## Debye screening

Static charge is screened in plasmas, because it attracts opposite charges and keeps away charges of the same sign.

*Note. Debye derived screening in the theory of electrolytes.*

We shall assume that the electron temperature  $T_e$  need not be in general equal to the ion temperature  $T_i$ . This is frequent in plasmas as (we show it later) the energy transfer between electrons and ions is rather slow.

We shall assume that plasma may be multiply ionized (in difference from textbooks [*Chen, Nich*]); we denote mean ion charge by  $Z$ . Thus, the electron charge is  $q_e = -e$  and the ion charge is  $q_i = Ze$ .

Electrostatic field around the charge  $q_T$  placed in the coordinate origin is described by Poisson equation

$$\Delta\varphi = -\frac{\rho}{\varepsilon_0} = \frac{e}{\varepsilon_0}(n_e - Zn_i) - \frac{q_T}{\varepsilon_0} \delta(\vec{r}) \quad (7)$$

Let in  $\infty$  (where  $\varphi = 0$ ) the charge density is  $\rho = 0$ . Thus,  $n_e = n_0 = Zn_i$  in  $\infty$ .

The thermal electron energy must greater than the Fermi energy so that the Boltzmann statistics for electrons would be applicable. Thus,

$$k_B T_e > E_F = \frac{\pi^2 \hbar^2}{2 m_e} \left( \frac{3 n_e}{\pi} \right)^{2/3}$$

*Note. In metals, typical electron density is  $n_e = 10^{29} \text{ m}^{-3}$ , then  $E_F = 7.9 \text{ eV}$ , for singly ionized gas of density  $n_e = 2.7 \times 10^{25} \text{ m}^{-3}$  is  $E_F = 0.038 \text{ eV} = 440 \text{ K}$ .*

In Boltzmann statistics, the probability of state occupation is  $\sim \exp(-U/k_B T) \Rightarrow$

$$n_e = n_0 \exp\left(\frac{e \varphi}{k_B T_e}\right) \quad n_i = \frac{n_0}{Z} \exp\left(-\frac{Z e \varphi}{k_B T_i}\right) \quad (8)$$

Electron and ion densities may now be substituted into the Poisson equation.

We shall simplify equation by linearization, we shall assume potential energy  $\ll$  kinetic. For  $|x| \ll 1$ , it holds  $\exp(x) \approx 1 + x$  and equation (7) is converted to

$$\Delta \varphi = \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\varphi}{dr} \right) = \frac{e^2 n_0}{\epsilon_0} \left( \frac{1}{T_e} + \frac{Z}{T_i} \right) \varphi \quad \text{pro } r \neq 0 \quad (9)$$

After substitution  $\varphi = \tilde{\varphi} / r$  Poisson equation has the form  $\frac{d^2 \tilde{\varphi}}{dr^2} = \frac{\tilde{\varphi}}{\lambda_D^2}$

Potential of a static charge  $q_T$  in plasma is thus expressed, as follows

$$\varphi = \frac{q_T}{4 \pi \epsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right) \quad (10)$$

**At the distance  $\lambda_D$  potential is screened to 1/e of its vacuum value.**

Screening is the sum of electron screening with  $\lambda_{De}$  and ion one with  $\lambda_{Di}$ .

**Debye length  $\lambda_D$  is**

$$\lambda_D^{-2} = \lambda_{De}^{-2} + \lambda_{Di}^{-2} \quad \lambda_{De} = \sqrt{\frac{k_B T_e \epsilon_0}{n_e e^2}} \quad \lambda_{Di} = \sqrt{\frac{k_B T_i \epsilon_0}{n_i Z^2 e^2}} = \sqrt{\frac{k_B T_i \epsilon_0}{n_e Z e^2}} \quad (11)$$

For  $T_e > T_i/Z$  ion screening of a static charge dominates.

There is certain screening around every charged particle in plasma, so called dynamic screening. The static ion screening occurs only if the particle velocity is  $\ll$  ion thermal velocity. If the particle is faster than the thermal ions, but much slower than the thermal electron velocity, electron static screening is formed, but ion screening is  $\ll$  than for a static charge.

