INSTITUTE OF PLASMA PHYSICS
OF THE CZECH ACADEMY OF SCIENCES

Introduction to gyro-fluid turbulence in tokamaks

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MINISTRY OF EDUCATION,
YOUTH AND SPORTS

- Small-scale structures in turbulence
- From single-particle to fluid models
- From gyrokinetics to gyrofluid models
- Gyro-Landau fluid models (GLF, TGLF)

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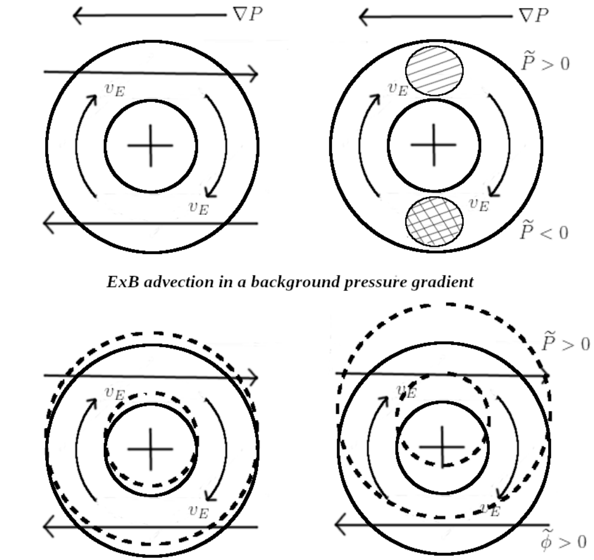
Drive of turbulent modes

- Departure of the distribution function from thermodynamic equilibrium (Maxwell-Boltzmann) provides drive for instabilities
- The combination of drift particle motions and pressure gradients can drive turbulent modes (drift-wave driven turbulence)
- Note: diamagnetic motions do not cause advection (diamagnetic cancellation)



nearly-adiabatic electrons = small net transport
large phase shift between $\tilde{P}$ and $\tilde{\phi}$ = grandient provides drive for perturbations

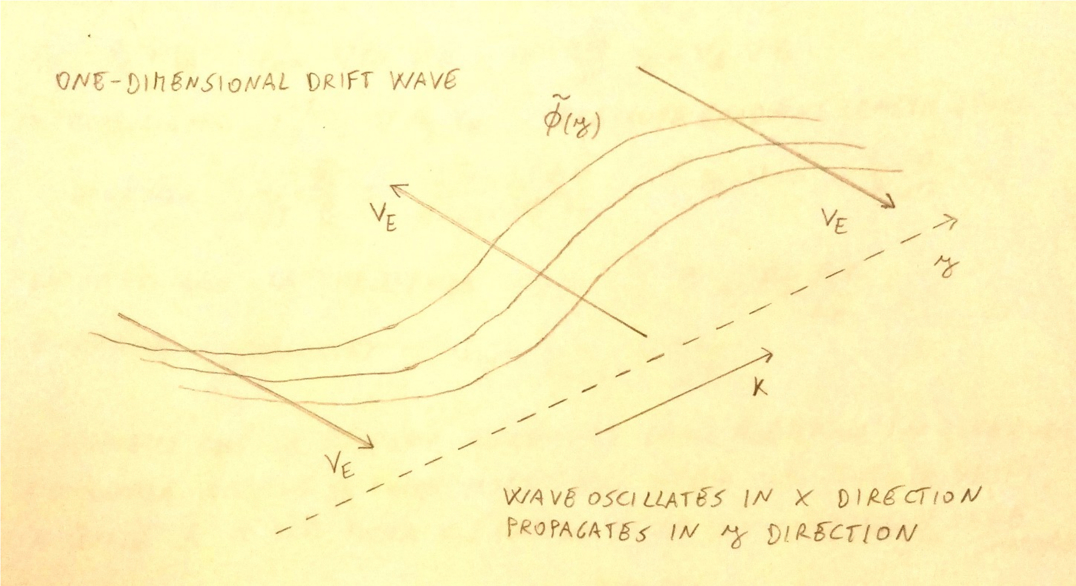
Horton et al., AIP Conference Proceedings (2008)