Assumption included in the derivation

- We have used densities of charged particles that can be used with reasonable accuracy only when the distances ( $\lambda_D$  in this case) are large compared to average interparticle distance. It is usually required that number  $N_D$  of electrons in electron Debye sphere

$$N_D = \frac{4\pi}{3} \lambda_{De}^3 n_e = \frac{4\pi}{3} \frac{\epsilon_0^{3/2} k_B^{3/2} T_e^{3/2}}{e^3 n_e^{1/2}} \gg 1 \quad (12)$$

Quantity  $N_D$  or its some small multiple is called **plasma parameter**. For  $N_D \gg 1$  plasma is ideal and screening is collective process.

*Note.* When  $N_D < 1$ , screening also exists, but its fluctuations are  $>$  mean screening value.

- When Poisson equation was linearized, potential energy of charged particles  $|e\phi| \ll$  than their thermal energy  $k_B T_e$  was assumed. Surely, this does not hold near to origin, but even the previous assumption is not valid there. It is enough to assume that  $q_T$  is so small that the inequality holds at the average distance among electrons  $R_e = [3 / (4\pi n_e)]^{1/3}$ .

## Collective behavior

By the term collective behavior one denotes mutual particle interactions via **macroscopic** electromagnetic field in contrast to **microscopic fields** that cause particle interaction during binary collisions.

Due to screening, binary interactions are efficient in plasmas only up to the distance of Debye length, but interactions on longer distances are present in plasmas due to macroscopic electromagnetic fields formed by macroscopic collective charges and currents. Fluctuations with wavelength  $>$  Debye length have mainly collective character, while short-wavelength fluctuations are mainly controlled by motion of individual particles with the predominance of binary interactions (more in detail in the book by *Ichimaru*).

The speed of system variations due to binary collisions is given by the collision frequency  $\nu_c$ . The significance of binary interactions increases with the collision frequency  $\nu_c$ .

There exists a large variety of collective motions in plasmas, but the fastest is the motion of electron cloud with respect to ions due to their mutual attraction. For simplicity we consider ions as static homogeneous neutralizing background (OCP approximation).

We use the model of planar layers again (Fig. 2). The velocity of ordered electron motion is  $v = d\Delta / dt$  and the electron equation of motion is

$$m_e \frac{dv}{dt} = -eE = -\frac{e^2 n_e \Delta}{\epsilon_0} \Rightarrow \frac{d^2 \Delta}{dt^2} = -\frac{e^2 n_e}{\epsilon_0 m_e} \Delta \quad (13)$$

Thus, plasma oscillations occur with the **electron plasma frequency**

$$\omega_{pe} = \sqrt{\frac{e^2 n_e}{\epsilon_0 m_e}} \quad (14)$$

**Electron plasma frequency**  $\omega_{pe}$  characterizes the strength of the collective action and for  $\omega_{pe} > \nu_c$ , the collective behavior dominates in the plasma.

The electron plasma frequency  $\omega_{pe}$ , the electron Debye length  $\lambda_{De}$  and the electron thermal velocity  $v_{Te}$  meet the simple relation

$$v_{Te} = \sqrt{k_B T_e / m_e} = \omega_{pe} \lambda_{De}$$

*Note.* When one includes also ion motion, then the frequency of plasma oscillations is  $\omega_p^2 = \omega_{pe}^2 + \omega_{pi}^2$ , where  $\omega_{pi}^2 = Z^2 e^2 n_i / (\epsilon_0 M_i) = Z \omega_{pe}^2 m_e / M_i$ .



## Collision frequency of charged particles

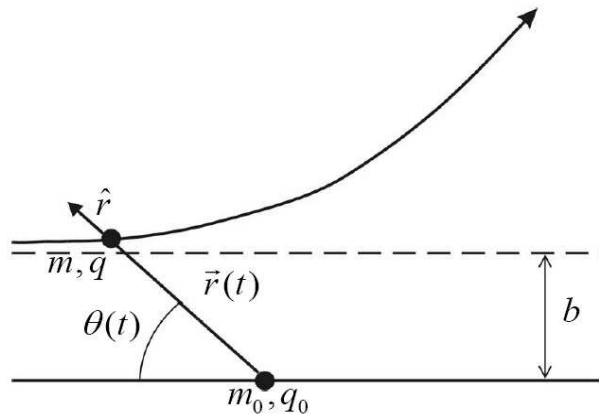


Fig. 3 Collision schematics ( $\hat{r}$  unit vector in direction  $\vec{r}$ ,  $b$  collision parameter)

We shall assume for simplicity that the component of velocity  $v_0$  in the direction of motion of flying-in particle before collision is constant (valid for large  $b$  when the alteration of the particle motion is small).