Drift wave mechanism

$$\frac{\partial p_e}{\partial t} + \boldsymbol{v}_E \cdot \nabla p_e = 0$$

$$\frac{\partial \tilde{p}_e}{\partial t} = -\boldsymbol{v}_E \cdot \nabla p_e$$

$$\boldsymbol{v}_E|_x = -\frac{c}{B}\frac{\partial \tilde{\phi}}{\partial y}$$



$$\frac{\partial}{\partial t}\frac{\tilde{p}_e}{p_e} = -\frac{cT_e}{eBL_p}\frac{\partial}{\partial y}\frac{e\tilde{\phi}}{T_e}$$

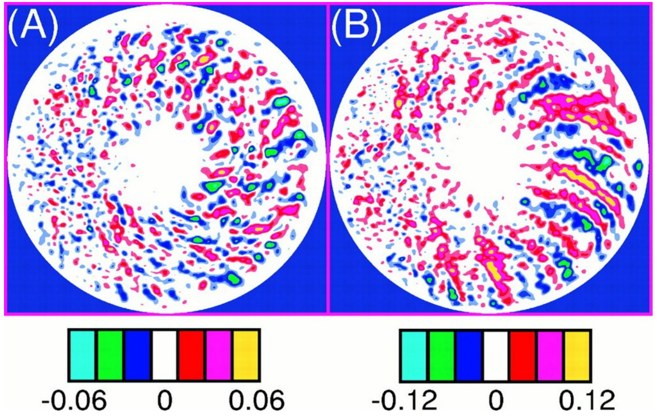
$$v_D = \frac{cT_e}{eBL_p}, \quad \frac{\tilde{p}_e}{p_e} = \frac{e\tilde{\phi}}{T_e}$$

$$\left(\frac{\partial}{\partial t} + v_D\frac{\partial}{\partial y}\right)\frac{e\tilde{\phi}}{T_e} = 0$$

Structure of turbulent modes

Examples: ion temperature gradient (ITG), electron temperature gradient (ETG)

- ETG lead to formation of elongated radial streamers
- ITG lead to formation of zonal flows
- Streamers cause large cross-field transport while zonal flows reduce it

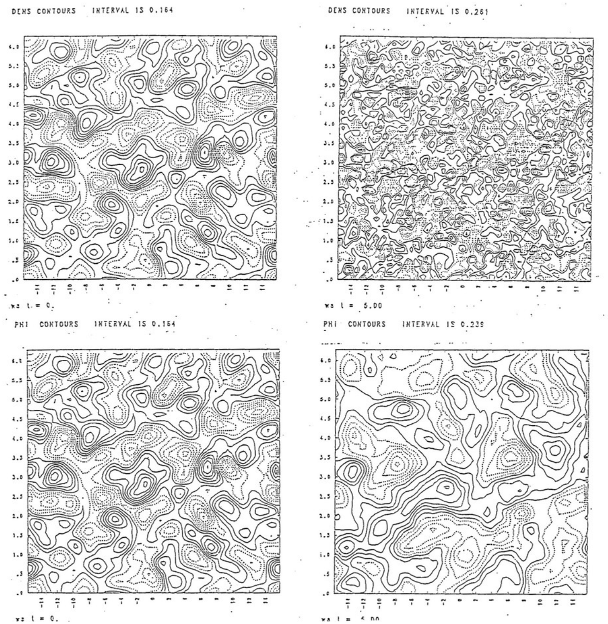


Horton et al., New Journal of Physics (2003)
Li et al., Science (1998)