The normal component of particle momentum is obtained by temporal integration of force impulse

$$m v_{\perp} = \int_{-\infty}^{\infty} F_{\perp}(t) dt \quad .$$

The normal force component is expressed via the relation

$$F_{\perp} = \frac{q q_0}{4 \pi \epsilon_0 r^2} \sin \theta = \frac{q q_0}{4 \pi \epsilon_0 b^2} \sin^3 \theta \quad ,$$

where the relation  $r = b/\sin\theta$  was utilized.

Time dependence of  $F_{\perp}$  is given by the dependence on angle  $\theta$ . A steady motion in the direction  $x$  of incident particle is assumed, and thus  $t = x/v_0 = -r \cos\theta/v_0 = -b \cos\theta/(v_0 \sin\theta)$  and  $dt = b d\theta / (v_0 \sin^2 \theta)$ . Consequently

$$v_{\perp} = \frac{q q_0}{4 \pi \varepsilon_0 m b^2} \int_{-\infty}^{\infty} \sin^3 \theta(t) dt = \frac{q q_0}{4 \pi \varepsilon_0 m b v_0} \int_0^{\pi} \sin \theta d\theta = \frac{v_0 b_0}{b} ,$$

where  $b_0$  is the **Landau length**  $b_0 = q q_0 / (2 \pi \varepsilon_0 m v_0^2)$

The collision parameter  $b_0$  corresponds to scattering to  $90^\circ$ , thus to the loss of original direction of velocity. The effective cross-section for  $\geq 90^\circ$  is  $\sigma = \pi b_0^2$ .

The **collision frequency** (for large-angle scattering) is thus

$$\nu_L = \pi n_0 v_0 b_0^2 = \frac{n_0 q^2 q_0^2}{4 \pi \varepsilon_0^2 m^2 v_0^3} \quad (15)$$

### Small-angle scattering

Electrostatic field – long-range force –sum of small-angle scatterings often dominates over large-angle scattering.

The loss of original direction of motion happens probably due to many small changes of the velocity vector earlier than one large-angle scattering event occurs.

The collision frequency is then defined as 1 over the average time in which the particle loses the original direction of velocity.

History of the particle motion may be considered a random walk in the velocity space. If  $N$  collisions happen in a certain time interval, the variation of e.g.  $y$  component of velocity is

$$\Delta \mathbf{v}_y = \Delta \mathbf{v}_{y1} + \Delta \mathbf{v}_{y2} + \dots + \Delta \mathbf{v}_{yN} \quad ,$$

and the average value is  $\langle \Delta \mathbf{v}_y \rangle = \langle \Delta \mathbf{v}_{yi} \rangle = 0$ . As individual collisions may be considered uncorrelated, the dispersion of  $v_y$  can be expressed

$$D_{v_y} = \left\langle (\Delta \mathbf{v}_y)^2 \right\rangle = \left\langle \left( \sum_{i=1}^N \Delta \mathbf{v}_{yi} \right)^2 \right\rangle = \sum_{i=1}^N \left\langle (\Delta \mathbf{v}_{yi})^2 \right\rangle = N \left\langle (\Delta \mathbf{v}_{y1})^2 \right\rangle$$

For one collision with collision parameter  $b$  there holds

$$\left\langle v_{\perp}^2 \right\rangle = \left\langle (\Delta \mathbf{v}_y)^2 \right\rangle + \left\langle (\Delta \mathbf{v}_z)^2 \right\rangle = \frac{v_0^2 b_0^2}{b^2} \quad \Rightarrow \quad \left\langle (\Delta \mathbf{v}_{y1})^2 \right\rangle = \frac{v_0^2 b_0^2}{2b^2}$$