Energy cascade in turbulence

- Turbulence is a non-equilibrium, dissipative system with many degrees of freedom, with short correlation times and lengths
- Nonlinear mode coupling transfers energy from large to small scales (direct cascade)
- Conservation of enstrophy (mean square vorticity) causes transfer of energy from small to large scales (inverse cascade)

Gang, Scott & Diamond, Physics of Fluids B (1989)



Density

Electrost. potential

Energy cascade in turbulence

Turbulence is the saturated state that follows from the incoherent interaction between the triplets of modes that take part in the three-wave coupling process

$$\frac{\partial \tilde{p}_{ek}}{\partial t} \propto \sum_{k',k''} \int k^2 e^{i(k+k'+k'')x} k' \wedge k'' \tilde{\phi}_{-k''} \tilde{p}_{e-k'} dx$$

The direct cascade process stops at some small scale, determined by dissipation; the inverse cascade, that causes vortex merging, stops at some large scale

Description of turbulence involves dealing with both the small and the large scales

- Small-scale structures in turbulence
- From single-particle to fluid models
- From gyrokinetics to gyrofluid models
- Gyro-Landau fluid models (GLF, TGLF)

From single-particle description to fluid equations

The most fundamental description of plasma dynamics requires single-particle approach

$$m_i \frac{d\boldsymbol{v_i}}{dt} = q_i \left(\boldsymbol{E} + \frac{\boldsymbol{v_i} \wedge \boldsymbol{B}}{c} \right)$$

This approach is computationally expensive → use statistical description

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{v} \cdot \nabla_{\boldsymbol{r}} f_\alpha + \frac{q_\alpha}{m_\alpha} \left(\boldsymbol{E} + \frac{\boldsymbol{v} \wedge \boldsymbol{B}}{c} \right) \cdot \nabla_{\boldsymbol{v}} f_\alpha = C_\alpha$$

$$\nabla \cdot \boldsymbol{E} = 4\pi \sum_\alpha q_\alpha \int f_\alpha d^3 v \qquad \nabla \wedge \boldsymbol{B} = \frac{4\pi}{c} \sum_\alpha q_\alpha \int \boldsymbol{v} f_\alpha d^2 v + \frac{1}{c} \frac{\partial \boldsymbol{E}}{\partial t}$$

From single-particle description to fluid equations

Statistical approach reduces the number of degrees of freedom, but it is still computationally expensive → moments of the distribution function

$$n_\alpha = \int f_\alpha d^3v \qquad \boldsymbol{P}_\alpha = m_\alpha \int \boldsymbol{v}\boldsymbol{v} f_\alpha d^3v$$

$$n_\alpha \boldsymbol{V}_\alpha = \int \boldsymbol{v} f_\alpha d^3v \qquad \boldsymbol{Q}_\alpha = \frac{1}{2} m_\alpha \int v^2 \boldsymbol{v} f_\alpha d^3v$$

When this is applied to the kinetic equation, it produces the hierarchy of fluid equations

Equations of each order depend on quantities of higher order → problem of closure

Fluid models of plasma dynamics

Different fluid models have been deduced:

- Single-fluid MHD
- Two-fluid MHD
- Reduced MHD
- Three-four fields etc.

$$\frac{\partial n}{\partial t} + n\nabla \cdot \boldsymbol{V} = 0$$

$$m_i n \frac{\partial \boldsymbol{V}}{\partial t} = \boldsymbol{J} \wedge \boldsymbol{B} - \nabla p$$

$$n = n_i = n_e$$

$$V \approx V_i$$

One or the other model can be used depending on the different applications

Single-fluid models are appropriate for low-frequency and large-scale dynamics

Fluid models of plasma dynamics

Two-fluid models, three-four field models and more advanced ones can deal with more complicated physics, but the finite Larmor radius (FLR) effects are not included

For edge plasma regions, fluid models work well; for the core region they are not sufficient

Turbulence displays structures on a small scale, comparable to the ion Larmor radius → extensions of fluid equations with FLR effects to address study of turbulence

- Small-scale structures in turbulence
- From single-particle to fluid models
- From gyrokinetics to gyrofluid models
- Gyro-Landau fluid models (GLF, TGLF)

Why use a gyrokinetic equation

Plasma dynamics involves processes which posses very different space and time scales
To accurately resolve the small scales, one needs huge computational power

Zero-order simplification: average over particle gyromotion → neglect scales smaller than Larmor radius ρ and times shorter than the inverse of the gyro-frequency Ω

Gyrokinetic (GK) theory provides a rigorous method for including the FLR effects in the kinetic equation to the lowest order ($\delta = \rho/L = \omega/\Omega \ll 1$)

Purpose of gyrokinetic theory

Describe plasma dynamics at frequencies $\omega \lll \omega_{ci}$ while keeping FLR effects

Principles of gyrokinetics:

- Without field perturbations, the magnetic moment μ is a constant of motion and all physical quantities are independent of the gyro-phase ϕ ; perturbations break the symmetry by introducing a dependence on ϕ $\rightarrow$ μ is no longer a constant of motion
- A change of coordinates is introduced which makes μ a constant of motion again
- This is achieved with Lie-transform perturbation theory, which eliminates the fast dynamics

Gyrokinetic equation

The GK equation can be written as:

$$\frac{\partial f}{\partial t} + (\boldsymbol{b}_0^* v_\parallel + \boldsymbol{v}_d) \cdot \nabla f + \frac{1}{m v_\parallel} (\boldsymbol{b}_0^* v_\parallel + \boldsymbol{v}_d) \cdot (Ze\bar{\boldsymbol{E}} - \mu \nabla (\boldsymbol{B}_0 + \bar{\boldsymbol{B}}_{1\parallel})) \frac{\partial f}{\partial v_\parallel} = 0$$

$$\boldsymbol{b}_0^* v_\parallel + \boldsymbol{v}_d = \left(\boldsymbol{b}_0 + \frac{\bar{\boldsymbol{B}}_{1\perp}}{B_0} \right) v_\parallel + \frac{1}{B_0} \boldsymbol{b}_0 \wedge \bar{\boldsymbol{E}} - \frac{\mu}{ZeB_0} \boldsymbol{b}_0 \wedge \nabla (\boldsymbol{B}_0 + \bar{\boldsymbol{B}}_{1\parallel})$$

$$\bar{\boldsymbol{E}} = J_0(\rho \nabla_\perp) \boldsymbol{E} \qquad \bar{\boldsymbol{B}} = J_0(\rho \nabla_\perp) \boldsymbol{B}$$

The FLR effects enter through the nonlinear operators J_0 that operate on the fields

Brizard, Princeton University (1990)

Why use gyrofluid equations

Solving the GK equation is computationally expensive:

- GYSELA: nonlinear, global GK code, comp. time ~10^6 CPU hours
- GENE, GKW: nonlinear, flux-tube GK codes, comp. Time ~ 10^4 CPU hours

Quasi-linear GK codes or gyrofluid codes were developed

- QuaLiKiz: quasi-linear GK code; TGLF: gyrofluid code, comp. time ~ 1 minute
- GLF23 gyrofluid code, comp. time ~ 1 second

Breton, Aix Marseille Université (2018)

Derivation of gyrofluid equations

Gyrofluid equations → moments of the GK equation

- the distribution function is separated in equilibrium+perturbation: $f = F_M + \tilde{f}$
- terms containing velocity dependent quantities are re-arranged properly
- velocity average of expressions involving J_0 introduce nonlinear operators: $\langle J_0 \rangle \approx \Gamma_0(\rho_i^2 \nabla_\perp^2)^{1/2}$
- Physical quantities are expressed in the coordinates of the particles → pull-back

Example: gyrofluid quasi-neutrality $n_e = \Gamma_0^{1/2} n_i + n_0 \frac{Ze}{T_i}(\Gamma_0 - 1)\phi$