Number of collisions with parameter in the interval  $db$  is  $dN = n_0 v_0 2\pi b db$  and thus the total dispersion of the normal velocity component is expressed

$$\frac{d}{dt} \left\langle (\Delta \mathbf{v}_y)_{tot}^2 \right\rangle = \pi n_0 v_0^3 b_0^2 \int \frac{db}{b} = \pi n_0 v_0^3 b_0^2 \ln \frac{b_{\max}}{b_{\min}}$$

We had to restrict the diverging integral. The lower boundary is due to the assumption of small-angle scattering, and this is not valid for collision parameters  $b$  less than Landau length  $b_0$ . The assumption of Coulomb interactions does not hold for large collision parameters  $b$  as the field is reduced by the Debye screening, therefore the upper boundary is  $b_{\max} = \lambda_{De}$ .

Let us denote  $\Lambda$  this ratio for electron collision with thermal velocity  $v_{Te}$

$$\Lambda = \frac{\lambda_{De}}{b_0} = \frac{2 \pi \epsilon_0 \lambda_{De} m_e v_{Te}^2}{e^2} = 2 \pi n_e \lambda_{De}^3 = \frac{3}{2} N_D \quad (16)$$

If the plasma parameter  $N_D$  is large, then  $\Lambda$  is also large. The quantity  $\ln \Lambda$  is called the **Coulomb logarithm**. It is the ratio of collision frequency due to small-angle scattering to collision frequency of scattering to angles  $\geq 90^\circ$ .

Collision frequency for collisions of electrons of velocity  $v_0$  with electrons is

$$\nu_{ee} = \frac{8 \pi n_0 e^4}{(4 \pi \epsilon_0)^2 m_e^2 v_0^3} (1 + \ln \Lambda) \approx \frac{8 \pi n_0 e^4}{(4 \pi \epsilon_0)^2 m_e^2 v_0^3} \ln \Lambda \quad (17)$$

Collision frequency of Coulomb collisions is  $\sim v^{-3}$  and mean free path is  $\sim v^4$ , thus relatively fast electrons from velocity distribution tail do not collide frequently and they can fly relatively long distance without direction alteration.

The collision frequency of electrons with thermal velocity  $v_0 = v_{Te} = (k_B T_e / m_e)^{1/2}$  is called the **effective collision frequency**

$$\nu_c = \frac{8 \pi n_e e^4}{(4 \pi \epsilon_0)^2 m_e^{1/2} (k_B T_e)^{3/2}} (1 + \ln \Lambda) \approx \frac{8 \pi n_e e^4}{(4 \pi \epsilon_0)^2 m_e^{1/2} (k_B T_e)^{3/2}} \ln \Lambda \quad (18)$$

The ratio of the effective collision frequency to the plasma frequency is

$$\frac{\nu_c}{\omega_{pe}} = \frac{1}{2 \pi n_0 \lambda_{De}^3} \frac{\ln \Lambda}{3 N_D / 2} = \frac{\ln(3 N_D / 2)}{3 N_D / 2} \quad (\ll 1 \text{ for } N_D \gg 1) \quad (19)$$

For large  $N_D$ , the collective behavior characterized by  $\omega_{pe}$  dominates over the impact of binary interactions characterized by  $\nu_c$ . Such plasma is called **ideal plasma**. Some phenomena can be then described in the approximation of **collisionless plasma**.

**Ideal plasma** – is **quasi-neutral** and **collective** effects caused by macroscopic charges and currents **dominate** in it.