Snyder, Princeton University (1999)

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Gyrofluid models for turbulence

Different systems of gyrofluid equations have been developed to study plasma turbulence

$$\frac{\partial n}{\partial t} + \nabla \cdot \Big(n u_\parallel \hat{\mathbf{b}} + n \left\langle J_0 \right\rangle \mathbf{v}_E \Big) = 0,$$

$$\frac{\partial (n u_\parallel)}{\partial t} + \nabla \cdot \Big((\frac{p_\parallel}{m} + n u_\parallel^2) \hat{\mathbf{b}} + n \left\langle v_\parallel J_0 \right\rangle \mathbf{v}_E \Big) + \frac{e}{m} \hat{\mathbf{b}} \cdot n \left\langle J_0 \right\rangle \nabla \Phi = 0,$$

$$\frac{\partial (p_\parallel + m n u_\parallel^2)}{\partial t} + \nabla \cdot \Big((q_\parallel + 3 p_\parallel u_\parallel + m n u_\parallel^3) \hat{\mathbf{b}} + n \left\langle m v_\parallel^2 J_0 \right\rangle \mathbf{v}_E \Big)$$
$$+ 2e \hat{\mathbf{b}} \cdot n \left\langle v_\parallel J_0 \right\rangle \nabla \Phi = 0,$$

$$\frac{\partial (q_\parallel + 3 u_\parallel p_\parallel + m n u_\parallel^3)}{\partial t} + \nabla \cdot \Big((R_\parallel + 4 u_\parallel q_\parallel + 6 u_\parallel^2 p_\parallel + m n u_\parallel^4) \hat{\mathbf{b}} + n \left\langle m v_\parallel^3 J_0 \right\rangle \mathbf{v}_E \Big)$$
$$+ 3e \hat{\mathbf{b}} \cdot n \left\langle v_\parallel^2 J_0 \right\rangle \nabla \Phi = 0,$$

$$\frac{\partial p_\perp}{\partial t} + \nabla \cdot \Big((q_\perp + n u_\parallel T_\perp) \hat{\mathbf{b}} + n \left\langle \tfrac{1}{2} m v_\perp^2 J_0 \right\rangle \mathbf{v}_E \Big) = 0,$$

$$\frac{\partial (q_\perp + n u_\parallel T_\perp)}{\partial t} + \nabla \cdot \Big((R_\perp + 2 u_\parallel q_\perp + n u_\parallel^2 T_\perp) \hat{\mathbf{b}} + n \left\langle \tfrac{1}{2} m v_\perp^2 v_\parallel J_0 \right\rangle \mathbf{v}_E \Big)$$
$$+ \frac{e}{m} \hat{\mathbf{b}} \cdot \nabla n \left\langle \tfrac{1}{2} m v_\perp^2 J_0 \right\rangle \Phi = 0,$$

Dorland, Princeton University (1993), Snyder, Princeton University (1999)

Gyro-Bessel operators

How to implement nonlinear differential operators?

$$\Gamma_0(b) = I_0(b)e^{-b} \qquad \Gamma_0(b)^{1/2} = I_0(b)^{1/2}e^{-b/2}$$

where $b = k_\perp^2 \rho_i^2$ in Fourier transform; the Taylor expansion holds only in the limit b<<1

$$\Gamma_0(b) \approx 1 - b + \mathcal{O}(b^2) \qquad \Gamma_0(b)^{1/2} \approx 1 - b/2 + \mathcal{O}(b^2)$$

To extend it to the limit b≥1, one can use Padé approximation:

$$\Gamma_0(b) \approx \frac{1}{1+b} \qquad \Gamma_0(b)^{1/2} \approx \frac{1}{1+b/2}$$

In Fourier transform, the differential operators become algebraic functions of $k_\perp^2$

Parallel phase-mixing

Closure of the system of gyrofluid equations: parallel phase-mixing by Landau damping

Landau damping is a collisionless damping mechanism → non-dissipative

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} + \frac{e}{m}E\frac{\partial f}{\partial v} = 0$$

$$f = f_0 + \widetilde{f}, \;\; E = -\frac{\partial\widetilde{\phi}}{\partial z} \;\; \rightarrow \;\; \widetilde{f} = \frac{e\widetilde{\phi}}{m}\frac{\partial f_0/\partial v}{v-\omega/|k|}$$

$$\widetilde{n} = \int \widetilde{f}dv, \;\; T_0 = mv_t^2 \;\; \rightarrow \;\; \widetilde{n} = v_t^2\frac{e\widetilde{\phi}}{T_0}\int\frac{\partial f_0/\partial v}{v-\omega/|k|}dv$$

- Start from collisionless kinetic equation
- Expand f into equilibrium+perturbation
- Solve for perturbed part with Fourier
- Calculate perturbed particle density

Hammett & Perkins, PRL (1990)

Example: two-field system

Plasma dispersion function must be approximated with a multi-pole expansion

For a two-field systém, one can use a two-pole expansion

$$\frac{\partial n}{\partial t} + n_0 \nabla_{\|} u_{\|} = 0$$

$$m n_0 \frac{\partial u_{\|}}{\partial t} + \nabla_{\|} P_{\|} = 0$$

$$P_{\|} = n T_0 + n_0 T_{\|}, \quad T_{\|} = -\mu T_0 \nabla_{\|} (u_{\|} / v_{th})$$

where $\mu = \sqrt{\pi}/2$; for higher-order systems ne needs higher-order pole expansion

Brinca, Journal of Plasmas Physics (1973)

Gyro-Landau fluid (GLF) models

GLF models consist in a linear eigenvalue system obtained from a set of reduced gyrofluid equations
Tested against GK simulations to compare the growth rates

TGLF (Trapped GLF): includes effect of trapped particles, covers the entire spectrum of modes
Nonlinear saturation implemented; main saturation mechanisms are zonal flows and drift-wave mixing

Different saturation rules were implemented: SAT0 → 1D; SAT1 → 2D; SAT2 → 3D
SAT2 is able to match very well the fluxes obtained in nonlinear GK simulations

Waltz et al., PoP (1997); Staebler et al., PoP (2007)

Gyro-Landau fluid (GLF) models

GLF23 and TGLF: solvers for turbulent transport based on gyrofluid equations

GLF23

- Different equations for low-k (ITG, TEM) and high-k (ETG)
- Padé approximation
- Small trapped fraction
- Shifted circular geometry
- Usually only electrostatic
- 4 moment equations per species

TGLF

- All modes calculated from a same set of equations (coupling possible)
- Exact FLR integrals
- All trapped fractions
- Shaped geometry
- Full electromagnetic
- 15 moment equations per species

TGLF is 10-30 times slower than GLF23

Staebler et al., 22nd IAEA Fusion Energy Conference, Geneva (2008)

How does TGLF work

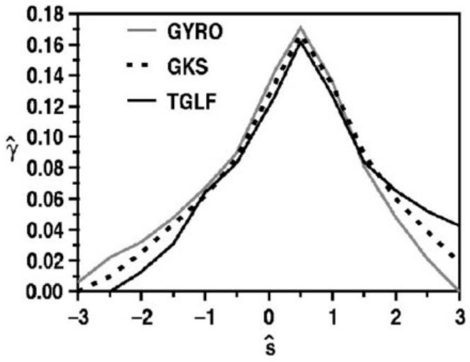
- System of 6 gyrofluid equations for 6 moments (N, $U_\parallel$, $P_\parallel$, P_T, $Q_\parallel$, Q_T)
- Particle number N separated in circulating and trapped fractions
- Full FLR integrals are calculated (no Padé approximation)
- Gyro-Landau closure with toroidal drifts implemented
- Fields are expanded in a series of Hermite polynomials
- Width of Gaussian envelope determined by fitting growth rates with GKS
- Weights for quasi-linear fluxes determined by fitting fluxes with GYRO

$$N = \frac{1}{n_0} \int_{\mathrm{tr}} d^3 v J_0 \tilde{F}$$

$$N^{\mathrm{kin}} = \frac{-\Phi}{n_0} \int_{\mathrm{tr}} d^3 v \frac{(k_\parallel v_\parallel + \omega_{dv} - \omega_*^T) J_0^2 F_0}{k_\parallel v_\parallel + \omega_{dv} - \omega}.$$