## The ratio of the potential to the kinetic energy

Let's compare electron energy in field of the nearest electron, placed in the distance  $R_e = [3/(4\pi n_e)]^{1/3}$  with its kinetic energy (*non-degenerate plasma is assumed*)

$$W_p \approx \frac{e^2}{4\pi \epsilon_0 R_e} = \frac{e^2 n_e^{1/3}}{3^{1/3} (4\pi)^{2/3} \epsilon_0} \quad W_k \approx \frac{3}{2} k_B T_e$$

$$\frac{W_p}{W_k} \approx \frac{2}{9} \left( \frac{3}{4\pi \epsilon_0^{3/2} k_B^{3/2} T_e^{3/2}} \right)^{2/3} = \frac{2}{9 N_D^{2/3}} \quad (20)$$

In ideal plasma  $N_D \gg 1$  and kinetic energy of particles is thus  $\gg$  their binding (potential) energy. **Ideal plasma is weakly coupled.** Ideal plasma is in this respect similar to a gas, one often speaks about *ionized gas*. The equation of state of ideal gas is then a good approximation of electron equation of state in ideal plasma. If even ions are weakly coupled ( $\Gamma_i \ll 1$ ), then the equation of state of ideal gas may be used also for ions.

## Number of particles (electrons + ions) in Debye sphere of radius $\lambda_D$

Adopted from R.P. Drake, *High-Energy-Density Physics*, Springer 2006

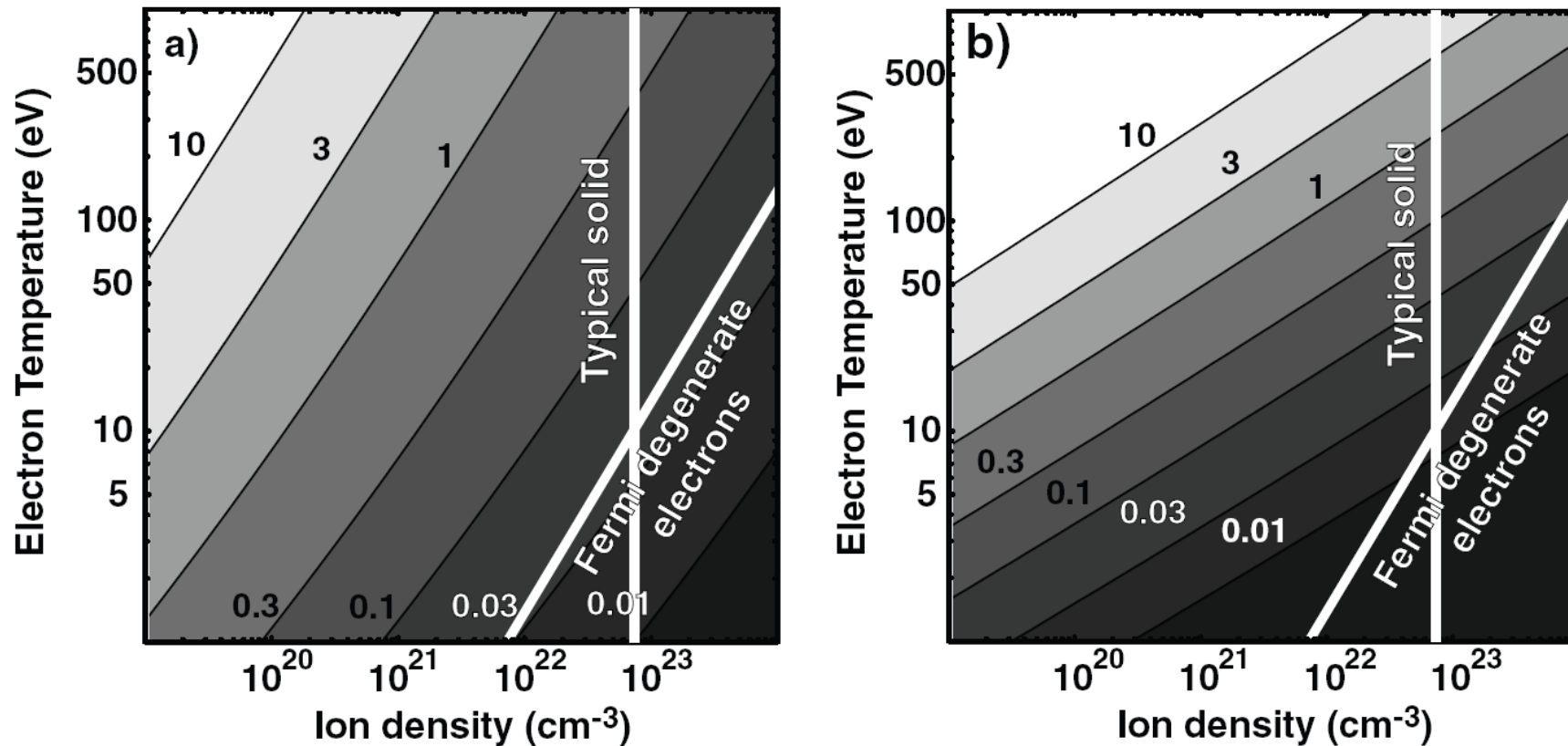


Fig. 4 (a) Plasma of materials with high atomic number, where mean ion charge  $Z = 0.63 \sqrt{T_e}$  is assumed ( $T_e$  is in eV).

(b) Plasma of materials with low atomic number, where mean ion charge  $Z=4$  is assumed

## Various types of plasmas

### Plasma in nature

**Ideal** - discharges; ionosphere; solar wind; outer layers of stars; interstellar gas

**Ideal or non-ideal** – star interiors (*center of sun is nearly ideal plasma*  
 $\rho = 150 \text{ g/cm}^3$ ,  $T = 1.35 \text{ keV}$ ,  $\Gamma = 0.14$ )