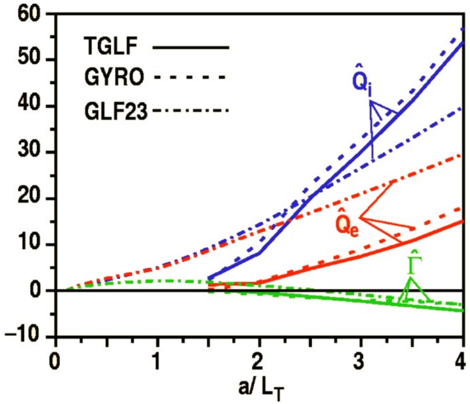
Staebler, Kinsey & Waltz, PoP (2005)

Benchmark with GK codes



Growth rate ITG versus magnetic shear s calculated with GK codes (GYRO, GKS) and with TGLF

Ion heat flux electron heat flux and particle fluxes; GLF23 always overestimates Q_e and deviates from Q_i



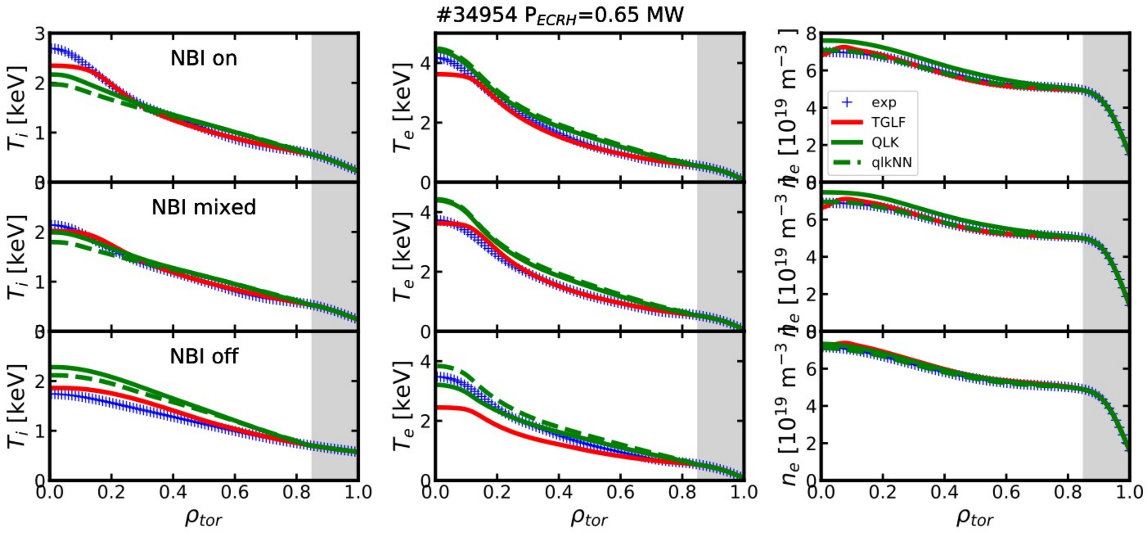
Staebler, Kinsey & Waltz, PoP (2005); Staebler et al., PoP (2007)

Implementation in ASTRA

5 MW on axis

2.5 MW on axis
2.5 MW off axis

5 MW off axis



Tardini et al., Nuclear Fusion (2021)

Implementation in ASTRA