**Non-ideal** - electron gas in metals (degenerate plasma), electrolytes, centers of large planets (Jovian planets)

### Plasma in laboratory

**Ideal** - discharges of various types (vacuum tubes, discharges for gas laser pumping, pinches, capillary discharge); MHD generators; ion engines, laser plasma from gas targets

**Ideal or non-ideal** - laser plasma from solid (or liquid) targets

**Non-ideal** – supercold plasma (plasma with temperature ca 1 K can be obtained by non-linear photoionization of laser-cooled vapor, electron densities of  $10^6$ - $10^9 \text{ cm}^{-3}$ )



**APPROXIMATE MAGNITUDES  
IN SOME TYPICAL PLASMAS**

Plasma Type	$n \text{ cm}^{-3}$	$T \text{ eV}$	$\omega_{pe} \text{ sec}^{-1}$	$\lambda_D \text{ cm}$	$n\lambda_D^3$	$\nu_{ei} \text{ sec}^{-1}$
Interstellar gas	1	1	$6 \times 10^4$	$7 \times 10^2$	$4 \times 10^8$	$7 \times 10^{-5}$
Gaseous nebula	$10^3$	1	$2 \times 10^6$	20	$10^7$	$6 \times 10^{-2}$
Solar Corona	$10^9$	$10^2$	$2 \times 10^9$	$2 \times 10^{-1}$	$8 \times 10^6$	60
Diffuse hot plasma	$10^{12}$	$10^2$	$6 \times 10^{10}$	$7 \times 10^{-3}$	$4 \times 10^5$	40
Solar atmosphere, gas discharge	$10^{14}$	1	$6 \times 10^{11}$	$7 \times 10^{-5}$	40	$2 \times 10^9$
Warm plasma	$10^{14}$	10	$6 \times 10^{11}$	$2 \times 10^{-4}$	$10^3$	$10^7$
Hot plasma	$10^{14}$	$10^2$	$6 \times 10^{11}$	$7 \times 10^{-4}$	$4 \times 10^4$	$4 \times 10^6$
Thermonuclear plasma	$10^{15}$	$10^4$	$2 \times 10^{12}$	$2 \times 10^{-3}$	$10^7$	$5 \times 10^4$
Theta pinch	$10^{16}$	$10^2$	$6 \times 10^{12}$	$7 \times 10^{-5}$	$4 \times 10^3$	$3 \times 10^8$
Dense hot plasma	$10^{18}$	$10^2$	$6 \times 10^{13}$	$7 \times 10^{-6}$	$4 \times 10^2$	$2 \times 10^{10}$
Laser Plasma	$10^{20}$	$10^2$	$6 \times 10^{14}$	$7 \times 10^{-7}$	40	$2 \times 10^{12}$

Typical parameters of various types of plasmas – here always  $n \lambda_D^3 > 1$  and  $\omega_{pe} > \nu_{ei}$ .

Fig. 5 Typical temperatures and densities of various plasma types

